

**Computer Aided Power System Analysis**  
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**Lecture - 24**  
**AC - DC Load Flow (Contd.)**

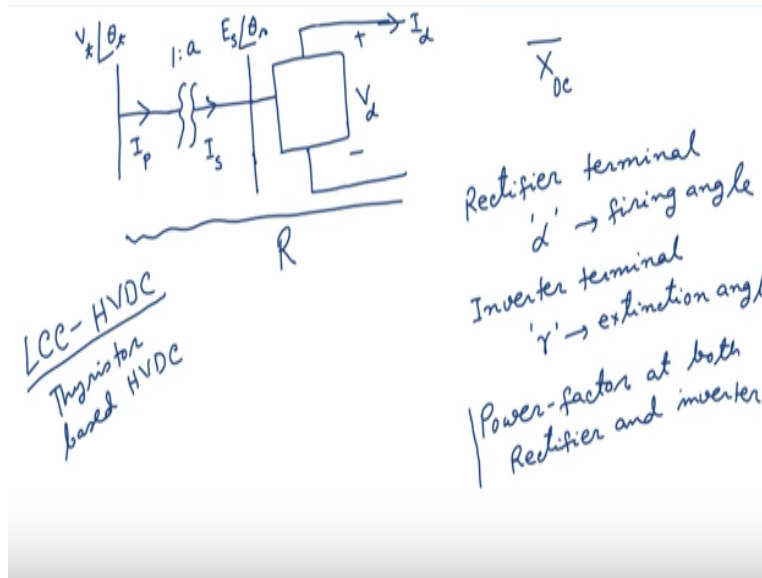
Hello, welcome to this lecture of this course computer aided power system analysis. In the last class we have discussed about the two methods of the AC – DC load flow. First one we discussed about is the simultaneous method of AC – DC load flow and the second one was the sequential method of AC – DC load flow and we have seen that in the case of sequential method of AC – DC load flow there is no change in the Jacobian matrix.

There is no change in the number of unknown variables. There is no change in the number of mismatch quantity. So then therefore there is very little amount of change which is to be done in the code of the power flow solution as compared to the case when we are trying to solve this AC – DC load flow by utilizing the simultaneous method.

In the case of simultaneous method we have seen that the Jacobian matrix involves 5 more Jacobian sub matrices. Also the number of unknowns increased and also the number of mismatch quantities increased. So then as a result the overall size of the Jacobian matrix increases. So as a result the computation burden increases quite significantly in the case of simultaneous AC – DC load flow.

On the other hand in the case of sequential AC – DC load flow because there is no change in the size of the Jacobian matrix, so then therefore the enhancement in the computation time as well as the computation complexity is quite minimal in the case of sequential load flow. So as a result you would be pursuing the sequential load flow. Let us look at the schematic diagram of

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So you have got the, this is the AC terminal bus. Then from this AC terminal bus there is one transformer. From this transformer, then this is the converter terminal bus. Then from this converter then we have got this is the converter and then this DC site. So we call it as V terminal angle theta terminal. This transformer has got a ratio 1: a. So this has got a tap changing transformer. This is I prime, sorry this is I primary, this is I secondary.

This is the converter terminal voltage  $E_s \theta_s$ . this is the DC current and this is the DC voltage. Please note that in the case of an HVDC link this arrangement is present at both the ends. That is if this is the rectifier side, for example if you are saying that this is the rectifier side identically the same management is also present in the inverter side.

So then therefore if we discuss in detail about one particular terminal so then automatically that discussion extends to the other terminal. So now what happens as we have said that in the case of AC – DC load flow we said that we have got some quantities which are unknown quantities and we said that this unknown quantities are given by this set R, by the vector X DC. So now we have to first understand that what are those unknown quantities.

And then subsequently we have to understand that what are the equations available to solve this unknown quantities. Now what happens that, now here in this course we are

talking about as we have already said that LCC - HVDC only. So line commutated converter based HVDC. This is nothing but this thyristor based HVDC. Now in the case of any terminal I have got different unknown quantities.

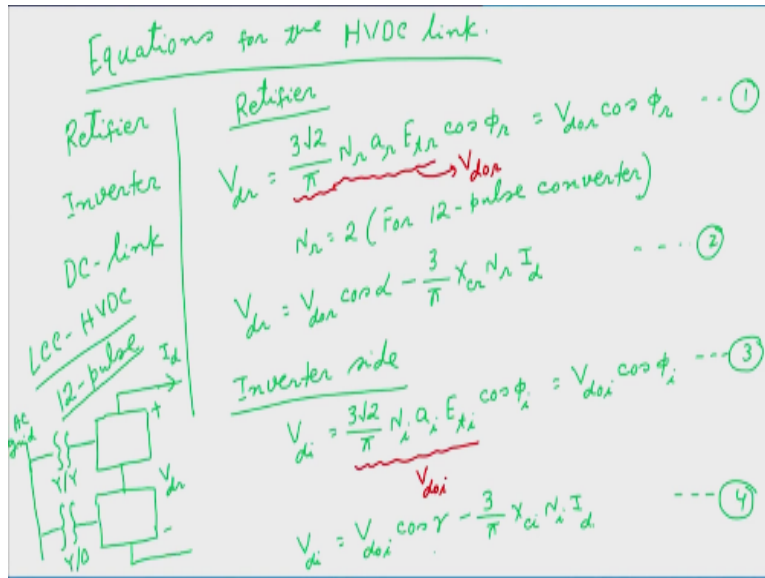
For example, now for any rectifier terminal there is something called quantity alpha which is called the firing angle. This should be apparent for anybody who has done the basic course on HVDC transmission. Similarly, for the inverter terminal there is a quantity called gamma which is called extinction angle. And we also defined something called power factor at both rectifier and inverter.

This is not defined. This quantity is not defined. So this also we take as unknown. Now here if we look at this, so then what are the things we can say that which are unknown. Here this  $V_t$   $\theta_t$  are nothing but the AC terminal voltage. So then therefore this  $V_t$  and  $\theta_t$  would be solved by the AC load flow code.

So then therefore as far as the DC system is concerned  $V_t$  and  $\theta_t$  need not be included as their unknown quantity because this  $V_t$  and  $\theta_t$  would be solved as part of the AC load flow code. And here we are left with  $I_p$ ,  $I_s$ ,  $a$ ,  $E_s$ ,  $\theta_s$ ,  $V_d$ ,  $I_d$ . Now here there are many quantities. Now the question is out of these quantities which one should we take as an unknown quantities and which one we should actually leave out.

Now to understand that first let us look at the equations for any HVDC terminal.

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So let us look at the equations of the, we have to understand that what should be our unknown quantities. But to understand that first let us look that what are the equations available to us and then by looking at these equations it would be relatively easy for us to identify what should be your unknown quantity. So equations for the HVDC link. Now we will write 3 parts of the HVDC links.

Now there are actually 3 parts. One is rectifier, one is inverter, and one is DC-link. So now the equations corresponding to rectifier are  $V_{dr}$ .  $V_{dr}$  is nothing but the DC voltage at the rectifier side for example here in this diagram if this is a rectifier so then this  $V_d$  is nothing but  $V_{dr}$ . So then  $V_{dr}$  it is given by  $3 \sqrt{2} / \pi N_r a_r E_{tr} \cos \phi_r$ . It is called  $V_{dor} \cos \phi_r$ .

Now what are these quantities?  $E_{tr}$  is nothing but the terminal voltage. For example in this diagram, here we have actually denoted this terminal voltage to be  $V_t$  but in this equation this  $E_{tr}$  is nothing but the terminal voltage at the rectifier side;  $a_r, a_r$  is nothing but the tap changing ratio at the rectifier side. For example here this transformer has got a tap changing ratio of  $1: a$ . So then corresponding to rectifier this is  $a_r$ .

$\phi_r$  is nothing but the power factor angle.  $N_r, N_r$  is nothing but the number of converters at the rectifier side. Now what happens that to understand this just I mean let

us simply recollect that for an LCC - HVDC, for LCC – HVDC usually it is 12 - pulse, 12 - pulse HVDC. Because it is a 12 – pulse HVDC so then what happens that at the DC side we always connect two 6 – pulse converters in series.

So two 6 – pulse converters in series, they are connected in series at the DC side. So this is the total  $V_{dr}$  and this is  $I_d$  and at this side one is, so at this side, so this is basically AC voltage. So this is AC voltage, AC grid. So this is AC grid. And then from this AC grid there are 2 converters which are fed by two separate transformers and these 2 converters are connected in series.

As we know very well I mean whoever has gone this very basic course on HVDC, one of the transformers will be connected in star - star another one will be connected in star – delta. This is just to element the (( )) (11:22) etc. But I mean these are actually beyond the scope of this particular course. So we are not going into this detail of this. Now here what happens that when we say  $N_r$ ,  $N_r$  is nothing but the number of 6 – pulse converters used in an HVDC terminal.

Now for a 12 – pulse converter we are using two 6 – pulse converters in series. So then therefore for 12 – pulse converter  $N_r = 2$ . So  $N_r = 2$  for 12 – pulse converter, right? Then the another equation is also  $V_{dr} = V_{dor} \cos \alpha$ . This alpha we have just now said that this alpha is nothing but the firing angle minus  $3/\pi \times X_{cr} \times N_r \times I_d$ .  $X_{cr}$  is nothing but the transformer leakage reactance at the rectifier side.

This is the individual transformer reactance. Please note that this is the transformer reactance of one transformer. For example here we are using 2 transformers. So then this  $X_{cr}$  denotes the leakage impedance of one transformer. It is assumed that this leakage impedance of both these transformers are identically same and etc. So when we do this calculation, we always assume this case that this, that basically these two transformers are identical in nature.

So then therefore their  $X_{cr}$  are all same. In case they are different so then in that case this analysis becomes too complicated. But usually whenever an HVDC link is put into practice it is always endeavored that these two transformers are identical in nature. So then therefore there is not much of an error to assume that this  $X_{cr}$  is same for actually both these transformers.

So  $X_{cr}$  denotes this, denotes the transformer leakage impedance of one transformer. Similarly, at the inverter side, same equations prevail. So  $V_{di}$  would be given by  $3 \sqrt{2} / \pi N_i$ ;  $N_i$  is the number of converters or rather number of 6 – pulse converters at the inverter side \*  $a_i$ ;  $a_i$  is nothing but the transformer tap ratio at the inverter side.  $E_{ti} \cos \phi_i$  that is given by  $V_{doi} \cos \phi_i$ .

Now here we must for the case of this quantity is actually  $V_{doi}$ .  $V_{doi}$  denoting is that this is  $V_{doi}$  and here we are denoting that this quantity is actually  $V_{dor}$ . So  $V_{dor} \cos \phi_i$ .  $\phi_i$  is nothing but the power factor angle at the inverter side. So another equation at the identically same type of equation  $V_{doi} \cos \gamma$ . So this, please note this difference in the case of rectifier this is  $\alpha$ .

But for the case of inverter this is  $\gamma$  minus  $3 / \pi X_{ci} N_i I_d$ . Please note that this we are not writing  $I_{dr}$  and  $I_{di}$  because you see for an HVDC link the current at the rectifier side and the inverter side they are same. So then therefore  $I_d$  is same. So then therefore we are not at all differentiating between the currents at the rectifier side and the inverter side. So then we have got 4 equations; 1, 2, 3, 4. Suppose I should write down all this quantities whichever we have not.

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$$\begin{aligned}
 N_r(N_i) &\rightarrow \text{number of } 6\text{-pulse converters connected in series at rectifier (inverter) side} \\
 a_r(a_i) &\rightarrow \text{transformer tap ratio at rectifier (inverter)} \\
 E_{tr}(E_{ti}) &\rightarrow \text{AC terminal voltage at rectifier (inverter)} \\
 X_{cr}(X_{ci}) &\rightarrow \text{Individual transformer leakage impedance at rectifier (inverter)} \\
 \phi_r(\phi_i) &\rightarrow \text{Power factor angle at rectifier (inverter)}
 \end{aligned}$$

HVDC link  

$$I_d = \frac{V_{dr} - V_{di}}{R_{dc}} \quad \text{---(5)} \quad R_{dc} \rightarrow \text{DC link resistance}$$

So we write that  $N_r$  or  $N_i$  is the number of 6 – pulse converters connected in series at rectifier or inverter side. Then  $a_r$  or  $a_i$  is transformer tap ratio at rectifier (inverter) and then we have  $E_{tr}$  or  $E_{ti}$ . Then we have  $E_{tr}$  or  $E_{ti}$  that is at AC terminal voltage at rectifier inverter and then we have  $X_{cr}$  or  $X_{ci}$  individual transformer leakage impedance at rectifier and inverter.  $\phi_r$  ( $\phi_i$ ) are the power factor angle at rectifier inverter.

So these are the different variables we have defined. And then what is left is for HVDC link there is only one  $I_d = V_{dr} - V_{di} / R_{dc}$  where  $R_{dc}$  is the DC link resistance. And  $V_{dr}$ ,  $V_{di}$  is the DC voltage at the rectifier side inverter side, these are understood. So then what we have? So we have got 5.

So for total in an HVDC link we have got 5 equations. Now let us look once you understand that these are the equations now let us try to find out that what should be my unknown quantities. For the purpose of convenience that how these are convenient we will show a little later in this discussion only.

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At each side, the unknowns are chosen as:

$$\begin{bmatrix} V_{dr} & I_d & a_r & \cos \alpha & \phi_r \end{bmatrix} \rightarrow \text{At rectifier}$$

$$\begin{bmatrix} V_{di} & I_d & a_i & \cos \gamma & \phi_i \end{bmatrix} \rightarrow \text{At inverter}$$

Total unknowns

$$\begin{bmatrix} V_{dr} & V_{di} & I_d & a_r & a_i & \cos \alpha & \cos \gamma & \phi_r & \phi_i \end{bmatrix} \rightarrow \text{total 9 unknown quantities.}$$

But for the purpose of convenience at each converter at each side the unknowns are chosen as, unknowns are chosen as follows. At the rectifier this is  $V_{dr}$ ,  $I_d$ ,  $a_r$ ,  $\cos \alpha$  and  $\phi_r$ . So this is the unknown quantities. These are at the rectifier. So  $V_{dr}$ ,  $I_d$ ,  $a_r$ ,  $\cos \alpha$ ,  $\phi_r$ . Similarly, at the inverter  $V_{di}$ ,  $I_d$  please note that  $I_d$  would be the same because the HVDC link the current at the rectifier and the inverter they are the same.

So  $a_i \cos \gamma \phi_i$ . So this is at inverter. So then therefore total unknowns are, so then total unknowns, please note that these are chosen just for the sake of convenience. Now how these are really convenient that we will be looking into at when we will be trying to solve this 5 equations to solve for this unknowns. So right now at this stage it is not apparent. But these are chosen just for the sake for convenience of calculation.

So the total unknowns combining these two are  $V_{dr}$ ,  $V_{di}$ ,  $I_d$  is common then  $a_r$  then  $a_i$  then  $\cos \alpha$  then  $\cos \gamma$  then  $\phi_r$   $\phi_i$ . So these are the total unknowns. So how many unknowns are there? Total 1, 2, 3, 4, 5, 6, 7, 8, 9. So total 9 unknowns. So total 9 unknown quantities. So now what we have got? We have got total 9 unknown quantities.

However, we have just now seen that for this HVDC link altogether we have got only 5 equations; 1, 2, 3, 4, and 5. So then therefore I have got 9 unknowns and 5 equations. Obviously, we cannot solve for 9 unknowns by utilizing 5 equations. By utilizing 5



equations we at best can only solve for 5 unknowns. So then therefore out of this 9 unknown quantities, 4 quantities need to be pre-specified and the rest 5 quantities actually need to be calculated.

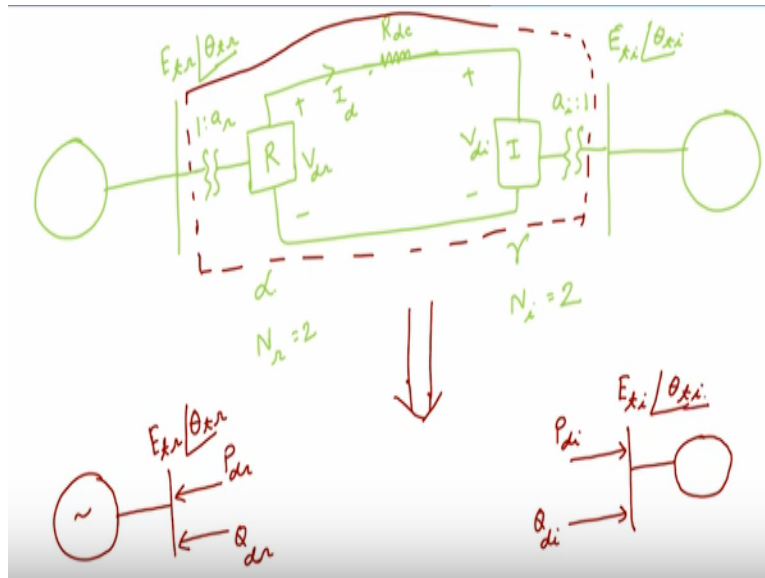
Now the question is that which 4 quantities need to be specified? There is, as such there is no hard and fast rule. Any 4 quantities can be pre-specified and once these 4 quantities are pre-specified then actually rest of the 5 quantities can easily be calculated by utilizing these 5 equations. Now we may think that all these equations are actually nonlinear equations because this involve cosine term. This also involve cosine term.

This also involve cosine term. This also involve cosine term. Only the last equation does not involve any cosine term. But out of this 5 equations 4 equations involve cosine term. So then therefore altogether this 5 equations seem to be basically nonlinear in nature. So you may think that after we pre-specify 4 quantities, then to solve this 5 remaining quantities we probably need to apply Newton – Raphson technique again to solve this 5 quantities.

But this is not so. We can solve for this 5 quantities just by applying simple algebraic manipulation and these algebraic manipulations are actually iteration free. That means we can simply calculate the quantities of our interest analytically without going through any iterative process.

Now before we pre-specify the known quantities and the unknown quantities, again just for the sake of our recollection let us just recollect for a while that what is the final objective of our calculation at the DC side in the case of sequential AC – DC load flow.

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So what we discussed that in the case of sequential AC – DC load flow what we do is that suppose we have, that we have one rectifier bus and one inverter bus. Please note that here there is a big grid and there is a big grid. So there is a big grid and there is a big grid and in this in between there is for example this transformer then rectifier and then here also transformer then inverter. So this is rectifier and this is inverter.

And in between this rectifier and inverter there is a DC line. So this is please note that there is some DC is also there. So this is the DC link resistance. So this is  $V_{dr}$ . This is  $V_{di}$  and current is  $I_d$ . And this has got  $1 : a_r$  and this has got  $a_i : 1$ . Please note that this tap ratio is always at the side of this converter and this is  $E_{tr}$ , angle  $\theta_{tr}$ . This is  $E_{ti}$ , angle  $\theta_{ti}$ . This rectifier has got an firing angle  $\alpha$ .

This inverter has got an extension angle  $\gamma$ . This rectifier has got number of 6 – pulse converter =  $N_r$ . Inverter has got number of 6 – pulse converter =  $N_i$ . Usually these are two because here we are only talking about 12 – pulse HVDC link. So  $N_r = N_i = 2$ . And this has got  $X_{cr}$ ,  $X_{ci}$  etc. So and this is the arrangement. Now what we are trying to do?

Basically what we are trying to do is that, that we do want to replace this entire circuit, we want to replace this entire circuit by some equivalent injection and please note that

some equivalent injection we call it  $P_{dr}$ , we call it  $Q_{dr}$  and here at this side also this is the and here also we  $P_{di}$   $Q_{di}$ . We are simply trying to replace everything from this to this, everything.

From this part to this part, everything from this part to this part **by 2** by 4 equivalent real and reactive power injections. So everything here, so this net effect of this HVDC link would be reflected by this  $P_{dr}$ ,  $Q_{dr}$ ,  $P_{di}$ ,  $Q_{di}$  at this two buses. This is again  $E_{tr}$ ,  $\theta_{tr}$  and this is again  $E_{ti}$ ,  $\theta_{ti}$ .

So my objective is to replace this entire HVDC link by solving this 5 equations to arrive the values of  $P_{dr}$ ,  $Q_{dr}$ ,  $P_{di}$  and  $Q_{di}$  such that after we get this equivalent power injections, after this we can simply treat them as far as this AC system is concerned this entire DC link is gone. We have got only 4 equivalent power injections and then after that standard AC load flow will continue.

Now, so then therefore our task is now to solve for this 5 unknown quantities. As we have just now said that although there are total 9 unknown quantities but because we have got 5 equations so then therefore we have to actually pre-specify 4 quantities and after that the left over 5 quantities need to be solved by utilizing this 5 equations and after we solve this 5 quantities then we will arrive at this  $P_{dr}$ ,  $Q_{dr}$ ,  $P_{di}$  and  $Q_{di}$ .

And then subsequently we will solve the AC load flow. So in the next lecture we will be looking at I mean some cases that how to arrive at these values of  $P_{dr}$ ,  $Q_{dr}$ ,  $P_{di}$ ,  $Q_{di}$  whenever we I mean when we do specify 4 quantities and then we do solve the rest of the 5 quantities utilizing these 5 equations. Thank you.