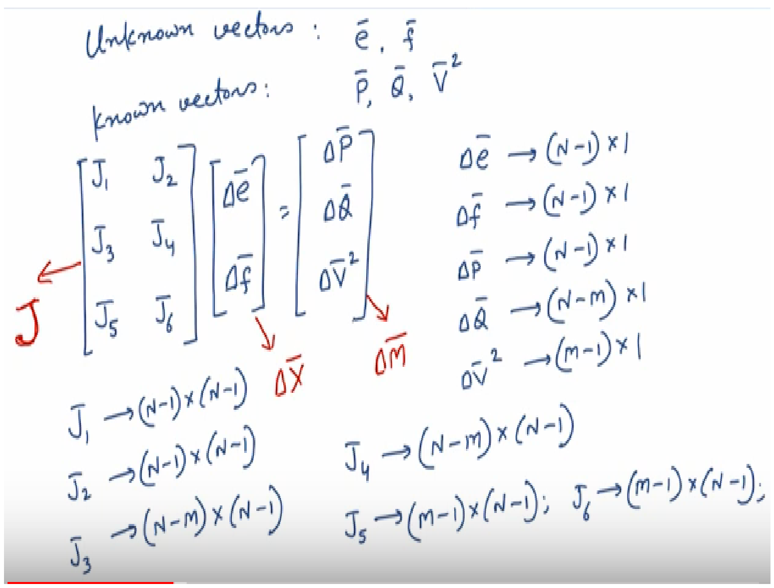
Computer Aided Power System Analysis
Prof. Biswarup Das
Department of Electrical Engineering
Indian Institute of Technology-Roorkee

Lecture - 17
NRLF in Rectangular Coordinate (Contd.)

Welcome to this another lecture of the course computer aided power system analysis. In this lecture we would be again continuing with our NRLF rectangular method. Now in the last lecture we have defined the unknown as well as the known vectors.

(Refer Slide Time: 00:49)



So we are talking about unknown vectors. Unknown vectors are vector e and vector f and known vectors are vector P, vector Q, vector V square and we have already defined what these vectors are.

So then therefore and now we have also looked into that what are the equations for this P. what are the equations for this Q and what are the equations for this V. And all these equations are actually represented in terms of this unknown values e and f. So then therefore following the standard procedure of Taylor series expansion and etc. we can formulate the load flow problem in this case also in the similar way as you have done in the case of NRLF polar.

As that we have got now unknown vector that is delta e, delta f. So these are the basically the correction vector. So it would be delta e and delta f and our mismatch vector would be delta P, delta Q and delta V square. Now here we can see that our unknown vector has got actually 2 sub vector and my known vector have got actually 3 sub vector. So then therefore there are altogether 2 * 3 = 6 sub matrices should be there.

So then therefore we can write down that it should be J 1, J 2, J 3, J 4, J 5, J 6. All these are Jacobian sub matrices. How did we arrive at this? We are simply taking the case of this Newton – Raphson polar coordinate where in that case we had this unknown vector delta theta as the delta V. So then therefore there are actually 2 unknown sub vector. And we had got 2 known sub vector delta P and delta Q.

And we have seen that connecting this vectors delta theta and delta V delta P and delta Q are there are 4 Jacobian sub matrices J 1, J 2, J 3, J 4. But here we have got 2 unknown sub vector and there are 3 known sub vector. So then therefore there should be 2 * 3 that is 6 Jacobian sub matrices. Now we have also seen that delta e vector would be dimension (N – 1) * 1. Delta f vector would be dimension (N – 1) * 1. Delta P vector would also be dimension (N – 1) * 1.

Delta Q vector would be a dimension of (N – M) * 1 and delta V square vector would be dimension of (M – 1) * 1. So if we have this dimension, so then therefore automatically J 1 would be a dimension of (N – 1) * (N – 1). J 2 would also be the dimension of (N – 1) * (N – 1). This cross this. J 3 would be, what is J 3? J 3 is actually connecting delta Q with delta e and delta Q has got a dimension (N – M). So it has got dimension (N – M) * (N – 1).

J 4 is also connecting delta Q vector with the vector delta f. So then therefore it also should have dimension (N – M) * (N – 1). J 5 is connecting the delta V square vector with delta e. So then it should be (M – 1) * (N – 1). Similarly, J 6 also should have a dimension of (M – 1) * (N – 1). Now what should be the size of this Jacobian matrix. Now, this is a big Jacobian matrix. Now according to our parlance, we say that this is a big Jacobian matrix.

This is a Jacobian matrix J, big Jacobian matrix. This is the correction vector delta X and this is the mismatch vector delta M. Now let us look what would be the dimension of this matrix J, dimension of this matrix delta X, dimension of this matrix delta M.

(Refer Slide Time: 06:54)

$$\Delta\bar{X} : 2(N-1)\times 1 \;:\; \Delta\bar{M} \rightarrow 2(N-1)\times 1$$

$$J \rightarrow 2(N-1)\times 2(N-1)$$

$$\Rightarrow J.\Delta\bar{X} = \Delta\bar{M}$$

$$J_1 = \frac{\partial\bar{P}}{\partial\bar{e}} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \frac{\partial P_2}{\partial e_N} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \cdots & \frac{\partial P_3}{\partial e_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_N}{\partial e_2} & \frac{\partial P_N}{\partial e_3} & \cdots & \frac{\partial P_N}{\partial e_N} \end{bmatrix} \rightarrow (N-1)\times(N-1)$$

$$\text{diagonal} : \frac{\partial P_i}{\partial e_i} \quad \forall\, i = 2, \cdots N$$

Dimension of delta is pretty easy. It would be 2 * (N – 1) * 1 because you see this is also (N – 1), this is also (N – 1). So then therefore altogether there will be 2 (N – 1) rows. This would have (N – 1) here (N – M) here and (N – 1) here. So then if we add them up it will also be 2 (N – 1). So delta M vector also would be a size of 2 (N – 1) * 1 and matrix J would be a dimension of 2 (N – 1) * 2 (N – 1).

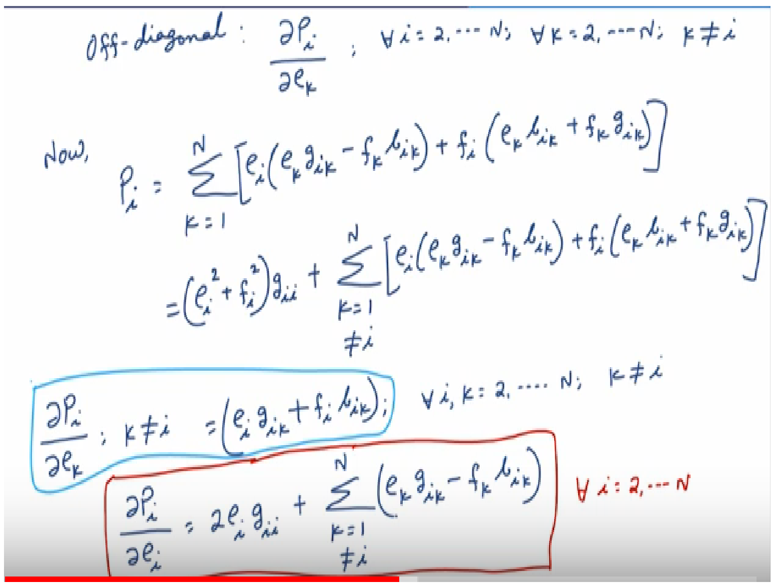
It is obvious because if this has got a dimension 2 (N – 1) and if this has got a dimension 2 (N – 1). So then therefore this matrix J has to have a dimension of this cross this; it is obvious. Now let us again crosscheck. So J 1 has got a row (N – 1). J 3 has got a row (N – M) and J 5 has got a row (M – 1). So then therefore we have got total number of rows as 2(N – 1). J 2 has got a row of (N – 1), (N – M) and (M – 1). So then we have got those number of columns.

I have also got 2(N – 1). So J has got also. So then therefore our J delta X is delta M. So now what we have? Now we have to define what is J 1 as we have done. J 1 would be delta P/delta e vector. So then therefore if I expand them it will be delta P 2/delta e 2, delta P 2/delta e 3…delta

P 2/delta e N. delta P 3/delta e 2, delta P 3/delta e 3…delta P 3/delta e N. Then we continue delta P N/del e 2, del P N/del e 3…del P N/del e N. This is the matrix J 1.

So this has got (N – 1) * (N – 1). So this is a square matrix. Because it is a square matrix so it has got diagonal elements, is delta P i/delta e i for all i = 2 to N. And off diagonals are delta P i/delta e j or e k where k is not equal to i.

(Refer Slide Time: 10:33)



And off diagonals are delta P i/delta e k for all i varies from 2 to N; k varies from 2 to N with the rider k not equal to i. So now therefore we have to now derive the expressions of these 2 elements. That is the 2 general elements that is one is delta P i/del e i and another is del P i/delta e k. So that is what we have to do. Now to do that we have to now first write down the expression of P i.

(Refer Slide Time: 11:35)

Also, $\bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = (e_i + jf_i)(a_i - jb_i)$

$\Rightarrow P_i + jQ_i = (a_i e_i + f_i b_i) + j(f_i a_i - e_i b_i)$

$\Rightarrow P_i = e_i a_i + f_i b_i$

$$\Rightarrow \boxed{P_i = \sum_{k=1}^{N} \left[e_i (e_k g_{ik} - f_k b_{ik}) + f_i (e_k b_{ik} + f_k g_{ik}) \right]}$$

Similarly, $Q_i = f_i a_i - e_i b_i$

$$\Rightarrow \boxed{Q_i = \sum_{k=1}^{N} \left[f_i (e_k g_{ik} - f_k b_{ik}) - e_i (e_k b_{ik} + f_k g_{ik}) \right]}$$

Now when we write down the expression of P i we have already written this expression of P i. So we again write down this expression P i. So P i is k = 1 to N e i(e k g ik – f k b ik). So we write it again. So now P i = k = 1 to N e i(e k g ik – f k b ik). e i(e k g ik – f k b ik) + f i (e k b ik + f k g ik). So this is the expression. So let us then crosscheck. f i (e k b ik + f k g ik) and e i(e k g ik – f k b ik) okay. Now as we have done earlier we will separate the term when k = i.

So then therefore when we separate the term k = i what I find? So when k = i what I got e i square into g ii – e i f i b ii + f i e i b ii. So then therefore this f i e i b ii and this minus e i f i b ii will cancel out and then it becomes f i (f i * g ii). So it becomes f i square into g ii. And here we have got e i square * g ii. So then therefore what we have is we have got e i square + f i square * g ii + k = 1 to N not equal to i.

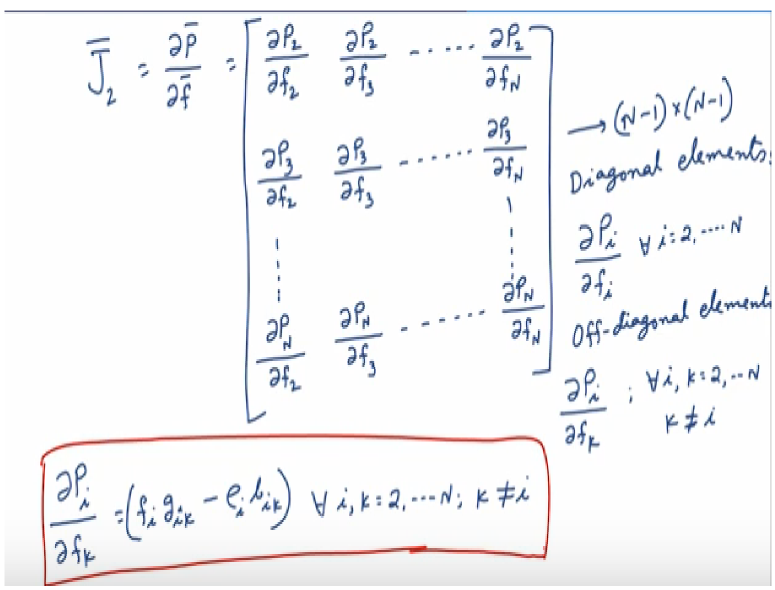
Then this entire expression e i(e k g ik)– f k b ik) + f i (e k b ik + f k g ik). So this is the expression. So now let us try to find out the expression of del P i, easier expression del P i/dl e k k is not equal to i. Now when k is not equal to i so then if I keep on expanding this remember in this entire expression k is not equal to i in all this expression.

So if I so when we are trying to do the partial derivative of P i with respect to any specific e k where k is not equal to i we would be only getting one element and that element would be equal to it is e i g ik – e i f k b ik would cancel out + f i b ik. So it would be e i g ik corresponding to this e k and from here i get f i b ik and f k f k and etc. they will all cancel out. So this is for all i k 2 to N k not equal to i. What would be the expression of del P i/del e i?

From here I will get 2 e i g ii + k = 1 to N not equal to i. I got only one e i, this. Please remember this e k would never be equal to e i. So then therefore this term will not come into picture, this term will not come into picture. But for any e i this term will come into picture. So it would be e k g ik – f k b ik. So this is the expression of del P i/del e i. This is for i = 2 to N. And this is the expression of del p i/del e k.

So once we have the expression of del P i/del e k and del P i/del e i so then in this expression by utilizing this expression we can find out this matrix J 1. So these are the, so this gives the expressions of all the elements or all analytical expressions of all the elements of this matrix J 1. Let us go to J2. What would be the J 2? J 2 would be from this J 2 would be del P/del f.

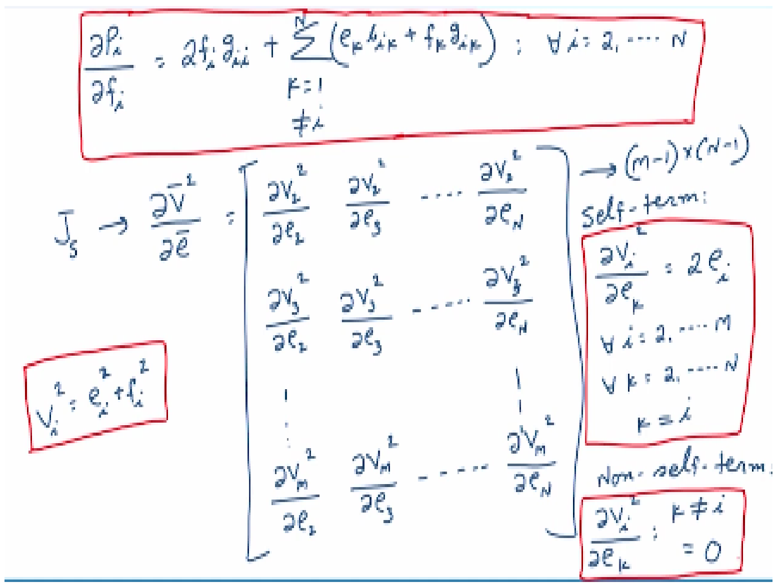
(Refer Slide Time: 17:51)



So we have got J 2 it is del P/del f. So it would be again del P 2/del f 2, del P 2/del f 3. So this is the total structure of this matrix J 2. Again we can see that this is also an we have already seen that this is an (N – N) * (N – N). So I have got diagonal elements are del P i/del f i for all i = 2 to N and off diagonal elements are del P i/del f k for all i k = 2 to N with the rider k = not equal to i. So here also we have to derive all this expressions of del P i, del V i and del P i/del f k.

So del P i/del f k would be from this expression, now P i expression if we look at, we have to pick up the elements containing f k. So we are trying to do this partial derivative with respect to f k. So one term will come from f k. So it is f i g ik. From here we will not get anything. From here also we will not get anything. And another I will get – e i b ik. So one would be f i g ik. So it would be f i g ik – e i b ik. This is the expression. This is for all i k 2 to N, k not equal to i.

(Refer Slide Time: 21:54)



And what would be the expression of, so this is the expression of, so the expression of del P i/del f i. It would be, if we look at this expression one is 2f i g ii + k = 1 to N not equal to i. We have to find out the elements with respect to f i. So with respect to f i we get this because here f k will not have any value of f i because it is k is 1 to N not equal to i. and of course this will have a 0 derivative. This also will have a 0 derivative because f k will never take the value of f i.

So only partial derivative will have only from this. So it is e k b ik + f k g ik. So it is e k b ik + f k g ik. So this is the case. This is for all i = 2 to N. So then therefore this is the expression of. So by this we have this matrix J 2 is also evaluated because for matrix J 2 this is an diagonal so I mean so for matrix J 2 we can evaluate this diagonal elements by utilizing this expression and we can evaluate this off diagonal elements by utilizing this expression.

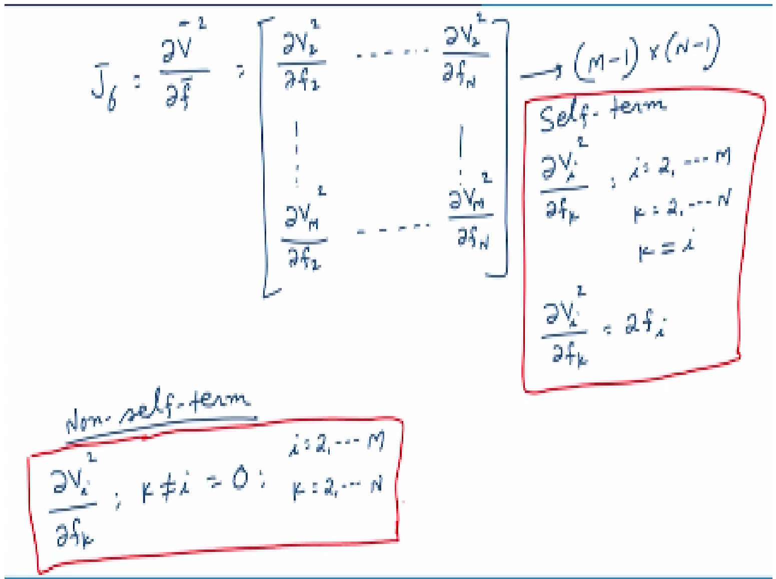
So once you evaluate all these elements by utilizing these 2 expressions this matrix is now defined. Now we come to the other elements. So what we have got? Now J 3 and J 4 would be del Q/del for example J 3 would be del Q/del e and J 4 would be del Q/del f. J 5 would be del V square/del e and J 6 would be del V square/del f. So let us first look at J 5 and J 6 because this is easier to complete. So J 3 and J 4 we would be doing a little later. So J 5.

J 5 is del V square/del e. So what would be this matrix? It would be del V 2 square/del e 2, del V 2 square/del e 3 del V 2 square/del e N. Similarly, del V 3 square/del e 2, del V 3 square/del e 3 del V 3 square/del e N. Similarly, del V M square/del e 2, del V M square/del e 3, del V M square/del e N. now this is an rectangular matrix (M – 1) * (N – 1). So I have got self-term. What is this self-term? Where del V i square/del e i. So what would be this expression?

Please remember that we have got our equation V i square = e i square + f i square. So then therefore it would be del V i square/del e i would be = 2e i, right? So this is for i varying from 2 to M. actually we should write it a little different way, k; k varies from 2 to N and the rider is k = i. So then therefore we are only trying to find out del V 2 square/del e 2, del V 3 square/del e 3, del V M square/del e M nothing else. And that expression would be del V i square/2e i.

And the non-self-term are del V i square/del e k for k not equal to i. They will be all identically = 0. Because V i square is only a function of e i square and f i square. No V i square is a function of e k where k is not equal to i. So then therefore this particular matrix J 5 would be always have only self-term which are very easy to evaluate, which are nothing but 2e i and all the non-self-term would be 0. So we have got this equation V i square this and self-terms are this and non-self-terms are this.

(Refer Slide Time: 28:57)



Similarly, for J 6 is del V square/del f. So it will also be something like this that del V 2 square/del f 2 Then …del V 2 square/del f N. Then it will go up to del V M square/del f 2. And it will go up to del V M square/del f N. it is also an (M – 1) * (N – 1) matrix. So I have got self-term. That is del V i square/del f k, right i varies from 2 to M; k varies from 2 to N; k = i. So this is equal to 2f k or rather we can also 2f i, it is all the same.

And non-self-term del f k. Same condition, only k is not equal to i. So it would be 0. Here also i varies from 2 to N, k varies from 2 to N. So these are the expressions of this J 5 and J 6. You can see that J 6 is also very easy to evaluate. It has also got only the self-term. All the non-self-terms are all 0 because V i square is only a function of f i square. It is not function of f k, any other f k where k is not equal to i.

So then here so far today you have what we have done, we have first defined what would be the Jacobian sub matrices. There will be 6 Jacobian sub matrices; J 1, J 2, J 3, J 4, J 5, J 6. We have also defined what are those I mean what are those sub matrices are. And out of this 6 sub matrices, we have finished looking at the expressions of J 1, J 2 and J 5, J 6. In the next lecture we would be looking at the expressions of the other two sub matrices and then further we will continue. Thank you.