Computer Aided Power System Analysis
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Lecture - 19
Example of NRLF (Rectangular) Method

Welcome to another lecture of this module computer aided power system analysis. In the last lecture we have looked into the basic algorithm of the Newton – Raphson rectangular method. Now in this lecture we would be looking into the complete algorithm of this Newton – Raphson rectangular method in which we would be also taking into consideration the generator reactive power limits.

Please remember that in the last lecture where we have talked about the basic Newton – Raphson rectangular method we did not consider the generator reactive power limits but in this lecture we would be considering the generator reactive power limits so as to make this algorithm complete. So let us start.

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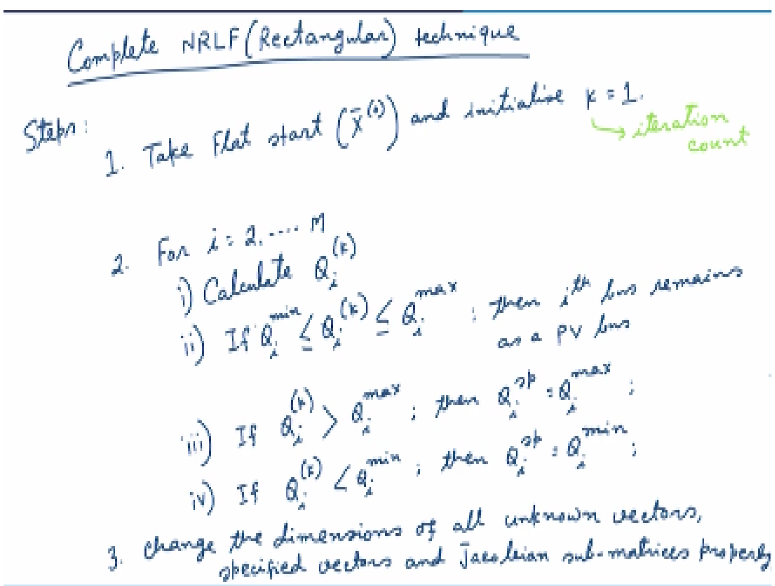

So complete NRLF rectangular technique. So steps, first step is usual step. Take flat start. Initialize this vector X (0) and initialize (k) = 1. As we have said, k is nothing but the iteration count. So it is the iteration count. Then from the second step onward in the second step we will first check the generator reactive power. So for i = 2 to M. Please remember in our convention if there are M generators in the system, first bus is always the slack bus.

And buses 2 to M are nothing but the PV buses. So we have to crosscheck the generator reactive power limits from bus 2 to M. So we first calculate Q i (k). Now how to calculate Q i (k) that we have already seen in the last lecture. So we have to actually utilize that particular expression which you have already given in the last lecture.

Please remember in that particular expression we have to utilize the real and imaginary part of the voltages corresponding to the last iteration or rather the in this case when actually k = 1 so we have to utilize the real and imaginary part of the voltages corresponding to the initial guess, that is corresponding to X 0. So calculate Q i (k). If Q i (k) is less than equal to Q i max then greater than equal to Q i min then ith bus remains as a PV bus.

If Q i (k) is > Q i max then what happens? In that case as we have already seen that this PV bus would be changed to PQ bus, so then therefore for this bus also you will have some value of Q i specified and that Q i specified would be Q i specified would be is = Q i max. And if similarly if Q i (k) is less than it actually should be greater than, less than Q i min then also Q i specified becomes Q i min.

Now in these two cases there would be some change in dimension of several vectors and matrices. So let us first look at this that what would be the dimension change with an simple example. After that we will again continue with this algorithm. So we take an simple example.

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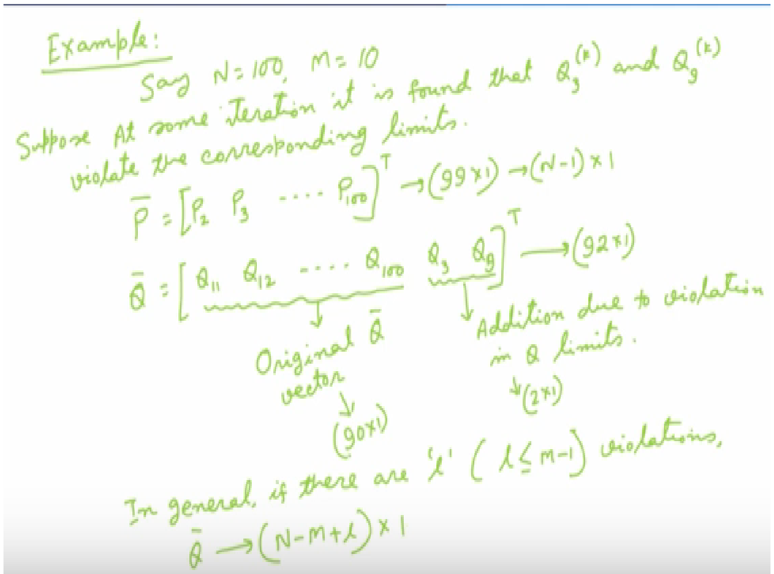

Say N = 100 and M = say 10. Now suppose at some iteration it is found, at some iteration it is found suppose some iteration it is found that Q say 3 k and Q say 9 k violate the limit, violate the corresponding limits. So then therefore what will happen? So now let us look at all the vectors and then let us look at all this corresponding Jacobian sub matrices. Now in this case P vector would be, it will remain unchanged.

P vector would be P 2, P 3 up to P 100 transpose. It would be still (99 * 1) which is nothing but (N – 1) * 1. So it will remain unchanged. What will happen to this Q vector? Earlier Q vector was Q 11, please remember because from (M + 1)th bus to Nth bus that is from 11 bus to 100 bus those buses are PQ buses. So then therefore this Q vector originally was Q 11, say Q 12 up to Q 100.

But now because Q 3 k and Q 9 k have also violated their reactive power limit, so then therefore for third bus also it has now become a PQ bus and for 9 bus also it has become a PQ bus. So then therefore for this we will have extra two elements. So this is the initial guess, so this is the initial original. This is the original Q vector and this is the addition due to violation in Q limits.

So then in this case, what is the dimension would be? So from here to here it is 90 + 2. So it is (92 * 1). Now what was the, now here what is the original Q vector dimension?

Original Q vector dimension is it is already, it is simply (90 * 1) and this addition is basically (2 * 1). So then therefore what we have got? Original plus addition. So that is basically 92 cross. So then in general if there are l violations where l would be (M – 1) l violations then Q vector would be (N – M + l) * 1 vector. So this is the new Q vector. What happens to the V square vector?

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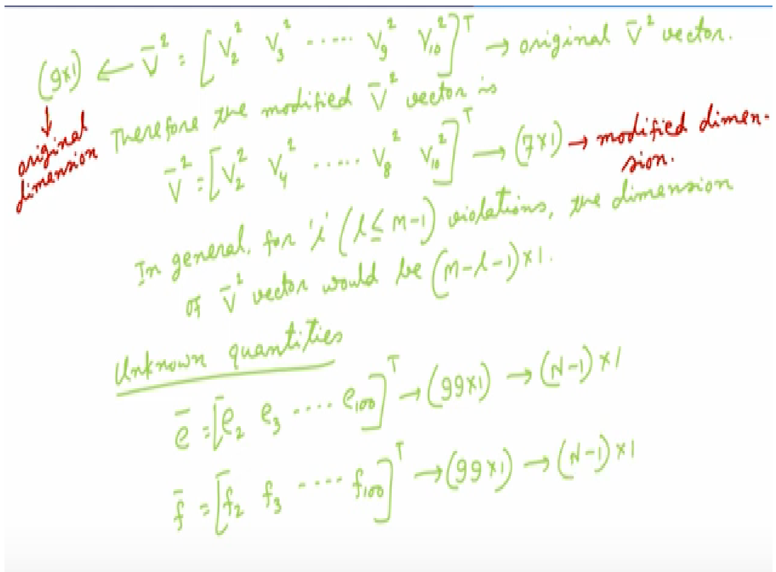

V square vector earlier was V 2 square, V 3 square…V 9 square, V 10 square T. This is the original V square vector. So this is the original V square vector. But now because of this generator reactive power violation third bus and ninth bus they are not anymore PV bus, they have now been converted to PQ bus. So as a result this have now been included into the Q vector as you have just now seen.

So then therefore the modified V square vector is V square = V 2 square; V 3 square will not be there, V 4 square, then V 8 square, then V 9 square would not be there, V 10 square. Now what was the original dimension of this? Original dimension was 9 * 1. This is the original dimension. What is the new dimension? The new dimension is 7 * 1 because bus 3 and bus 9 have now converted to PQ bus and their contribution has been taken into account into the Q vector. So then this is the modified dimension.

So then therefore in general for l violations, again l violations V vector, the dimension of the wave vector would be V square vector would be (M – l – 1) * 1, okay. So these are the dimensions of this known quantities. So what about the unknown quantities. Unknown quantities, unknown quantizes is vector e. Vector e would still be e 2, e 3 to e 100.

Because we have already discussed that whether any buses PV or PQ at the starting when we have started discussing this NRLF rectangular method we have already argued that whether any buses PV or PQ even for a PV bus although we know the voltage magnitude but because we do not know the angle so then therefore real and imaginary part are not known.

So then therefore irrespective of the case whether any buses PV or PQ, right for any bus except the slack bus both these real and imaginary part would be unknown. So then therefore e vector would be still be e 2, e 3, e 100 so then its dimension would be still 99 * 1 in this case. So in general it would be always (N – 1) * 1. Similarly, f vector would be also f 2, f 3, f 100 and it would be in this case (99 * 1). So in general would be (N – 1) * 1.

So then therefore we are taking an example here. So this is an example. So then now let us look into the Jacobian sub matrices.

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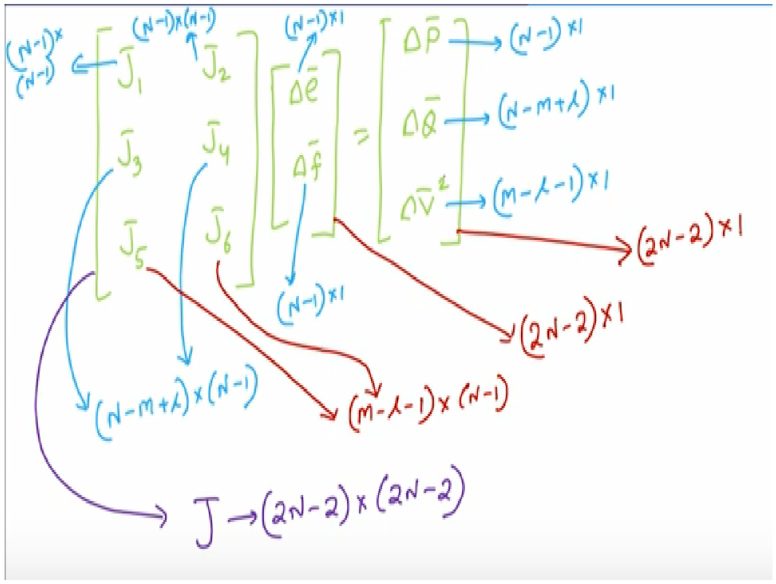

So Jacobian sub matrices were with J 1, J 2, J 3, J 4, J 5, J 6. These are all Jacobian sub matrices delta e, delta f. that would be delta P, delta Q, delta V square. So in case of violation we have already seen that this dimension is (N – 1) * 1. This dimension is (N – M + l) * 1. This dimension is (M – l – 1) * 1. This dimension is (N – 1) * 1. This dimension is (N – 1) * 1. So then therefore J 1 still remains (N – 1) * (N – 1).

So this dimension does not change. J 2 also remains (N – 1) * (N – 1). J 3 changes, J 3 it now becomes (N – M + l) * (N – 1). J 4 also the same. What happens to J 5 and J 6? J 5 becomes (M – l – 1) * (N – 1) and J 6 also the same. So these are the new changed dimensions of the different sub matrices. Now what happens to the total dimension of the J matrix? Now here it is (2N – 2). So this is actually (2N – 2) * 1.

This is also (N – 1) and plus, so this is also (2N – 2) * 1. So then overall this J matrix, so overall this J matrix, J matrix overall this dimension does not change, (2N – 2)* (2N – 2). So that dimension does not change. Overall dimension does not change. But this J 3, J 4, J 5, J 6, now in the case of J 3, J 4 what would be the structure of J 3 and J 4.

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$$J_3 = \begin{bmatrix} \frac{\partial Q_{11}}{\partial e_2} & \frac{\partial Q_{11}}{\partial e_3} & \cdots & \frac{\partial Q_{11}}{\partial e_{100}} \\ \vdots & & & \vdots \\ \frac{\partial Q_{100}}{\partial e_2} & \frac{\partial Q_{100}}{\partial e_3} & \cdots & \frac{\partial Q_{100}}{\partial e_{100}} \\ \frac{\partial Q_{3}}{\partial e_2} & \frac{\partial Q_{3}}{\partial e_3} & \cdots & \frac{\partial Q_{3}}{\partial e_{100}} \\ \frac{\partial Q_{9}}{\partial e_2} & \frac{\partial Q_{9}}{\partial e_3} & \cdots & \frac{\partial Q_{9}}{\partial e_{100}} \end{bmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} (90\times 1) \to (N-m)\times 1 \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} (2\times 1) \to (l\times 1) \end{array}$$

For example I am just showing the structure of J 3, that same structure would be also for J 4. So J 3 earlier was del Q let us say in this example del Q 11 to del e 2. Then del Q 11 del e 3…del Q 11 del e 100. Then it goes to del Q 100 del e 2, del Q 100 del e 3, del Q 100 del e 100. So this matrix is the original. But now because of the violation at bus 3 and bus 9, so then two extra rows would be added.

And these extra rows would be del Q 3/del e 2, del Q 3/del e 3, del Q 3/del e 100 and del Q 9/del e 2, del Q 9/del e 3 and del Q 9/del e 100. So then therefore up to this it is actually (90 * 1). So that is (N – M) * 1 and these two are (2 * 1). So that is (l * 1) right? Similar structure would also be for J 4. Only difference is that instead of e at the denominator it would be the imaginary part of the voltages. Rest of the structure is the same.

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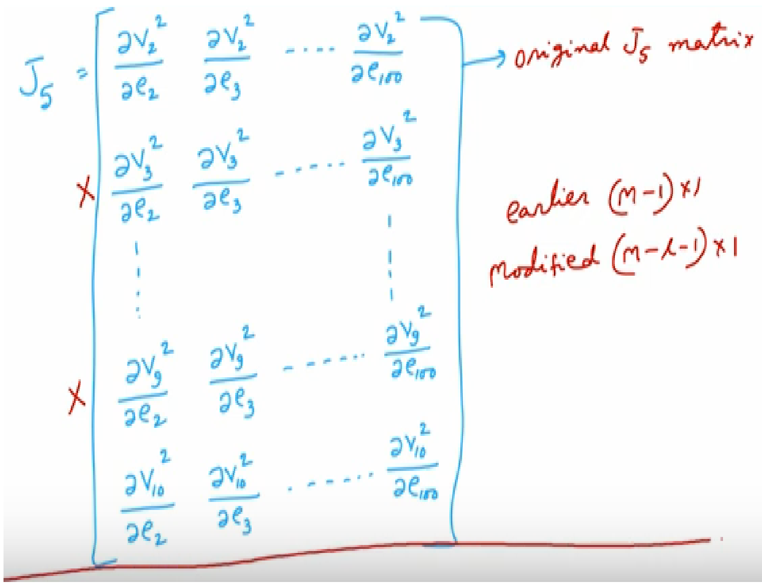

What happens to J 5? J 5 earlier was del V 2 square/del e 2 and then let us say del V 2 square/del e 3…del V 2 square/del e 100. Then del V 3 square/del e 2, del V 3 square/del e 3 then …del V 3 square/del e 100. And then we go up to this. Then we have del V 9 square/del e 2, del V 9 square/del e 3 then …del V 9 square/del e 100 and last is del V 10 square/del e 2, del V 10 square/del e 3 and del V 10 square/del e 100.

So this is the original. This is the original J 5 matrix. So we say that this is the original J 5 matrix. Because now bus 3 and bus 9 have not converted to PQ bus so then therefore this row would not be there and this row would not be there. So then therefore these rows would be del V 2 corresponding to one row would be V 2. After that it would be V 4, V 5, V 7, V 6 sorry V 2, V 4, V 5, V 6, V 7, V 8 and then V 9 would not be there and then V 10. So then therefore here it will be total 7 rows.

So earlier it was (M – 1) * N and modified is (M – l – 1) * 1. So this example ends here. Third step is that change the dimensions of all unknown vectors, specified vectors and Jacobian sub matrices properly as we have now discussed. So after we finish this for all this PV buses then accordingly we will change the dimension of all unknown vectors, specified vectors and the Jacobian sub matrices properly. So once you do that, so this algorithm continued. So algorithm continued.

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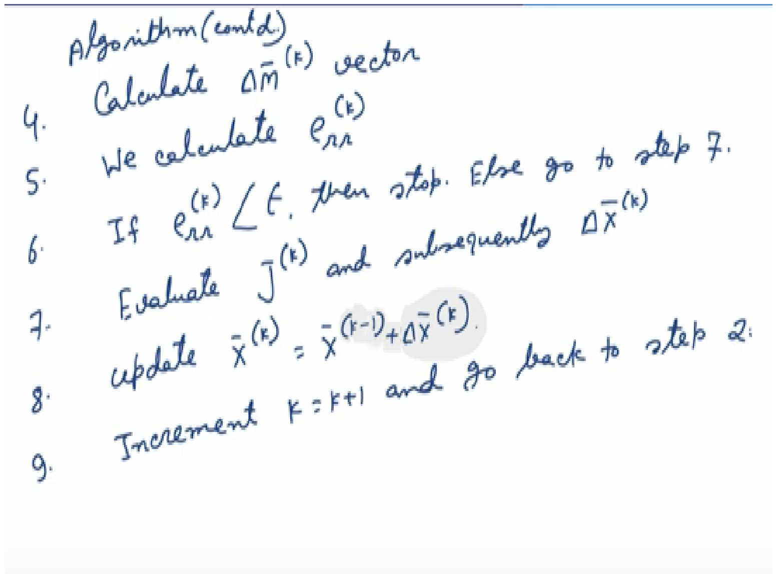

Once you do that in the fourth we compute delta M (k) vector. Now how to compute delta M (k) vector we have already seen in the last lecture. We would be following this identically the same procedure. Only with this exception that depending upon the number of violations the dimensions and the quantities corresponding to the Q vector as well as the sorry dimensions and the quantities corresponding to the delta Q as well as the delta V square vector would be changed.

But otherwise, we will calculate this delta M (k) vector as we have done in the last lecture. So after that we calculate error k as we have done in the last lecture we calculate error k. Error corresponding to the kth iteration. Remember this k at the subscript denotes that it is corresponding to the kth iteration. Then if error kth iteration is less than some epsilon or rather the threshold then stop, else go to step 7.

So what we do in step 7? We evaluate Jacobian matrix corresponding to iteration k and then and subsequently delta X k vector. How to do this that also we have seen in the last lecture and then we update X (k) vector = X nought vector + delta X (k). Then we update increment k = k + 1 and go back to step 2. So this is the complete algorithm. Of course here you can see that we have not filled up all the steps properly.

Because we have already all this steps we have already discussed in great detail in our last lecture. So basically many of this things would be just as common as we have done in the last lecture. So then therefore we have taken some liberty to actually omit some of the details here. So now let us look at some very small example. So some small example. Again, we are looking at this example of that same 5 bus system.

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NRLF (rectangular) results in 5 bus system without any generator Q limit

$$\mathbf{J}_1 \rightarrow (4\times 4);\quad \mathbf{J}_2 \rightarrow (4\times 4);\quad \mathbf{J}_3 \rightarrow (2\times 4);$$
$$\mathbf{J}_4 \rightarrow (2\times 4);$$
$$\mathbf{J}_5 \rightarrow (2\times 4);\quad \mathbf{J}_6 \rightarrow (2\times 4) \text{ and } \mathbf{J} \rightarrow (8\times 8)$$

Bus no.	e (p.u)	f (p.u)	V (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1	0	1	0	0.56743	0.26505
2	0.99958	0.02893	1	1.65757	0.5	-0.18519
3	0.99987	-0.01592	1	-0.91206	1	0.68875
4	0.89634	-0.13157	0.90594	-8.35088	-1.15	-0.6
5	0.94034	-0.08272	0.94397	-5.02735	-0.85	-0.4
Total iteration = 5						

So initially what we have that in this particular 5 bus system N = 5 and M = 3. So then therefore J 1 would be (N – 1) * (M – 1). So (4 * 4). Similarly, J 2 also (N – 1) * (M – 1). So (4 * 4). J 3 is (N – M) * (N – 1). So it is (2 * 4). Similarly, J 4. J 5 also this, J 6 also this and the total J is (8 * 8). So without any generator Q limit these are the result. It takes 5 iterations.

And these results are identically same as what we have obtained corresponding to this GSLF and NRLF polar. Only thing is to be noted that in this case also as compared to this GSLF, the number of iterations taken by Newton – Raphson rectangular technique also pretty low, it is only 5. Earlier it was roughly 69 or 70.

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NRLF (rectangular) results in 5 bus system with generator Q limit

In the third iteration, there is a violation of the reactive power limit of generator 3.

Bus no.	e (p.u)	f (p.u)	V (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1	0	1	0	0.56979	0.33935
2	0.99956	0.02961	1	1.69679	0.5	-0.04769
3	0.98244	-0.01097	0.9825	-0.63991	1	0.5
4	0.87973	-0.12927	0.88918	-8.35906	-1.15	-0.6
5	0.93092	-0.08123	0.93445	-4.98675	-0.85	-0.4
Total iteration = 5						

Now here also again then if we consider this generator Q limit so in the third iteration there is a violation of the reactive power limit of generator 3. So then accordingly what we have discussed in the just now in this lecture that accordingly this dimensions of all this Q vector, V square vector, J 3 sub matrix, J 4 sub matrix, J 5 sub matrix, and J 6 sub matrices are changed.

And then again these calculations are repeated and after when this calculations are repeated so then also this entire algorithm converges in 5 iterations. And as we have already said in this case that for this 5 bus systems this generator reactive power limit for generator 3 was taken to be 0.5 per unit. So if you look at this earlier it was actually generating 0.68 per unit. But now it is limited to 0.5 per unit.

So it is now being limited to 0.5 per unit. Because it is being limited to 0.5 per unit so then its voltage is not being able to be maintained at 1 per unit. It has now become less. Here also total iteration is 5 which is pretty less as compared to GSLF. So now in this lecture we have looked into the detailed example or rather we have looked into the detailed algorithm of this Newton – Raphson rectangular method followed by a one small example.

So from the next lecture onwards we would be looking into other aspects of this course.

Thank you.