Computer Aided Power System Analysis
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Lecture - 14
NRLF in Polar Co-Ordinate (Contd..)

Welcome to the another module of this course computer aided power system analysis. We have been discussing the Newton – Raphson load flow in the polar coordinate. So far we have discussed that after the linearization, the basic equation corresponding to the Newton – Raphson load flow in polar coordinate is given by

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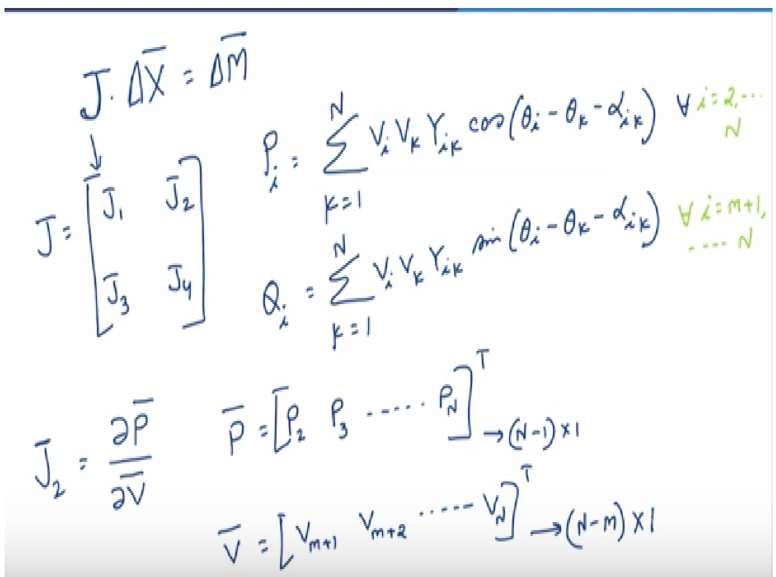

J, matrix J * delta X vector = delta M vector where this matrix J is nothing but the Jacobian matrix and this matrix J is given by J 1, J 2, J 3, J 4 where J 1, J 2, J 3, J 4 are the Jacobian sub matrices and we have also defined their dimensions. We have also defined what they stand for and we have also looked at the analytical expressions of the elements of the matrix J 1. So now we have to look at the analytical expressions of the elements of this matrices J 2, J 3, and J4.

So to do that let us again recollect our basic expression of the power flow equations. So it is given by P i = k = 1 to N V i V k Y ik cos (theta i – theta k – alpha ik). We have already defined what these quantities are. Q i = k = 1 to N V i V k Y ik sin (theta i – theta k – alpha ik). Here we again recollect that P i and Q i are nothing but the injected real and reactive power at bus i and this P i is for i = this is i = 2 to N and this Q i is for i = M + 1 to N.

And we have also defined what is N what is M. N is nothing but the number of buses. M is nothing but the number of generators and our convention is that bus 1 is the slack bus and then buses 2 to M are the generator buses and buses M + 1, M + 2 up to N are the PQ buses. Now J 2 we have also seen that J 2 is given by, J 2 matrix is given by del P/del V where P is a vector, V is a vector. Where vector P is given by P 2, P 3, P N transpose.

So this is an (N – 1)*1 vector. Vector V is given by V M + 1 V M + 2…V N transpose. So this is an (N – M)*1 vector.

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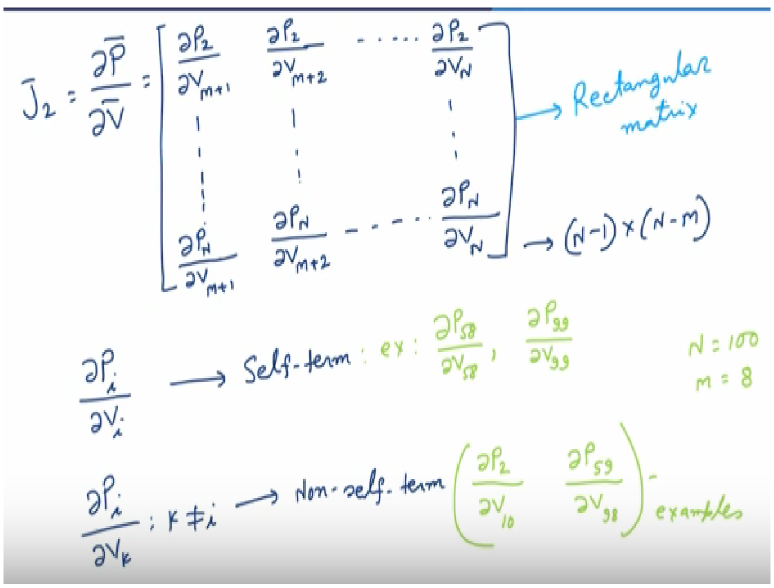

So then therefore del P by del V vector would be and we have also seen that del P the expressions of this the matrices are del P 2/del V + 1, del P 2/del V + 2…del P 2/del V N and then it goes up to then del P N/del V M + 1. Then del P N/del V M + 2. And then last is N. And it would be essentially an (N – 1) * (N – M) matrix. So then it is a rectangular matrix.

So then therefore there is nothing called a diagonal element and also there is nothing called an off diagonal element. But then there would be some terms where the index of P and the index of V would be same and there would be other terms where this index of P and the index of V would be different. So the terms will have in general in this nature del P i/del V i. You please note that this index of P that is i and this index of V that is i so they are same.

And then also there would be some term which is del P i by let us say del V k where k is not equal to i. So this we call let us say a self-term example and we say, let us say we should not say mutual term it is a non-self-term. It is just a matter of terminology, non-self-term. The examples of this non-self-terms would be something like this. Say del P 2 by let us say del V let us say 10. For example if say N is equal to say 100 and let us say M is equal to say 8.

So then del P 2/del V 10 del P 59/del V 98. So these are the examples, some of the examples are this. These are the examples. And here these examples are for example del P say 58/del V 58 say del P 99/del V 99 etc. So these are the self-term where this index of P and the index of V are same. Here this is 58, this is 58. Here this is 99, this is 99. But here this index of P and index of V are different.

So then we have to actually derive the analytical expressions of this two general terms. So now for that we again write the expressions of P i as we have done.

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$$P_i = \sum_{k=1}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$= V_i^2 G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

J_2

$$\frac{\partial P_i}{\partial V_k};\ k \neq i = V_i Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}); \quad i = 2, \ldots, N;\ k = M+1, \ldots, N;\ k \neq i$$

$$\frac{\partial P_i}{\partial V_i} = 2 V_i G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{N} V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}); \quad i = 2, \ldots, N;\ k = M+1, \ldots, N;\ k = i$$

P i is k = 1 to N V i V k Y ik cos. It is we have already seen, it is V i square G ii + = 1 not equal to i N V i V k Y ik cos (theta i – theta Q – alpha ik). So then if I do calculate del P i/del V k for k not equal to i. So then what I will get? This would be equal to V i Y ik cos (theta i – theta k – alpha ik). This would be for i going from 2 to N, k varying from M 1 to N and k is not equal to i. Then the second term that is this self-term del P i/del V i that would be from here 2 V i G ii + k = 1 to N not equal to i. It would be V k Y ik cos (theta i – theta k – alpha ik).

So here what will happen? Here also we have i varies from 2 to N. Here also k varies from M + 1 to N but the rider is that k = i. That is the only rider. So these 2 terms together they do define J 2 matrix. So this 2 terms they do define J 2 matrix. So now let us go to the matrix J 3.

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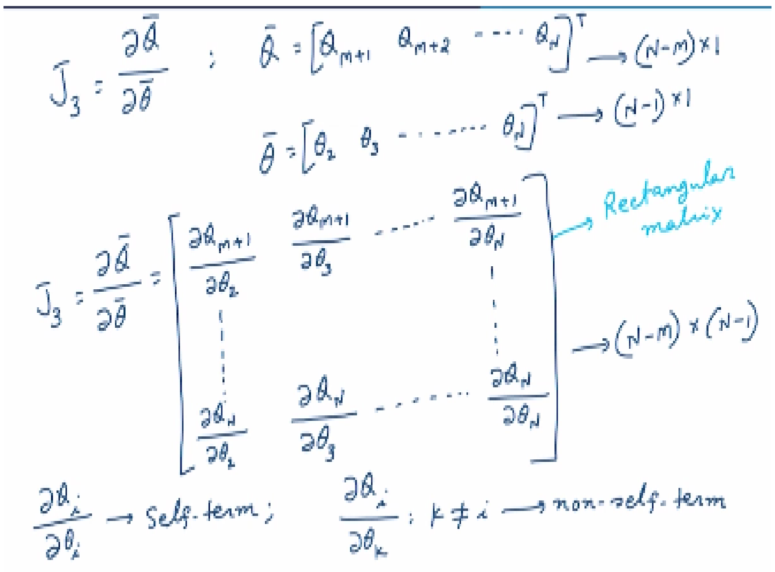

Matrix J 3 we have already seen it is del Q/del theta. Now vector Q is Q M + 1, Q M + 2, Q N transpose. So this is an (N – M) * 1 vector and vector theta is theta 2, theta 3, theta N transpose. This is (N – 1) * 1 vector. The definition of J 3 is del Q/del theta is del Q M + 1/del theta 2, del Q M + 1/del theta 3…del Q M + 1/del theta N. And it goes up to del theta sorry del Q N/del theta 2. Then del Q N del theta 3 and it goes up to…del Q N/del theta N.

So it would be also a rectangular matrix. It is a rectangular matrix (N – M) * (N – 1). So because it is a rectangular matrix it will also have a self-term and non-self-term. Self-terms would be del Q i/del theta i. So this is the self-term and non-self-terms would be in our own parlance it would be del Q i/del theta k. This is sorry k not equal to 1. These are non-self-term. So again we have to find out the analytical expression of this 2 elements.

And if we can do find that so we can then we can define this entire matrix. Now to do that let us write down the expression of Q i.

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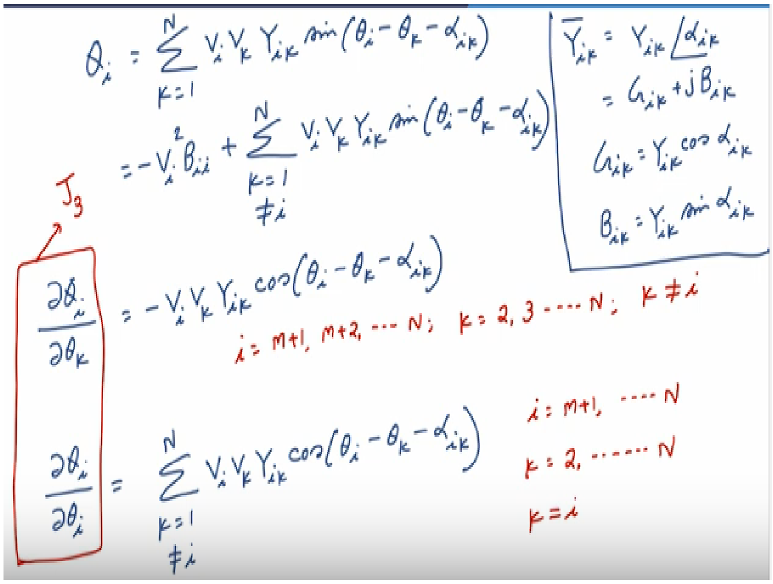

Q i is k = 1 to N V i V k Y ik sin (theta i – theta k – alpha ik). Now we have defined that Y ik = Y ik magnitude and angle. So this is G ik + J B ik. This J is the complex operator. And so then therefore we know that G ik = Y ik cos alpha ik and B ik = Y ik sin alpha ik. So this, all this we have already looked into. We are merely writing it for our recollection. So now here if we do take the ith term separately as we have done for the expression of P i. So then what I will get?

If k = i so it becomes V i square Y ii. So theta i and theta k cancels out. So this is sin of – alpha ik. Sin of –alpha ik is nothing but – sin alpha ii because here k = i. So then it will be sin of – alpha ii. So then it will be basically V i square Y ii * sin of – alpha ii and we know that the sin of – alpha ii is nothing but – sin alpha ii. So then if we do apply this we do get – V i square B ii + k = 1 to N not equal to i V i V k Y ik sin (theta i – theta k – alpha ik).

So this is the expression of Q i. So now you have to find out first non-self-terms del Q i/del theta k. So what it would be? Please kindly remember k is not equal to i. So the partial derivative of this would be 0. So sin derivative cos it has got a minus. So it would be – V i V k Y ik cosine (theta i – theta k – alpha ik). We are trying to differentiate this expression with respect to any particular specific k. So this would be 0. So this is the expression of delta Q.

Now what would be the range of. So for here i would be varying from M + 1, M + 2 to N. K would be varying from 2, 3, N and k is not equal to i. It would be clear from here. So k varies from 2, 3 to N and i varies from M + 1 to N. Now we have to find out the expression of del Q i/del theta i. So that would be is equal to again it would be 0. And it would be k = 1 to N not equal to i.

Only this term will come but because theta i exists in all the expressions in all this terms of this summation. So then we will get all this terms of the summation and it will be very simple. V i V k Y ik cosine (theta i – theta k – alpha ik). And here the i would be varying from again M + 1 to N. K would be varying from 2 to N. And the rider is k = i. So these two terms together define the matrix J 3. The last one is remaining, J 4. So let us do that.

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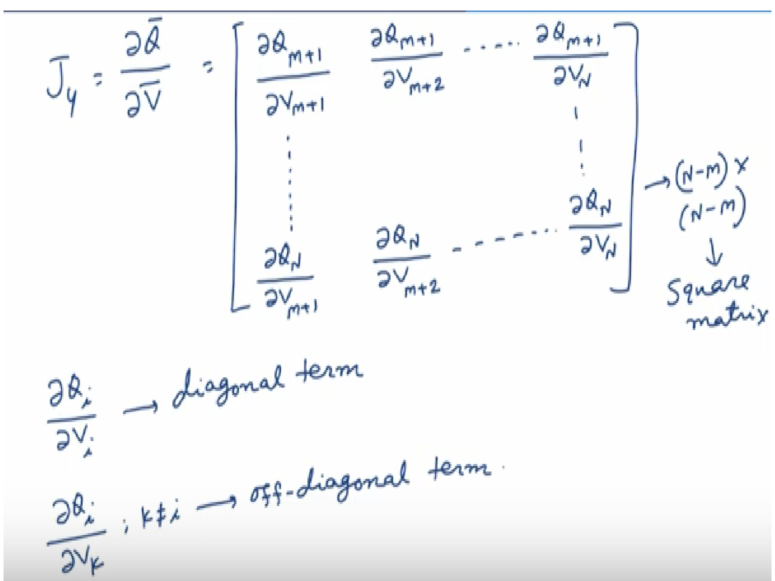

J 4 is del Q/del V. So if I do del Q M + 1/del V M + 1, del Q M + 1/del V M + 2…del V N. And it would go up to last one del Q N/del V M + 1. Then del Q N/del V M + 2… del Q N/del V N. So it would be a (N – M) * (N – M) matrix. So it is a square matrix. Last 2 were this I must say that this is a rectangular matrix. J 2 is also a rectangular matrix. Okay, so this is a square matrix.

Now because it is square matrix so then therefore it would have diagonal term as well as off-diagonal terms. Diagonal terms would be obviously del Q i/del V i. And that is the diagonal terms. And del Q i/del V k, k is not equal to i those would be off-diagonal term.

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$$Q_i = -V_i^2 B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{N} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

J_4

$$\frac{\partial Q_i}{\partial V_k} = V_i Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$i = M+1, M+2, \cdots N$

$k = M+1, M+2, \cdots N$

$k \neq i$

$$\frac{\partial Q_i}{\partial V_i} = -2V_i B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{N} V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$i = M+1, M+2, \cdots N$

$k = M+1, M+2, \cdots N$

$k = i$

So we again write down the expression of Q i is – V i square B ii + k = 1 to N not equal to i V i V k Y ik sin (theta i – theta k – alpha ik). So we first do the easier one. Del Q i/del V k. So this part derivative would be 0 and we will have only one term here and that term would be V i. So this V k will go. Y ik sin (theta i – theta k – alpha ik) and here the i would vary from M + 1, M + 2 to N. K would also vary from M + 1, M + 2 to N.

But the rider is K is not equal to i. The last one is del Q i/del V i. So here it is – 2V i B ii plus please note that in this expression of Q i in all the terms inside the summation symbol has got this quantity V i. So then therefore all this terms inside its summation symbol will come into picture. So k = 1 to N not equal to i. It would be V k Y ik sin (theta i – theta k – alpha ik). Please note that this V i is gone.

And here what would be the variation of i and k? i again would be varying from M + 1 to M + 2 to N; k also would be varying from M + 1 to M + 2 to N. And the rider is k = i. So then therefore these elements together define the matrix J 4. So then we have found out all the expressions of all the elements of this Jacobian matrix. So now what we will do? Now please remember that at each and every iteration, all these elements of this Jacobian matrix would be evaluated with the most updated values available for the quantities V and theta.

V means all this bus voltages and theta means all this bus voltage angles. So then therefore once we get the numerical values of all these elements of the Jacobian matrix after that we can simply invert. Now here one very interesting thing is that if we look at the expression of let us say J 2, J 3, J 4 apparently if we look at the equation of let us say J 2. Now the question is here for example there would be many such elements which will have this particular expression.

Now are all this expressions be nonzero. Now we need to understand that when we would be actually evaluating this expression or let us say this expression, what we will do? We will simply substitute the latest voltage magnitude and the angles available for all this V i and all this theta i and theta k. And remember Y ik and alpha ik are already known. So then it may happen that all this elements del P i/del V k and del P i/del V i would be nonzero.

So then as a result this matrix J 2 would be a completely full matrix. But then one very important point to notice is that, that this value Y ik would be 0 in the bus admittance matrix if bus i and k are not directly connected with each other. So then therefore if bus i and k are not directly connected with each other so then therefore many of this elements del P i/del V k would be 0.

Only those elements would be nonzero where bus i and bus k are connected to each other because in that case Y ik would be nonzero and of course alpha ik will also be nonzero. What about this element del P i/del V i? This element would be always nonzero because after all G ii is nothing but the real part of the diagonal element corresponding to bus i. So then therefore this particular diagonal element will always exist. So then therefore G ii will always exist.

So then therefore this part would be always nonzero. But then again depending upon the direct connection between bus i and bus k here Y ik may be 0, Y ik may be nonzero. So then therefore even it may happen that we have to add all this sum total together but then in effect probably only few terms will come. Similarly, if we look at the expressions of J 3 also for this also only those terms would be nonzero for which bus i and bus k are directly connected.

And here also basically those terms would be nonzero where bus i and bus k would be directly connected. Now obviously there will be some buses here where at least bus i would be directly connected to bus k so then therefore at least there would be one element here would be nonzero. So then usually this self-terms become always nonzero.

But this non-self-terms some of them would be nonzero but the rest of them would be 0 because in any power system usually any particular bus is directly connected to mostly 4 to 5 other buses. We would be actually discussing this issue in much more detail in some later classes. Similar observation also holds good for this matrix J 4. So here also depending upon the case that where bus i and bus k are directly connected or not, this term would be either 0 or nonzero.

But then this term would be always nonzero and then again depending upon the fact that whether bus i and bus k are directly connected or not some of these terms would be nonzero and some of these terms would be 0. So then therefore what we can see is that in this matrix J2, J 3, and J 4 many of the elements would be 0. So as a result this matrix J 2, J 3 and J 4 would be actually sparse in nature.

Similar observation also holds good for this matrix J 1. A matrix is called sparse when most of this elements are 0. So now because this matrices J 1, J 2, J 3, J 4 are sparse in nature so then therefore this matrix J is also sparse in nature. So this is one interesting observation. But now we have already covered all the mathematical necessities corresponding to Newton – Raphson polar coordinate. In the next lecture we would be looking at the complete algorithm as well as one small example of NRLF in polar coordinate. Thank you.