Computer Aided Power System Analysis
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Lecture - 16
NRLF in Rectangular Coordinate

Welcome to another module of this course computer aided power system analysis. Till the last lecture we have looked into the Newton – Raphson load flow method in the polar coordinate. Today onwards we would be looking at the Newton – Raphson load flow method in the rectangular coordinate and we have already explained what is actually meant by rectangular coordinate as well as the polar coordinate. So we start today.

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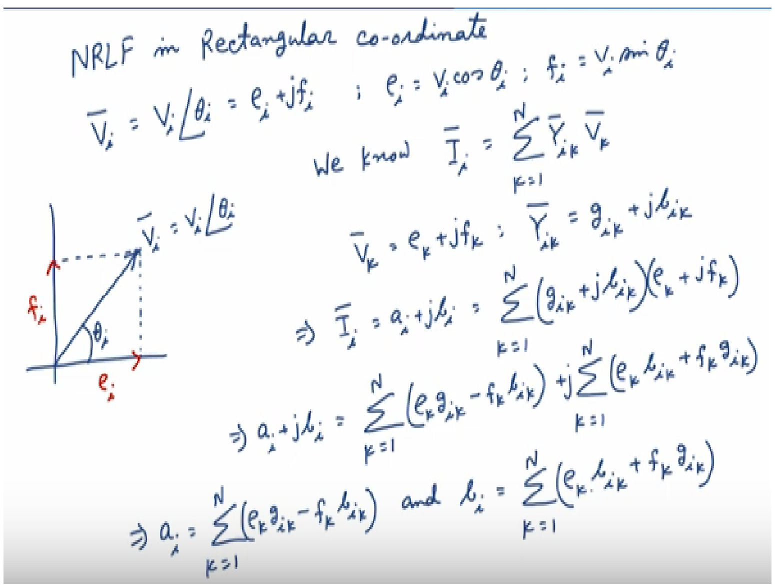

So we today is NRLF in rectangular coordinate. So when we say that NRLF in a rectangular coordinate, we denote for example V i that is the ith bus voltage, complex bus voltage is given by we have already seen V i angle theta i. So it can be written as e i + jf i where e i is V i cos theta i and f i is V i sin theta i. The basic concept is that if I have an Argand plane, so in the complex plane we do represent the voltage vector V i. This is a complex voltage vector.

So this is the magnitude and this is the angle. So then when we do represent it V i angle theta i. So this is nothing but the standard polar representation. But then when we do resolve this into the two coordinates. So this is the and when you do resolve this into 2 coordinates, so this is e i and this is f i. So when we are actually resolving it into 2 coordinates, one across the real part and another across the imaginary part this is nothing but the rectangular coordinate representation of the bus voltage.

Now what we have is we know for any bus I i, I i is nothing but the injected current at bus i, N is the number of bus and Y ik is nothing but the element of the Y bus matrix corresponding to ith row and kth column. Now according to this rectangular representation we have V k we represent as e k + jf k and Y ik we represent as g ik + jb ik. So please note that we are also representing the elements of the bus admittance matrix which are also themselves a complex quantity also in the rectangular coordinate.

So then therefore we can write down I i let us say that it is a i + jb i, that is we are also representing I i as I i in the rectangular coordinate of which this real part is a i and for which **this** our imaginary part is b i. That is equal to given by k = 1 to N (g ik + jb ik) * (e k + jf k). So then therefore we can write down a i + jb i = k = 1 to N write (e k g ik – f k b ik) + j k = 1 to N (e k b ik + f k g ik) e k b ik + f k g ik and this is e k g ik minus.

So then therefore a i that is the real part of the current can be given by k = 1 to N (e k g ik – f k b ik) and b i can be given as k = 1 to N e k b ik + f k g ik. So these are the expressions of the real and imaginary part of the injected current V i.

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$$\text{Also, } \bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = (e_i + jf_i)(a_i - jb_i)$$

$$\Rightarrow P_i + jQ_i = (a_i e_i + f_i b_i) + j(f_i a_i - e_i b_i)$$

$$\Rightarrow P_i = e_i a_i + f_i b_i$$

$$\Rightarrow \boxed{P_i = \sum_{k=1}^{N} \left[e_i (e_k g_{ik} - f_k b_{ik}) + f_i (e_k b_{ik} + f_k g_{ik}) \right]}$$

$$\text{Similarly, } Q_i = f_i a_i - e_i b_i$$

$$\Rightarrow \boxed{Q_i = \sum_{k=1}^{N} \left[f_i (e_k g_{ik} - f_k b_{ik}) - e_i (e_k b_{ik} + f_k g_{ik}) \right]}$$

Now we known, also we know complex power at bus S i that is P i + jQ i. That we know as V i * I i* that we already know. So then therefore this can be written as (e i + jf i) I i* would be (a i – jb i). So then therefore we can write down P i + jQ i = (a i e i + f i b i) + j(f i a i – e i b i). So then therefore P i = e i a i + f i b i. Now we do substitute the expression of a i and b i. So it is e i(e k g ik – f k b ik). So (e k g ik – f k b ik) + f i b i is b ik + f k g ik. So it is (e k b ik + f k g ik).

So this is this and similarly Q i from this expression f i a i – e i b i. So then therefore k = 1 to N f i into a i, a i is this; (e k g ik –f k b ik) – e i and b i is (e k b ik + f k g ik). So then therefore these are the two expressions of injected real power at bus i and injected reactive power at bus i in rectangular coordinate. Now what happens is, now we have to look at that what are the unknowns and known quantity in the rectangular coordinate.

So after we do this so now we have to look at what are the unknown quantity. So what are the unknown quantities?

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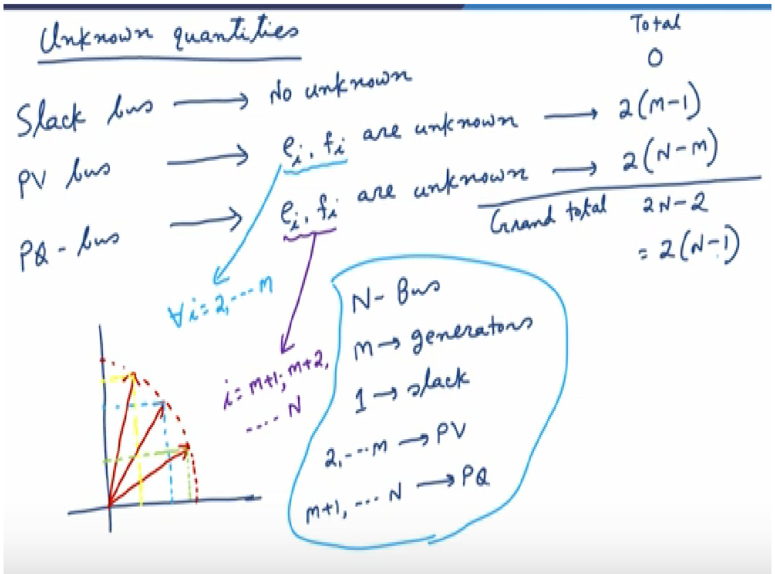

Now we have got different types of bus, slack bus then PV bus then PQ bus. Now in the slack bus we always define what is the voltage magnitude and the angle. So then once we know the voltage magnitude as well as the angle so then therefore its real part as well as the imaginary part of the voltage at slack bus is known. So then therefore there is no unknown. So in slack bus, no unknown. In the PV bus what we have?

In the PV bus we do specify bus voltage magnitude and the injected real power. Now we do not at all specify the angle. So then therefore because we do not specify the angle so then therefore in a PV bus both the real part as well as the imaginary part are unknown. For example if I have a plane and if my bus voltage magnitude is let us say 1 per unit and if I do let us say draw a circular arc having let us say 1 per unit for example.

And so then therefore bus voltage magnitude can be anywhere like this, can be anywhere. Each of this bus voltage magnitude is maintained at 1 per unit but for this bus voltage, but for this vector our real part is this and the imaginary part is this. For this vector, our real part is this and my imaginary part is this and for this vector my real part is this and my imaginary part is this.

So then therefore what happens even though my bus voltage magnitude is maintained at 1.0 per unit but depending on its angle both the real part as well as the imaginary part will vary. So then therefore even though we do pre-specify the bus voltage magnitude but because we do not pre-specify the bus voltage angle, so then therefore both e i and f i are unknown. So then therefore in PV bus at every PV bus e i and f i are unknown.

And at PQ bus we do only pre-specify the real power as well as the reactive power; we do not pre-specify neither the bus voltage magnitude nor the angle. So then therefore both e i and f i are unknown. Now let us see what is the total number of unknowns. Now for that again we take our same assumptions or rather that we have got an N- Bus system. There are M- generators. Bus 1 is the slack bus. Bus 2 to M are PV, the standard whatever you have been doing; M + 1 to PQ.

So then therefore here the unknowns would be, the total number would be 2 (M – 1) and here the total number would be 2(N – M). So it is total, it is 0. So then therefore my grand total is 2N – 2. That is 2(N – 1). And for PV bus this i varies from 2 to M and for PQ bus this i varies from M + 1, M + 2 to N. So this you all know. Now this result is also quite obvious to determine because you see except at the slack bus at all the other buses both the real part as well as the imaginary part are unknown.

So then therefore if I have total number of buses in the system is N so then therefore at each of this N – 1 buses both the real part as well as the imaginary part of the voltages are unknown so then therefore at each of this buses there are 2 quantities which are unknown so then therefore total of 2(N – 1) number of unknowns are there for an N- Bus system because at each of this N – 1 buses there are 2 quantities which are unknown.

So these are the unknown quantities and we have also specified this. So now let us look at what are the known quantities.

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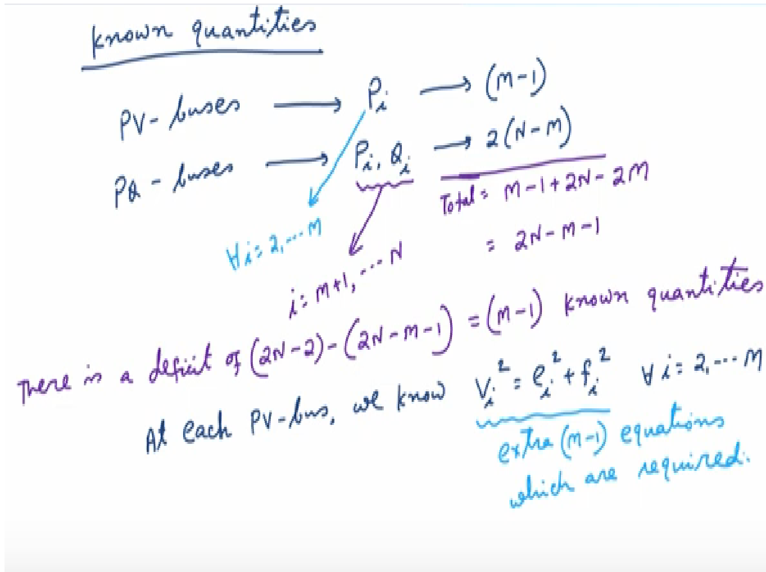

So now our task is to, known quantities. Now what are the known quantities? Now the known quantities are so far at PV buses there are P i unknown and at PQ buses P i and Q i are known. So there are total of (M – 1) and there are total of 2(N – M) because there are total of (N – M) PQ buses and each of this buses their P i and Q i are unknown. So then therefore total known quantities would be 2(N – M) and my total number of PV buses is (M – 1).

And at each bus I know P i. So then therefore there are altogether (M – 1) known quantities. So as again we note that for all i varies from 2 to M as usual and for this i varies from M + 1 to N. So then what is the total number? Total is it is M – 1 + 2N – 2 M. So then therefore it is 2N – M – 1. So now what happens? I have got unknown quantities which is equal to 2N – 2 and my known quantities are only 2N – M – 1.

So then therefore there is a deficit of (2N – 2) – (2N – M – 1). That is = (M – 1) known quantities. So then therefore what we need? I still need M – 1 known quantities because otherwise I cannot solve this set of equations because in this case what is happening? We have got total 2N – 2 unknown quantities but our known quantity is only 2N – M – 1. So then therefore there is a deficit of M – 1 known quantities.

Now from where this M – 1 equations will come? This M – 1 equation will come from the fact that at each of this PV buses we do know the bus voltage magnitudes. So then therefore at each and every PV bus this bus voltage magnitudes are given by at each PV bus we know V i square = e i square + f i square. So this is nothing but the bus voltage magnitude. This is the real part, this is the imaginary part. So then at each and every bus these equations hold good.

So then therefore these equations together this is for i = 2 to M. So from here we get extra (M – 1) equations which are required.

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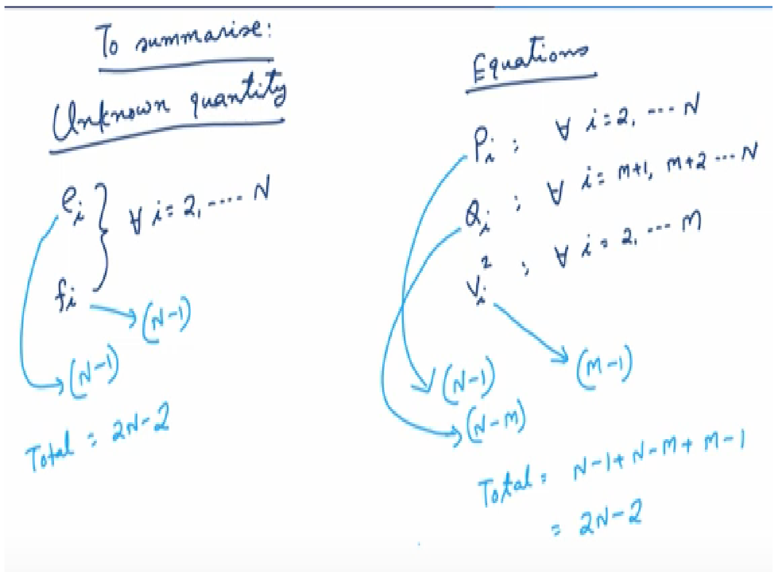

So then therefore to summarize, unknown quantities are e i and f i. So this is for all i = 2 to N. And my equations P i for all i = 2 to N Q i for all i = M + 1, M + 2 to N and V i square equation that is for all i = 2 to M. So the total number of unknowns if we do note, so the unknown here is N – 1. Here is unknown N – 1. So total = 2N – 2. Here the total number of known quantities is (N – 1). Here it is (N – M) and here it is (M – 1). So total = N – 1 + N – M + M – 1.

So that becomes equal to 2N – 2. So then therefore the number of unknowns and number of equations they are exactly matched. Total number of unknowns is 2N – 2. Total number of equations is also 2N – 2. Now if we do recollect our discussion of Newton – Raphson polar method, what we do find?

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Recollect NRLF (Polar)

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\bar{\theta} \\ \Delta\bar{V} \end{bmatrix} = \begin{bmatrix} \Delta\bar{P} \\ \Delta\bar{Q} \end{bmatrix}$$

NRLF (Rectangular)

We define:

$$\left.\begin{aligned} \bar{e} &= [e_2 \;\; e_3 \;\; \cdots \;\; e_N]^T \rightarrow (N-1)\times 1 \\ \bar{f} &= [f_2 \;\; f_3 \;\; \cdots \;\; f_N]^T \rightarrow (N-1)\times 1 \end{aligned}\right\} \text{unknown vectors}$$

$$\left.\begin{aligned} \bar{P} &= [P_2 \;\; P_3 \;\; \cdots \;\; P_N]^T \rightarrow (N-1)\times 1 \\ \bar{Q} &= [Q_{M+1} \;\; Q_{M+2} \;\; \cdots \;\; Q_N]^T \rightarrow (N-M)\times 1 \\ \bar{V}^2 &= [V_2^2 \;\; V_3^2 \;\; \cdots \;\; V_M^2]^T \rightarrow (M-1)\times 1 \end{aligned}\right\} \text{known vectors}$$

That in our Newton – Raphson polar method recollect NRLF polar. So in NRLF polar what we had? We had equations as J 1, J 2, J 3, J 4 and delta theta vector, delta V vector = delta P vector, delta Q vector. That was our equation what we have seen. These equations we have found out from the linearization of the all this nonlinear equations around an operating point or rather at some operating point X nought.

So now here in this, now this J 1, J 2, J 3, J 4 are nothing but these are Jacobian sub matrices. But here what we need to note down is that here our unknown quantities are delta theta and delta V and my quantities were P and Q. So then what we had got? We had got actually 2 unknown vectors theta and V and we also had got 2 known vectors P and Q. So then therefore this has got let us say 2 unknown vectors and this has also got 2 known vectors.

So then therefore 2 * 2 that is = 4. So then therefore we had a total of 4 Jacobian sub matrices. Now what happens in the case of NRLF rectangular. Now NRLF in rectangular we define my unknown vector as e as [e 2, e 3, e N] T. So this has got dimension (N – 1) * 1. Then my vector f has [f 2, f 3, f N] transpose. It has also got (N – 1) * N and our standard vector P is [P 2, P 3, P N] this is already known but we are just defining it. So this is (N -1) * 1.

Vector Q as [Q M + 1, Q M + 2, Q N] transpose T. this is (N – M) * 1 and then we also define something called vector V square which is nothing but vector V 2 square, V 3 square, V M square transpose. So this has got dimension of (M – 1) * 1. So these are unknown and these are known vectors. So then therefore in this lecture what we have first again recollected what is meant by this NRLF rectangular method.

After that we have derived the equations for real and reactive injected power at bus i in the rectangular coordinate. After that we have defined that what are the unknown quantities and what are the known quantities and we have also looked at their dimensions. So now in the next lecture onwards we would be again continuing this particular topic. Thank you.