

Computer Aided Power System Analysis
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Lecture - 20
Fast Decoupled Load Flow (FDLF)

Welcome to another lecture of this course of computer aided power system analysis. Till the last lecture we have discussed about the Newton – Raphson load flow both in polar coordinate as well as in the rectangular coordinate. We have seen that both in polar coordinate as well as in the rectangular coordinate all the equations are coupled with each other. Not what do we mean by this statement. So let us look at that.

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NRLF (POLAR)

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{V} \end{bmatrix} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix}$$

$$\Rightarrow \begin{aligned} J_1 \Delta \bar{\theta} + J_2 \Delta \bar{V} &= \Delta \bar{P} \\ J_3 \Delta \bar{\theta} + J_4 \Delta \bar{V} &= \Delta \bar{Q} \end{aligned}$$

$$g_{ij} = \frac{r}{r^2 + x^2}; \quad b_{ij} = -\frac{x}{r^2 + x^2}$$

Simplifying assumptions:

- i) All voltage magnitudes are nearly equal to 1.0 p.u. i.e. $V_i \approx 1.0$ p.u.
- ii) For all transmission line $g_{ij} \ll b_{ij}$

$$\bar{y} = \frac{1}{z} = g_{ij} + jb_{ij}$$

$$\bar{z} = r + jx \quad r \ll x$$

$$\bar{y} = \frac{1}{r + jx} = \frac{r - jx}{r^2 + x^2} = \frac{r}{r^2 + x^2} - j \frac{x}{r^2 + x^2}$$

So in the polar coordinate for example NRLF polar, the equations we have is J 1, J 2, J 3, J 4 and then it is delta theta, delta V. Please note that this J 1, J 2, J 4 all are matrices of appropriate dimensions. Then we have delta P, delta Q vector. So then if we do expand this what equations I get? I get J 1 delta theta + J 2 delta V = delta P and we get J 3 delta theta + J 4 delta V = delta Q.

So if we solve them what we get? We would be able to solve for theta and V but then what I can see is that both theta and V they are actually dependent on both P and Q which is obviously the case. But then for all practical purpose we can show which we will

actually show that with some simplifying assumptions it can be shown that the variation of theta is largely dependent on the variation of P that is the real power.

While the variation of voltage magnitude that is this vector ΔV is largely dependent on the variation of Q that is the reactive power. In other words, the variation of theta is not dependent on Q while the variation of V is not dependent on P. So it will actually help us to expedite the solution. Now, to do that we have some simplifying assumptions. So we have some simplifying assumptions.

So let us look at those simplifying assumptions, under which assumptions this is true. So first look at this simplifying assumptions. So first assumption is all voltage magnitudes are nearly equal to 1.0 per unit. That is we should say that all V_i is roughly 1.0 per unit which is almost true because we have already seen in our example that for this 5 bus system for example we have seen that all this voltage magnitudes are more or less equal to 1.0 per unit.

That is they are very close to 1.0 per unit. We may ask the questions that is it a kind of general situation. To some extent it is because in any power system you would like to maintain the voltage magnitudes at all the buses equal to their nominal voltage, nominal voltage means nothing but this rated voltage which in turn in per unit is nothing but equal to 1.0 per unit.

So then therefore because we wish to maintain the voltage profile of any power system network at the rated voltage so then therefore we can say that all the voltage magnitudes are more or less equal to 1.0 per unit. Second simplifying assumptions is for all transmission line g_{ij} is $\ll b_{ij}$. What is g_{ij} and b_{ij} ? g_{ij} and b_{ij} are nothing but the conductance and the susceptance.

That is if we take the inverse of the impedance that is y so that is if we compute y that is the line admittance that is $1/z$ so that is $= g_{ij} + j b_{ij}$. Now what is the justification of this? Now we have that $J = r + jx$ where r is the resistance and x is the

inductance and in a transmission system usually the realistic part is much less than reactive part. So then therefore we know that r is much less than x .

Usually r is much less than x . Now y is $1/r + jx$. So you have got $r - jx/r$ square + x square. So it is r/r square + x square - jx/r square + x square. So then therefore we can write down that $g_{ij} = r/r$ square + x square or rather this magnitude of g_{ij} and b_{ij} is $-x/r$ square + x square. As r is very less than x so then therefore g_{ij} would be very less than b_{ij} because this denominator is same for both g_{ij} and b_{ij} .

And as r is very less than x so then therefore g_{ij} would also be much less than b_{ij} . So this is the second assumption.

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iii) For any Transmission line between bus 'i' and 'j',
 $(\theta_i - \theta_j) \approx 5-10^\circ$ (approximately equal to zero).

iv) In per unit, $Q_i \ll B_{ii} V_i^2$

$B_{ii} = \sum_{j \in \Omega} b_{ij}$
 $\Omega \rightarrow$ is the set of lines directly connected to bus i

$i = 3$
 $\Omega = \{4, 7, 13\}$
 $j \in \Omega$

50 MVAR
 $S = 100$ MVA.
 In p.u. $|Q_i| = 0.5$

Third assumption is for any transmission line between bus i and j the angular difference between them $\theta_i - \theta_j$ is quite small. So we say that it is roughly 5 to 10 degree. We say that in all parlance we say that is approximately equal to 0. Now what is the justification for this? We know from our equal area criteria that for maintaining stability the angular difference between two terminals of a line must be as small as possible.

Because if the angular difference of the voltages between the two terminals of a line is increased so then therefore the possibility of instability for any fault in that line increases

severely. So then therefore for maintaining the stability of the system we need to maintain the angular difference between the two terminal voltages as low as possible.

So then therefore according to that so we take that assumption and we say that for any transmission line between bus i and j the angular difference of the voltages between two terminals are approximately equal to θ . It is not equal to 0, it is approximately equal to θ and the fourth one is in per unit Q_i that is the injected reactive power at bus i is much less than $B_{ii} V_i^2$. Now what is the justification for this?

Please note that it is in per unit. So Q_i is much less than as compared to $B_{ii} V_i^2$. Now what is the justification for this? The justification is that in per unit for example at any bus let us say the reactive power load is 50 MVR at a bus the reactive power load is 50 MVR. Let us say our base MVA S is 100 MVA. So then therefore in per unit Q_i that is magnitude = 0.5. So this is basically the numerical value, 0.5 in per unit.

Now we know that B_{ii} is nothing but the sum total of all b_{ij} where j belongs to the set ω where ω is the set of lines directly connected to bus i , right? So then therefore j is actually basically or rather we should say that ω is the set of the indices of the lines directly connected to bus i . For example, just to give an example. Suppose I do have bus, let us say bus 3 in some system.

And let us say this bus 3 is directly connected to bus 4 and let us say this is bus 7 and here it is connected to let us say bus 13. So then therefore in this case i is 3 and set ω would be 4, 7, and 13 and J is ω and J belongs to ω . So then therefore here in this case this J would be 4, 7, and 13. And then therefore this B_{ii} which is nothing but the imaginary part of the diagonal element corresponding to bus i .

That would be nothing but we know from the basic formulation of the Y- Bus matrix that this any diagonal element is nothing but the sum total of all the admittances of the lines directly connected to bus i . So then therefore the imaginary part of the admittance

corresponding to the diagonal element would also be nothing but the sum total of the imaginary part of the admittance of all the lines directly connected to bus i .

So then therefore now from this, now if we look at this expression b_{ij} , if I just look at this expression b_{ij} , r square is quite less in per unit. Please note that all this calculations are being done in per unit. So then therefore r is a value which is small in per unit, let us say 0.03 or let us say 0.0023, something like that; x is also a value which is also pretty less. So then therefore r square would be even reduced; x square would be even reduced.

So then therefore r square + x square would be an very small quantity and x although it is a small quantity but because this is square so then therefore this divided by this would be actually a very large quantity. Now because this b_{ij} in numerical value would be a large quantity so then therefore in numerical value numerically B_{ii} would be also a very large quantity.

And even though if we do assume that this V_i is almost equal to 1.0 per unit so even then, even if we take that this $V_i = 1$ so even then because B_{ii} numerically is very large value so then therefore numerically $B_{ii} V_i^2$ would be pretty large as compared to Q_i in per unit because of this numerical value. So with this 4 assumptions now if we apply this 4 assumptions to our equations of Newton – Raphson polar coordinate some very interesting results can be obtained.

So let us look at that. So we will apply this 4 simplifying assumptions one by one and we will see that what conclusion is drawn.

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NRLF (Polar)

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{V} \end{bmatrix} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix} \quad \begin{matrix} J_1 = \frac{\partial \bar{P}}{\partial \bar{\theta}} & ; & J_2 = \frac{\partial \bar{P}}{\partial \bar{V}} \\ J_3 = \frac{\partial \bar{Q}}{\partial \bar{\theta}} & ; & J_4 = \frac{\partial \bar{Q}}{\partial \bar{V}} \end{matrix}$$

$$J_2 = \frac{\partial \bar{P}}{\partial \bar{V}} ; \left. \frac{\partial P_i}{\partial V_i} ; \frac{\partial P_i}{\partial V_j} \right|_{j \neq i}$$

$$P_i = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$= V_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

So again we write. This is again we discuss about NRLF polar. And we again just it is recollection. So delta that delta V delta P delta Q. Now what is J 1? J 1 we know that it is del P/del theta. We have already defined what is meant by del P/del theta. J 2 is del P/del V. J 3 is del Q/del theta and J 4 is del Q/del V. So first let us look at this two off diagonal sub matrices. If we think of this particular big matrix J constituting of 4 components J 1, J 2, J 3, J4. So first let us look at J 2 and J3.

So J 2 is del P/del V. So we have got, as we have seen so I have got two general expressions. One is del P i/del V i and another is del P i/del V j; j not equal to i. So these are the two expressions we have. We have P i is equal to we know that V i V j Y ij cosine (theta i – theta j – alpha ij) j = 1 to N. N is the number of bus. So it is we know that it is V i square G ii. We have already done it, we are just merely recapitulating it.

J = 1 not equal to i goes up to N V i V j Y ij cosine (theta i – theta j – alpha ij). So this is the expression. So now again let us do this two and then we apply these 4 assumptions one by one judiciously. So let us start.

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$$\frac{\partial P_i}{\partial V_j} \Big|_{j \neq i} = V_i Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$= V_i Y_{ij} [\cos(\theta_i - \theta_j) \cos \alpha_{ij} + \sin(\theta_i - \theta_j) \sin \alpha_{ij}]$$

$$= V_i \left[\underbrace{G_{ij}}_{\approx 0} \cos(\theta_i - \theta_j) + \underbrace{B_{ij}}_{\approx 0} \sin(\theta_i - \theta_j) \right]$$

$G_{ij} = \sum_{j \in \Omega} g_{ij}$

$g_{ij} \ll b_{ij}$
 $(\theta_i - \theta_j) = 0^\circ$

$$\Rightarrow \frac{\partial P_i}{\partial V_j} \Big|_{j \neq i} \approx 0$$

$$\frac{\partial P_i}{\partial V_i} = 2V_i G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$\frac{\partial P_i}{\partial V_i} = 2V_i G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_j \left[\underbrace{G_{ij}}_{\approx 0} \cos(\theta_i - \theta_j) + \underbrace{B_{ij}}_{\approx 0} \sin(\theta_i - \theta_j) \right]$$

So first we look at the simpler one. So $\partial P_i / \partial V_j$ with the condition that j is not equal to i . So what it would be? So it would be basically we have already seen. So this will go and there will be only one term of V_j . So it would be $V_i Y_{ij}$. So it would be $V_i Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$. So now $V_i Y_{ij} \cos a \cos b \cos(\theta_i - \theta_j) \cos P$; $\cos a - b$. So $\cos a \cos b + \sin a \sin b \sin(\theta_i - \theta_j) \sin \alpha_{ij}$.

Now you know that $Y_{ij} \cos \alpha_{ij}$ is nothing but g_{ij} . So we write $V_i g_{ij} \cos(\theta_i - \theta_j)$ and we know that $Y_{ij} \sin \alpha_{ij}$ is B_{ij} . So we write $B_{ij} \sin(\theta_i - \theta_j)$. So this is the expression. Now we apply this simplifying assumptions. The simplifying assumption is we apply two assumptions. One is that g_{ij} is much less than b_{ij} and another assumption is $\theta_i - \theta_j$ is approximately $= 0$.

These two assumptions we apply. If we do apply this assumptions, so what do we get? $\sin \theta_i - \theta_j$ because $\theta_i - \theta_j$ is approximately $= 0$ so then therefore $\sin \theta_i - \theta_j$ is also equal to approximately $= 0$. So this is 0 this part. And because g_{ij} is much less than b_{ij} we can say that this part g_{ij} is also approximately $= 0$.

But remember because $\theta_i - \theta_j$ is approximately $= 0$ so $\cos \theta_i - \cos \theta_j$ would be approximately $= 1$ but because g_{ij} is very small so we can neglect it as compared to b_{ij} . So then therefore for all practical purpose we take g_{ij} to be $= 0$. So this

part becomes approximately = 0 and because of this assumption that $\theta_i - \theta_j$ is approximately = 0 so then therefore $\sin \theta_i - \theta_j$ becomes = 0.

So then therefore this term becomes = 0, this term becomes = 0. So then approximately $\frac{\partial P_i}{\partial V_j}$ where j is not equal to i becomes = 0. So this is a very interesting result. So we get this result. So this is the first result. Now look at this one. So it would be $2V_i G_{ii} + \sum_{j=1, j \neq i}^N V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$ because we are taking the derivative with respect to V_i and from this expression V_i would be everywhere.

So then this summation term will come. We have already looked into this. So now if we expand this part as we have done here, so we can write straightaway without any problem. This is $2V_i G_{ii} + \sum_{j=1, j \neq i}^N V_j$ then $G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)$. So this part is identical of this. So we have just taken this result as it is. So as usual as before what we get? As before we get that this is almost = 0.

This part is almost = 0 and G_{ii} because all G_{ij} is much less than B_{ij} so then therefore G_{ii} which is nothing but the sum total of all G_{ij} , we again repeat. We write that we again note that G_{ij} we write it here, just G_{ij} is sum total of g_{ij} where j belongs to the set ω . So then therefore because all g_{ij} 's are very negligible so then therefore their sum total would also be very small.

So then therefore G_{ii} would also be very small. So then for all practical purpose G_{ii} would also be 0. So then therefore $\frac{\partial P_i}{\partial V_i}$ would also be 0.

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$$\begin{array}{l}
 \boxed{\frac{\partial p_i}{\partial v_i} = 0} \quad J_2 : \frac{\partial p_i}{\partial v_i} = 0; \frac{\partial p_i}{\partial v_j} = 0 \\
 J_3 = \frac{\partial \bar{Q}}{\partial \theta} ; \left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j=i} ; \left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j \neq i} \quad \begin{array}{l} i = M+1, \dots, N \\ j = 2, \dots, N \end{array} \\
 \boxed{J_2 = 0}
 \end{array}$$

So then therefore what we get is we get $\frac{\partial p_i}{\partial v_i}$ is also = 0. So then for this matrix J_2 we have terms $\frac{\partial p_i}{\partial v_i}$ which is 0 and $\frac{\partial p_i}{\partial v_j}$ that is equal to 0. So then therefore we can straightaway say that J_2 matrix is a null matrix. So this is a very interesting result. So matrix J_2 becomes a null matrix. It becomes simply a null matrix. Now let us look at this matrix J_3 . So we have matrix J_3 that is basically $\frac{\partial Q}{\partial \theta}$.

So we know that it has got two $\frac{\partial Q_i}{\partial \theta_j}$. So one is $j = i$ and another is $\frac{\partial Q_i}{\partial \theta_j}$, j is not equal to i . We all know that how Q_i varies and how j varies. For example i would be varying from $M + 1$ to N and j would be varying from you know 2 to N and etc. etc. all this are known. So this is it. So then we have these two terms. So then therefore when i varies from $N + 1$ to N and j varies from 2 to N .

So then therefore there will be some occasion where j would be = i and also there would be some occasion and in basically most of the occasions j would be not = i . So then we again need to look at this 2 expressions as we have looked into J_2 and then deduce what happens to this matrix J_3 . So this we will look into this next lecture and also along with the other lectures. Thank you.