Computer Aided Power System Analysis
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Lecture - 13
NRLF in Polar Co-Ordinate (Contd.)

Welcome to this lecture on this course computer aided power system analysis. We are now discussing the Newton – Raphson load flow in the polar coordinate. So in the last lecture we have seen that for solving the power flow equations we do linearize those equations around the initial operating point and after we do the linearization we get this matrix equation.

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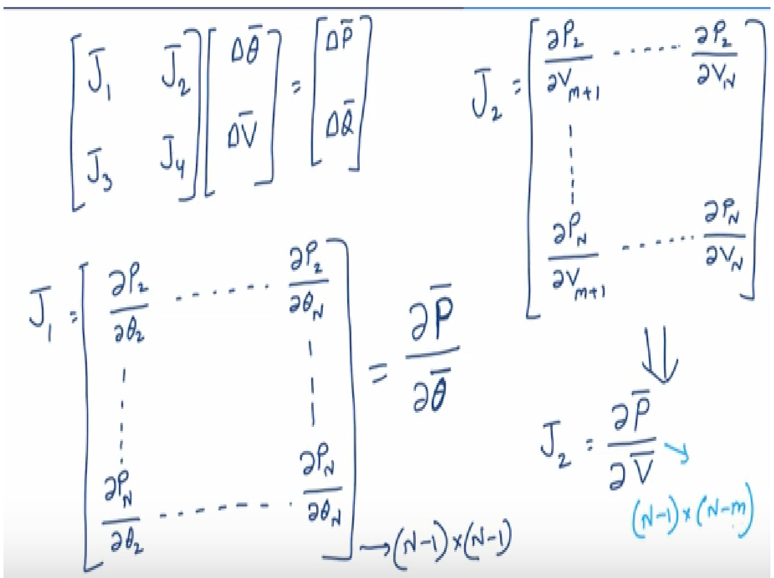

We get this matrix equation. Now we need to, we will now look into all this quantities, all this vectors and matrices more closely. Now what is J 1? J 1 if we do recall, it is actually del P 2/del theta 2. Then … del P 2/del theta N. Then … del P N/del theta 2… del P N/del theta N. So this is written as in the language of the multivariable calculus del P vector partial derivative of P vector with respect to the theta vector. Now what would be the dimension here?

Dimension would be you see P 2 is actually varying from P 2, P 3 up to P N. So then therefore there are N – 1 rows and theta 2 is also varying from theta 2, theta 3 to theta N. So there will be N – 1 columns. So it would be N -1 row and N – 1 column. What would be J 2, J 2 matrices? Del P 2/del V M + 1…del P 2/del V N. Then it is del P N/del V M + 1… del P N/del V N.

So again in the language of the multivariable calculus, we say that J 2 matrix is nothing but the partial derivative of P vector with respect to V vector. And we have already defined what is P vector and what is V vector.

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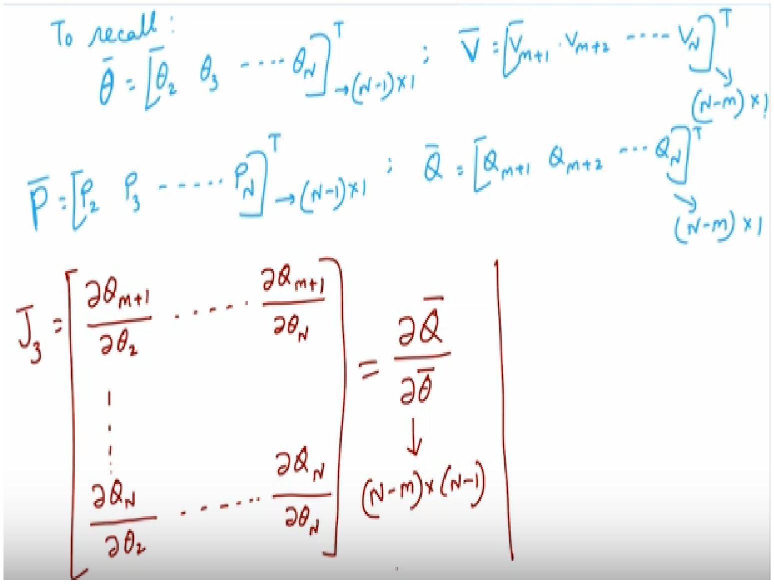

Just to recall, theta vector is theta 2, theta 3, theta N transpose is (N – 1) + 1. V vector is V M + 1, V M + 2 to V N transpose. So this is a (N – M)*1 vector. Vector P is P 2, P 3, P N. So this is (N – 1)*1 and vector Q is Q M + 1, Q M + 2, Q N transpose. This is also an (N – M)*1. So then therefore now here what would be the dimension of this? So dimension of this would be there are how many rows? N rows, P 2 to P N and how many columns? There are M + 1 to N.

So M – N columns. So it is N – 1. So (N -1) rows * (N- M) columns. What would be J 3? J 3 is del Q M + 1/del theta 2…del Q M + 1/del theta N…del Q N/del theta 2 del theta N. So we call it as del Q partial derivative of Q vector with respect to theta vector. And would be the dimension? Dimension would be how many rows? N – M rows and how many columns? N – 1. So (N – M) *(N – 1).

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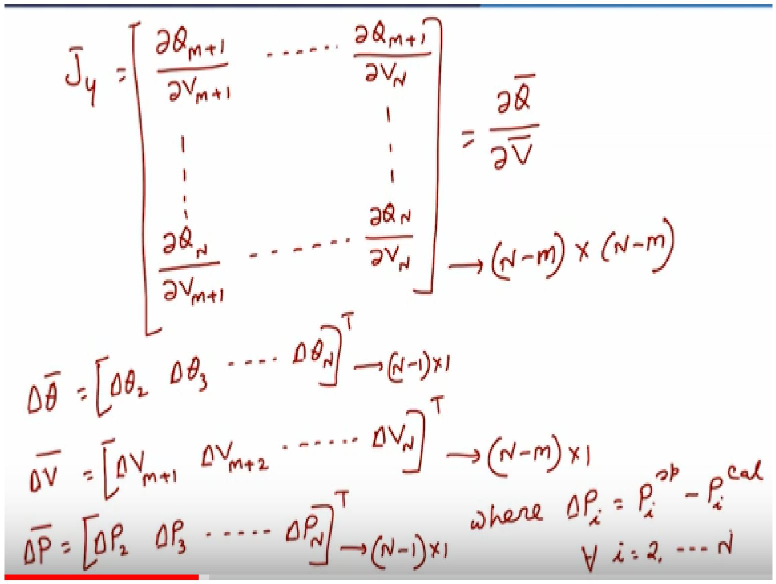

And J 4 would be del Q M + 1/del V M + 1 del V N. So it would be del Q/del V vector and its dimension would be (N – M)* (N – M). Now what is delta theta vector? Delta theta vector is delta theta 2, delta theta 3, delta theta N T. So then therefore it has got (N – 1)* 1 vector. What is delta V vector? Delta V M + 1 delta V M + 2 delta V N T. So it is (N –M)*1. What is delta P vector? Delta P vector is if you recall, it is P 2 specified – P 2 calculated.

Rather I should write actually delta P 2, delta P 3…delta P N transpose. It has also got (N – 1)*1 where delta P i is P i specified – P i calculated for all i = 2 to N. So then therefore delta P 2 is nothing but P 2 specified – P 2 calculated. Delta P 3 is nothing but P 3 specified – P 3 calculated and …similarly delta P N is nothing but P N specified – P N calculated.

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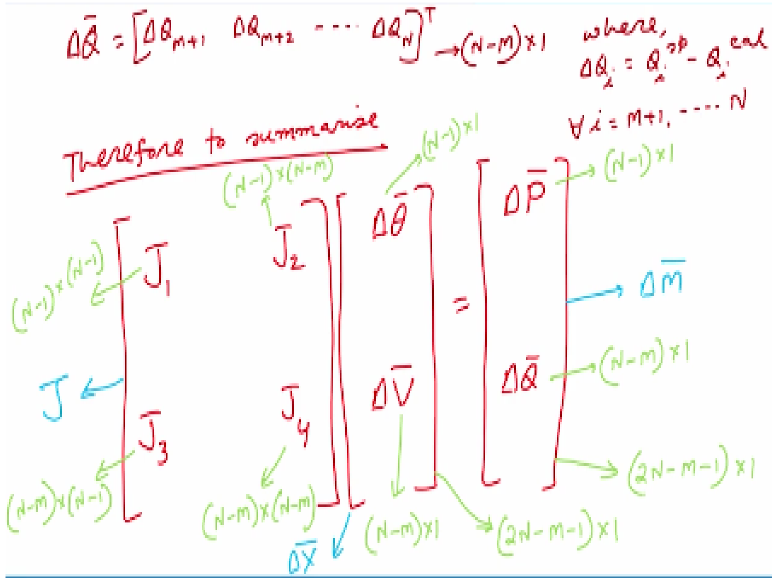

Lastly delta Q vector is actually delta Q M + 1 delta Q M + 2 transpose. So it is (N – M)*1 and where delta Q i = Q i specified – Q i calculated for all i = M + 1 to N. So then therefore we again write down summarize, what we have we have got J 1. We have got J 2. We have got J 3. We have got J 4. So these are the 4 matrices or the sub matrices into delta theta vector into delta V vector. That is equal to delta P vector and that is equal to delta Q vector.

So this we are again reproducing it. Now here we would be writing down their dimension. So dimensions would be so this is (N – 1)* (N – 1). J 2, what would be the J 2 dimension? (N – 1)*(N – M). It is (N – 1)*(N – M). What is the dimension of J 3? J 3 is (N – M)*(N – 1).So it is (N – M)*(N – 1) and this is (N – M)*(N – M). This has a dimension of (N – 1)*1. This has a dimension has got (N – M)*1. Delta P has got a dimension of (N – 1)*1.

Delta Q has a dimension of (N – M)*1. So then therefore this entire vector has got total dimension of (2N – M – 1)*1. This vector, this entire vector has got a dimension of (2N – M – 1)*1 and this matrix J 1, J 2, J 3, J 4 is called the Jacobian matrix.

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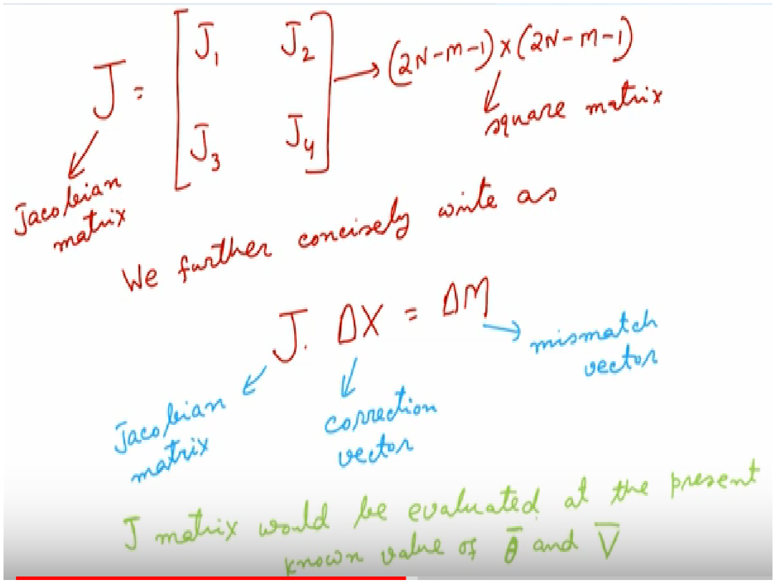

And the matrix J, we call it matrix J it is J 1, J 2, J 3, J 4. This is a Jacobian matrix. And this has got a dimension of (2 N – M – 1)*(2N – M – 1). So this is a square matrix. So this is irrespective of the number of buses and number of generators in the system this Jacobian matrix would always a square matrix. So this is called Jacobian matrix.

Now this equation, this big equation we do further write in a more succinct form as we further concisely write as this big equation, this big matrix equation, Jacobian matrix J into correction vector delta X = mismatch vector delta M. So this is the Jacobian matrix. This is called the correction vector and this is called the mismatch vector. And what is delta X? So this vector is called delta X vector.

This vector is called mismatch vector because this is the mismatch between the specified value and the calculated value. So this vector denotes the mismatch between the specified value and the calculated value. So this vector is called the mismatch vector. This vector is called the correction vector.

Because we are trying to correct or rather we are trying to calculate the corrected value of or rather the correction value of theta which should be added to the initial guess such that we can reach the final solution and this big matrix is called the Jacobian matrix. Now in this equation as we have said we have already seen in our basic Newton – Raphson load flow sorry basic Newton

– Raphson numerical method as we have already seen that to calculate delta X we can calculate as delta X = J inverse * delta M.

So now how to calculate this J matrix? Now this J matrix would be evaluated at we note that this J matrix would be evaluated at the present known value of vector theta and vector V. So this would be calculated, evaluated at the present known value of vector theta and vector V. Now when you start the iteration what is the known value of vector theta and vector V? This is nothing but the initial guess.

So then therefore when you start the iteration we know what is the vector theta nought and we also know what is the vector V nought. Basically those are nothing but the initial guess. So when we know that, so then we do simply evaluate this matrix J by substituting these values of theta nought and V nought. Now the point is now if we have to evaluate this matrix J that means we have to find out the each and every of this partial derivative.

So then how do we really evaluate this matrix J? So that means we have to evaluate each and every of this matrix J 1, J 2, J 3 and J 4. So then what we will do? We will simply first find out that what is the or rather what are the analytical expressions of all this partial derivatives as well as for all the partial derivatives of this matrices J 3 and J 4. And then in those matrices we would be simply substituting the values of theta nought and V nought.

And after we do substitute the values of theta nought and V nought in this partial derivatives we would be able to find out the numerical values of this matrices or rather this sub matrices J 1, J 2, J 3 and J 4 and once we know the numerical values of this matrices J 1, J 2, J 3 and J 4 we would be able to find out what is the matrix J and subsequently we can take the inverse.

So then therefore our next task would be to find out what are the analytical expressions of all this partial derivatives because until and unless we know this analytical expressions of all this partial derivatives we would not be able to find out the numerical value.

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To find analytical expressions of all elements of

J_1, J_2, J_3, J_4

J_1 : diagonal elements $\rightarrow \dfrac{\partial P_i}{\partial \theta_i}$

Off-diagonal elements $\rightarrow \dfrac{\partial P_i}{\partial \theta_k}$; $i \neq k$

$$P_i = \sum_{k=1}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$= V_i^2 Y_{ii} \cos(\alpha_{ii}) + \sum_{\substack{k=1 \\ \neq i}}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$= V_i^2 G_{ii} + \sum_{k=1, \neq i}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

So our next task is to find out the analytical expressions of all elements of J 1, J 2, J 3, J 4. Now we start with J 1. So J 1 has got, so it is a square matrix. So it has got some diagonal elements and some non-diagonal elements. So this diagonal elements are so if we look at this diagonal elements, this diagonal elements are nothing but delta P 2/del theta 2, del P 3/del theta 3…del P N/del theta N and the off diagonal elements are del P i/del theta k where i is not equal to k.

So therefore it has got J 1 has got it is a square matrix so diagonal elements are del P i/del theta i and off diagonal elements are del P i/del theta k where i is not equal to k. So now we have to find out this expressions. Now to find out this expression we have to write down the expression of P i. Now what do we do is we simply take the ith term separately. Now here you see when k is varying from 1 to N at some instant or rather at some step k would be = i.

So then would be taking that term k = i I mean separately. So now what we have? So when actually k = i so then this becomes V i* V i, V i square. This becomes Y i cos theta i – cos theta 1. So then basically this two cancels out and it only remains cos – alpha ii, cos – alpha ii is nothing but cos alpha iii. So then what we have got? V i square Y ii cos alpha ii. So then what I have got is V i square Y ii cos alpha ii + k = 1 to N not equal to i V i V k Y ik cosine of theta i – theta k minus.

Now what we have is for now we write it as V i square. We will just explain it G ii. Now what is G i we just explained. So k = 1 to N not equal to i V i V k Y ik cos theta 1 – theta k – alpha ik. Now here we have defined a term or we have introduced a term G ii. So now let us explain what is Gii.

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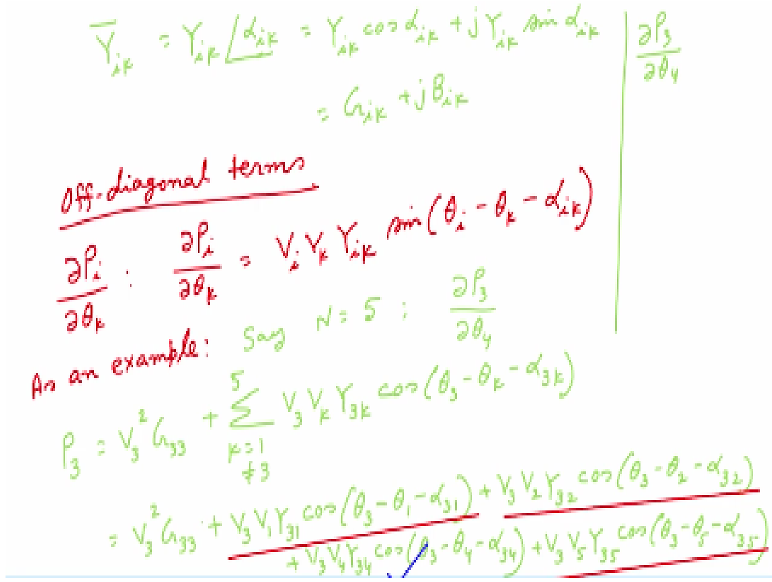

Now what I have got Y ik is the complex element of the Y- Bus matrix corresponding to ith row and kth column. So it is given by magnitude Y ik and angle alpha ik. So this can be written as Y ik cos alpha ik + J that is J is the J is nothing but the complex operator. So this real part we call as G ik and this imaginary part we call as B ik. So G ii is nothing but the real part of the diagonal element Y ii and B ii would be similarly the imaginary part of the diagonal element Y ii.

So then what we have? So we have here, so P i is equal to this. So now we have to evaluate. Now with this expression we have to evaluate now delta P i theta i and delta P i theta k. So we start with the easier one. So we first do this off diagonal terms because that is the easier one. What are the off diagonal terms? Off diagonal terms are delta P i/delta theta k. So what would be this, delta P i/delta theta k.

What would be this? It would be simply, there would be only one term because here there is absolutely no term as theta k so then therefore this term would be 0 or rather basically the partial derivative of this first term would be 0 and among this term there is going to be only one term which will come into picture. So then therefore what we will have is we will have it is V i V k Y ik sin (theta i – theta k – alpha ik).

To illustrate this take an example, to illustrate this take as an example say N = 5 and we are trying to find out let us say del P 3/del theta 4, we are trying to find out. So now from this expression, what is the expression of P 3. P 3 would be V 3 square G 33 + k = 1 to 5 not equal to 3 V 3 V k Y 3k cosine theta 3 – theta k – alpha 3k.

So if I just expand it what I get? I get V 3 square G 33 + V 3 + V 1 Y 31 cos (theta 3 – theta 1 – alpha 31) + V 3 V 2 Y 32 cos theta 3 – theta 2 – alpha 32 + V 3 V 4 Y 34 cos (theta 3 – theta 4 – alpha 34) + V 3 V 5 Y 35 cos (theta 3 – theta 5 – alpha 35). So this would be the complete expression of P 3. Now if I wish to take del P 3/del P 4. So del P 3/del P 4 del theta 4 from this. In this expression there is no theta 4 so then it will be 0.

In this expression also there is no theta 4 so then therefore this partial derivative would be 0. It also would be 0. It also would be 0. Only this term would come into picture and cosine has got a partial derivative minus and for this – theta will get another minus. So then clubbing everything together I will get this. So it would be V 3 V 4 Y 34 sin theta 3 – theta 4 – alpha 34. And that is what we have written.

So delta P i by delta theta k would be V i V k Y ik sin theta i – theta k – alpha ik. Let us look into the diagonal terms.

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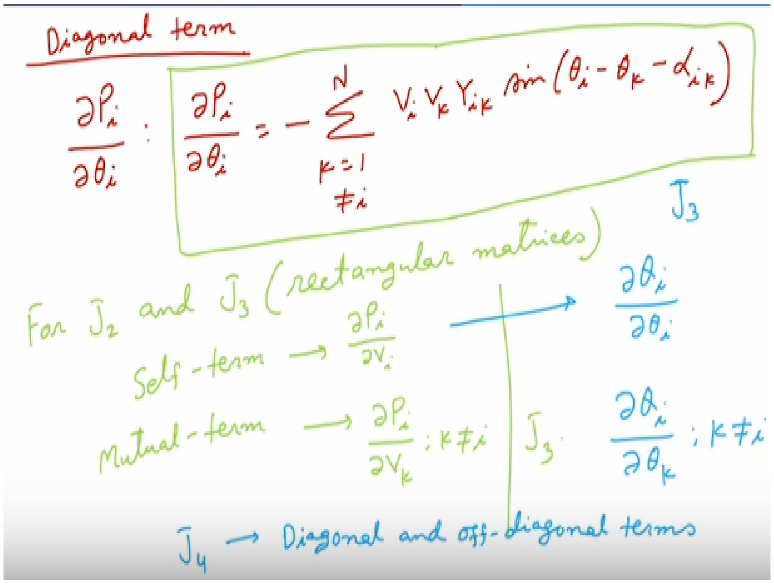

This diagonal term is delta P i/delta theta i. If we look at this expression, in this entire expression, only in the first term, in this term there is no theta i. So then therefore the partial derivative of this term with respect to that would be 0 but in all the other terms this quantity theta i exists. So then therefore there will be a partial derivative and cosine has got a derivative of minus sign so then it would be simply similar.

For example as an example here when you were writing down the expression of P 3, only in the first term there is no theta 3 but in all the other terms there is theta 3 available. So then therefore if we do delta P 3/delta theta 3 so all this terms will contribute to this partial derivative and all of them would be put together and all this cosine will have minus.

So therefore putting them together what we will have is delta P i/delta theta i = - k = 1 to N not equal to 1 V i V k Y ik sin (theta 1 – theta k – alpha ik). So this is the expression of the diagonal term and this is the expression of the off diagonal terms. So when we find out this diagonal term and off diagonal terms for all of them so then therefore we are able to find out the analytical expressions of all the elements of J 1.

Now for J 2 and J 3 matrices those are these are nothing but rectangular matrix. So then therefore for this two matrices there is nothing called a diagonal term or off diagonal term. But for our own parlance, for J 2 and J 3, for J 2 and J 3 which are rectangular matrices we call something called self-term. This is in our parlance and we call say we are abusing this term mutual but just for the sake of this course we call them mutual term.

Self-term for example it would be in the case of J 2 it would be delta P i/delta V i and the mutual term would be delta P i/delta V k where k is not equal to i. Self-term means when this expression of P i is being evaluated with respect to the voltage at its own bus. And mutual term means that when the expression of P i is being evaluated with respect or rather when the partial derivative of the expression of P i is taken with respect to the voltage of some other bus.

So this is for J 2 and for J 3 these terms would be delta Q i/delta theta i and it would be delta Q i/delta theta k for k not equal to i. So this is for J 3. So in the next classes we would be looking into these expressions that what would be these expressions and also we would be looking into the expressions of all the elements of J 4. For J 4 because it is a square matrix so then therefore there would be a diagonal and off diagonal terms.

So for J 2 we would be looking at the self-term as well as mutual term that what would be the expression. For J 3 also we would be looking at the self-term and the mutual term that what would be the expressions and for J 4 also we would be looking at the expressions of diagonal and off diagonal terms. So we will do in the next lecture. Thank you.