**Computer Aided Power System Analysis**
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**Lecture - 21**
**FDLF (Contd.)**

Hello, welcome to this lecture of this course computer aided power system analysis. So we have been discussing FDLF. So let us continue. So we have been discussing, we were talking about J 3.

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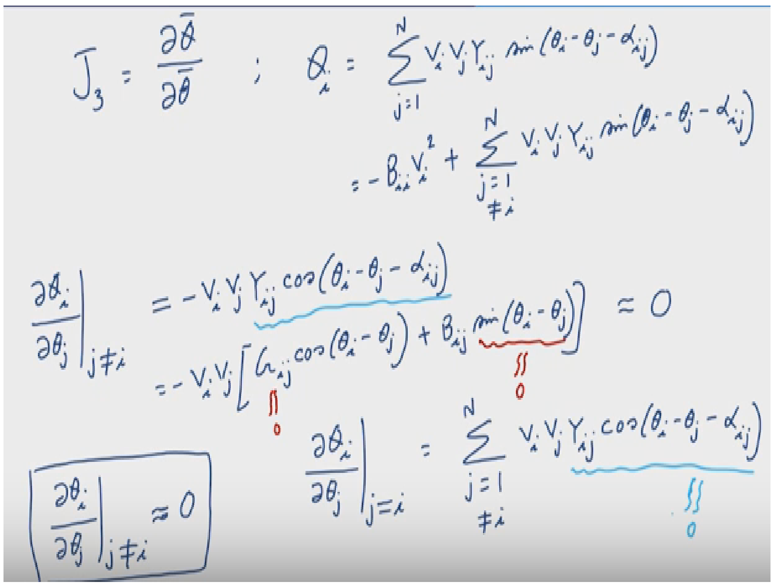

And J 3 is del Q/del theta. So let us look at the expression of del Q i. So before that let us look at the expression of Q i. Q i is V i V j Y ij sin (theta i – theta j – alpha ij) j = 1 to N and we as usual we take the ith term. So V i square – B ii V i square + j = 1 to N not equal to i V i V j Y ij sin (theta i – theta j) – alpha ij. Now we look at the expression del Q i/del theta j where j is not equal to i.

So that should be, this part would be gone and there would be only one term. So it would be –V i V j Y ij cosine (theta i – theta j – alpha ij). So it would be –V i V j and we have already seen that this part can be written as G ij cos (theta i – theta j) + B ij sin(theta i – theta j) and we have already seen in the last lecture that this part is 0 and this part is 0. So we have seen that this is = 0 and this is almost equal to 0.

So then therefore this is also almost = 0. So then therefore we have this result that delta Q i/delta theta j not equal to i = 0, approximately = 0. You should not say it is exactly equal to 0 but under these assumptions these are equal to 0. So this is 0. Now what we have? We have del Q i del theta j where j is = i. So then therefore all these terms will come into picture except this because this does not have any theta i. So this would go.

And it would be j = 1 not equal to i V i V j Y ij cos (theta i – theta j – alpha ij). Just now we have done this exercise and we have seen that this part after expansion becomes 0. So this part after expansion becomes 0.

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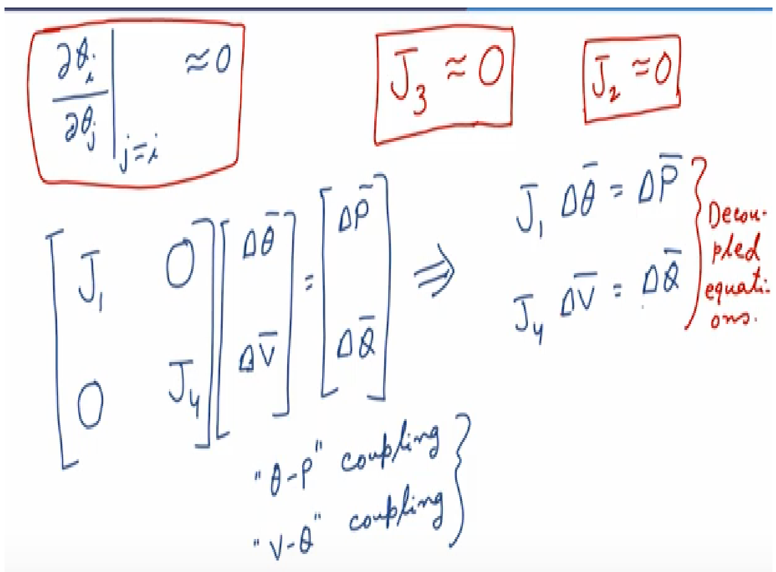


So then therefore we have got this another interesting result that del Q i/del theta j subject to j = i. That is when j = i this is also = 0 or in other words because both this term as well as this term are 0 so then therefore we write this matrix is also a null matrix. And we have already seen in the last lecture that J 2 is also null matrix. So J 2 is also a null matrix that we have already established in the last lecture.

So then therefore what we have is now, we have J 1 then it is 0 null matrix. Then it is another 0 null matrix J 4. J 4 we have not as yet looked at. But this is we are just delta theta, delta V = delta P delta Q. So then therefore if I simplify this or rather write down the equations individually what we get? J 1 delta theta = delta P and J 4 delta V = delta Q.

So now what happens? So from this 2 equations we can see that variation of theta is only dependent on variation of P and variation of V is only dependent on Q. Earlier when this matrix was a full matrix that is when J 2 and J 3 were not null matrix, so then in that case variation of theta is actually dependent on both P and Q. Similarly, the variation of V is also dependent on both P and Q.

But here in this case after this application of this simplifying assumptions we find that this variation of theta is only dependent on P. it is not at all dependent on Q and variation of V is only dependent on Q. It is not at all dependent on P. Or in other words, this theta and P and V and Q they are decoupled from each other. So theta is only dependent on P and V is only dependent on Q. So then therefore these equations are actually decoupled equations. So that is why we call them as decoupled equations.

And in normal power system terminology we call this that we call that it is theta and P coupling as well as V- Q coupling. That is theta is only dependent on P and V is only dependent on Q. So we call it theta P coupling and V- Q coupling. That is exactly where it is. Variation of theta is only dependent on variation P and variation of V is only dependent on variation on Q. So that is why it is called decoupled equation.

So our topic is fast-decoupled load flow. So we are trying to solve the load flow equation. So we have seen that after the application of this simplifying assumptions this load flow equations get decoupled from each other. So that is why these are called decoupled load flow. Now why it is fast? So for that we have to now look into in little more detail into the expressions of J 1 and J 4 and apply this simplifying assumptions again. So let us do that.

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$$J_1 = \frac{\partial \bar{P}}{\partial \theta} : \left.\frac{\partial P_i}{\partial \theta_j}\right|_{j \neq i} \quad \text{and} \quad \left.\frac{\partial P_i}{\partial \theta_j}\right|_{j = i} \qquad i, j = 2, \ldots N$$

$$P_i = V_i^2 G_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$\left.\frac{\partial P_i}{\partial \theta_j}\right|_{j \neq i} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= V_i V_j Y_{ij} \left[ \sin(\theta_i - \theta_j) \cos \alpha_{ij} - \cos(\theta_i - \theta_j) \sin \alpha_{ij} \right]$$

$$= V_i V_j \underbrace{G_{ij}}_{\approx 0} \underbrace{\sin(\theta_i - \theta_j)}_{\approx 0} - V_i V_j B_{ij} \cos(\theta_i - \theta_j)$$

$$\approx - V_i V_j B_{ij}$$

So we look at J 1. J 1 is del P/del theta, we know. So it has got del P i/del theta j. One is j is not equal to i and del P i/del theta j; j = i. So these two cases are only possible. This is a square matrix we already know and we also know that i and j both vary from 2 to N. So these are all known, we are just recollecting. So P i, we again write down that it is V i square G ii + j = 1 to N not equal to i V i V j Y ij cos (theta i – theta j – alpha ij).

So now we first do the simplified one. So we first do del P i/del theta j, j not equal to i. So what we will have? Of course this partial derivative of this term would be 0 and because j is not equal to i so then therefore there will be only one theta j term and it would be V i V j Y ij cosine – sin (theta i – theta j – alpha ij). We are doing the partial derivative with respect to theta j. So cosine have got a partial derivative minus.

And we have got another minus. So minus, minus plus. So now we have to expand this. V i V j Y ij. This is sin a – b. So sin a cos b – cos a sin b. So it is V i V j Y ij G ij sin (theta i – theta j) and it is becoming V i V j. It is B ij cos (theta i – theta j). Now here we again put this simplifying assumptions. So what are those assumptions? Assumptions are again the same assumptions. So we have already seen that this is almost = 0.

This is also almost = 0. This is not 0 and this is almost = 1. So then therefore because this is very small, this is very small so the entire term gets vanished. So then therefore what we are left with is it is now almost = V i V j B ij. So what assumptions we have taken here? We have taken the assumptions that theta i – theta j is almost = 0. So then therefore cosine theta i – theta j would be almost equal to 1. So then therefore we make it unity.

First of all this is neglected. So this is gone. So what is remaining is – V i V j B ij cosine (theta i – theta j) and because theta i – theta j is almost = 0 so then therefore cosine theta i – theta j almost = 1. So then ultimately we have got del P i/del so this is = - V i V j B ij, alright? So I have got this. So we have an expression.

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$$\frac{\partial P_i}{\partial \theta_j} \approx -V_i V_j B_{ij} \quad : j \neq i$$

$$\Rightarrow \frac{1}{V_i}\frac{\partial P_i}{\partial \theta_j} \approx -V_j B_{ij} \quad : j \neq i$$

$$\Rightarrow \boxed{\frac{1}{V_i}\frac{\partial P_i}{\partial \theta_j} \approx -B_{ij} \quad : j \neq i}$$

$$\frac{\partial P_i}{\partial \theta_i} = -\sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= V_i V_i Y_{ii} \sin(\theta_i - \theta_i - \alpha_{ii}) - V_i V_i Y_{ii} \sin(\theta_i - \theta_i - \alpha_{ii}) - \sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

So we write this expression. Delta P i/delta theta j = - V i V j B ij. Condition is that j is not equal to i. So from here I can write down that 1/V i delta P i/delta theta j = - V j B ij for the condition j is not equal to i. Now here we invoke the assumption that all voltage magnitudes = 1. But we only invoke the assumption only at the right hand side but not at the left hand side. So if we invoke this assumption at this right hand side.

So we get for the condition j is not equal to i. Now the question becomes that why are we only invoking the assumption at the right hand side but not at the left hand side. Because at the left hand side also we have got one voltage term V i. The answer is this. That when we invoke this assumption at the right hand side, this right hand side terms becomes constant. Because you see this quantity B ij is entirely dependent on the line parameters which is constant.

So then therefore once we calculate this parameter B ij, it remains constant. We really do not have to calculate it at each and every iteration. So to make the right hand side constant we invoke this assumption at the right hand side. But if we apply this assumption at the left hand side also, it will actually increase the error in our calculation. Because see after all this all this bus voltage magnitudes are equal to 1 this is just an assumption.

So then when we are invoking this assumption at the right hand side we are already incurring some amount of computational error just to make it constant and if we again invoke the same assumption here so then therefore it will actually increase the computational error as compared to this ideal solution. So then therefore we do not invoke this assumption at the left hand side just to reduce this computational error.

But we only invoke this assumption at the right hand side such that this right hand side is constant. Now because this is constant it has got a very good implication which we will see a little later. So we, this. Now we look at the expression del P i/del theta i. We get from the expression of minus, it is – j = 1 to N not equal to i V i V j Y ij sin (theta i – theta j – alpha ij). So what we do here is, we do an intelligent thing.

V i V i Y ii, so V i square Y ii sin (theta i – theta i – alpha ii). Because we have added this term so we must also subtract this term to maintain the sanctity of the expression. You may be wondering what is happening but it will be very clear in a moment. Alpha ii and then this entire expression j = 1 and not equal to i N V i V j Y ij sin (theta i – theta j – alpha ij). Now if we look at this two terms together.

Here in this entire term j is not equal to i and in this term j is equal to i. So you see this j = 1, this j = i, this j = i, this j = i. So then therefore if I take them together so what is get and this term is V i square * Y ii * sin – alpha ii and sin – alpha ii is – sin alpha ii. So then it is – V i square B ii.

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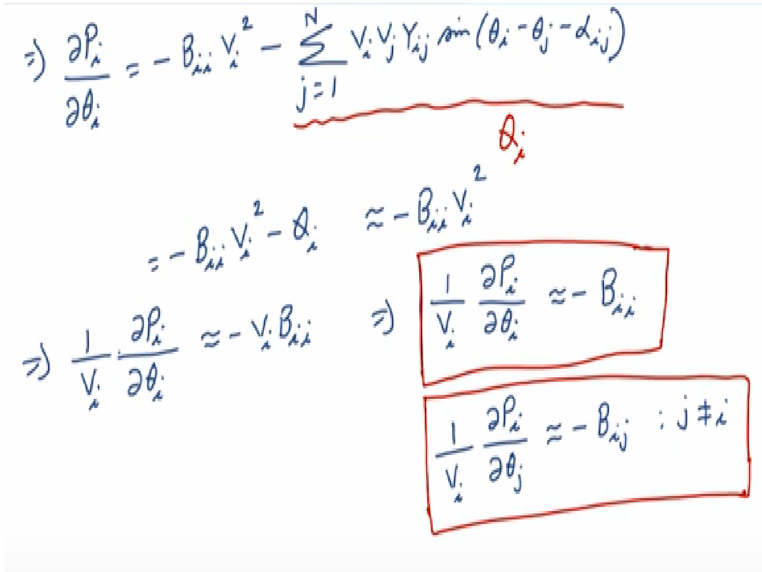


So then this term becomes, so del P i del theta i it becomes – B ii V i square then – j = 1 to N V i V j Y ij sin (theta i – theta j – alpha ij). So this is it. Now this term we get by combining this and this together. Now we know that this is nothing but Q i. This we already know. So then therefore we write it is – B ii V i square – Q i and please recollect out last assumption that in magnitude Q i much less than B ii * V i square.

So then therefore it becomes – B ii V i square. So then therefore we have 1/V i * del P i/del theta i = - V i B ii. Invoking this assumption that V i all voltage magnitudes are equal to 1.0 only on the right hand side as we have done just a little while ago we get 1/V i del P i/del theta i = - B ii. So this is the interesting result. So now what happens? So this is J 1. So this is the expression of J 1.

So this is the first and again we write again the second one just for the clarity. And for the second one is 1/V i del P i/del theta j = - B ij. This is for j is not equal to i. So this is the second condition which we have just now developed. So now what happens? So we have got some constant quantity. Now let us look at the last one J 4.

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$$\bar{J}_4 \Rightarrow \frac{\partial \bar{Q}}{\partial \bar{V}} \; ; \; \frac{\partial Q_i}{\partial V_i} \; ; \; \left.\frac{\partial Q_i}{\partial V_j}\right|_{j \neq i} \qquad i, j = M+1, \cdots N.$$

$$Q_i = -B_{ii} V_i^2 + \sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$\left.\frac{\partial Q_i}{\partial V_j}\right|_{j \neq i} = V_i Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= V_i \left[ G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right]$$

$$\approx -V_i B_{ij} \quad \Rightarrow \boxed{\frac{\partial Q_i}{\partial V_j} \approx -B_{ij} \; ; \; j \neq i}$$

J 4 is del Q i del V and it has got two terms del Q i/del V i and del Q i/del V j where j is not equal to i. Now you know that i and j all both varies from M + 1 to N. So Q i is B ii V i square it is minus + V i V j Y ij sin (theta i – theta j – alpha ij). So del Q i/del V j where J is not equal to i. It will become, this part would be 0. So it would be V i Y ij sin (theta i – theta j – alpha ij). We have already seen that this part is nothing but G ij * sin etc. etc.

It is G ij * sin (theta i – theta j) – theta j. Invoking this assumption we get, so then therefore if I again invoke this assumption that V i = 1.0 what I get is del Q i/del V j = - B ij j is not equal to i. So this is the assumption. So this is the equation. Again this side is constant.

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$$\frac{\partial Q_i}{\partial V_i} = -2V_i B_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$\Rightarrow V_i \frac{\partial Q_i}{\partial V_i} = -2V_i^2 B_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= -V_i^2 B_{ii} - V_i^2 B_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= -V_i^2 B_{ii} + \underbrace{\sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})}_{Q_i}$$

$$= -V_i^2 B_{ii} + Q_i$$

And the last one, it is 2V i B ii + j = 1 not equal to i to N V j Y ij sin. We multiply everywhere by this. We are multiplying everything, alpha ij; - V i square B ii and – V i square B ii then plus this minus alpha ij. So then therefore if I do add them together I get this. So I get – V i square B ii + j = 1 to N V i V j Y ij sin (theta i – theta j – alpha ij). This quantity is nothing but Q. So then therefore it is – V i square B ii + Q i.

This part is Q i and invoking this assumption that Q i is has magnitude much less than this so we get this.

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$$\Rightarrow V_i \frac{\partial Q_i}{\partial V_i} \approx -V_i^2 B_{ii}$$

$$\Rightarrow \frac{\partial Q_i}{\partial V_i} \approx -V_i B_{ii}$$

$$\Rightarrow \left.\begin{aligned} &\boxed{\frac{\partial Q_i}{\partial V_i} \approx -B_{ii}} \\ &\boxed{\frac{\partial Q_i}{\partial V_j} \approx -B_{ij}} \end{aligned}\right\} J_4$$

$$J_1 \left\{\begin{aligned} &\boxed{\frac{1}{V_i}\frac{\partial P_i}{\partial \theta_j} = -B_{ij} \quad j \neq i} \\ &\boxed{\frac{1}{V_i}\frac{\partial P_i}{\partial \theta_i} \approx -B_{ii}} \end{aligned}\right.$$

$$J_2 = 0$$

$$J_3 = 0$$

V i del Q i/del V i = - V i square B ii or in other words del Q i/del V i = - V i B ii or in other words del Q i/del V i again invoking this assumption that V i is almost equal to 1.0. So these are the expressions of J 4. So we have got these expressions of J 1, J 2, J 3. J 2 and J 3 are already made 0. For J 1 we have got these two expressions. And for J 4 we have got, so we have got one expression is this and another expression is.

So this is for J 4 and we again for J 1 we write that 1/V i del P i/del theta j = - B ij j not equal to i. That is this one, we are just recollecting them. 1/V i del P i/del –B ij, j is not equal to i and this is B ii. And we have got 1/V i del P i/del theta i = - B ii. So I have got this expression and I have got this expression. So this is for J 1. And we have already seen that J 2 is 0, that we have already seen and J 3 is 0.

So we have derived this expressions of J 1 and J 4 and we have already seen that J 2 is a null matrix, J 3 is a null matrix and J 1 and J 4 all these terms without the application of this assumptions become constant. So now we will see that how this expressions are applied for our, for the solution of our load flow equations and how the application of this really make the load flow solution process fast. So this we will look into the next lecture. Thank you.