Computer Aided Power System Analysis
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Lecture - 12
Newton - Raphson Load Flow (NRLF) in Polar Co-Ordinate

Hello welcome to this another lecture of this course computer aided power system analysis. In the last lecture we have looked at the basic procedure of the Newton – Raphson numerical method. Today, starting with this class, starting with this lecture as well as the for the next few lectures we would be discussing about how to apply this basic Newton – Raphson numerical method for the solution of the power flow equations.

Now as we have already indicated earlier Newton – Raphson method is actually of two type. One is called the polar method another is called the rectangular method. So first let us try to understand what is meant by the polar method and what is meant by the rectangular method and after that we will today start with the Newton – Raphson polar method.

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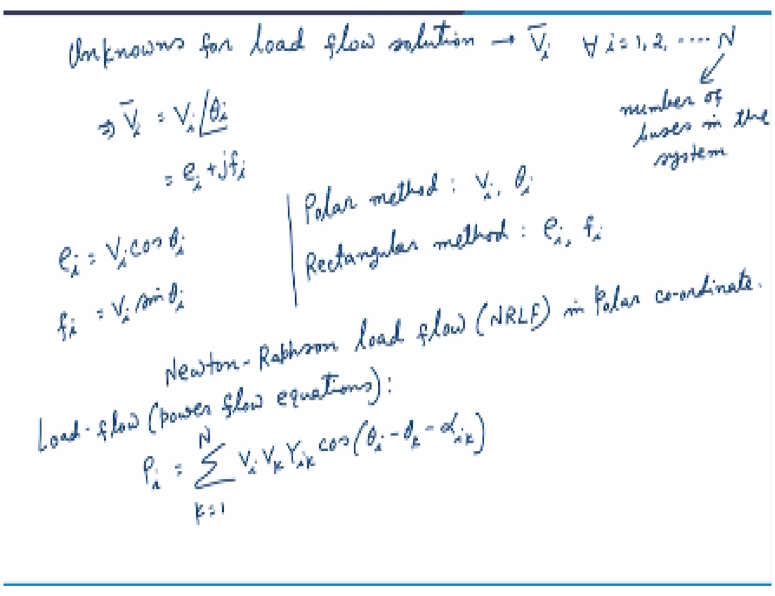

So in polar method now we have already discussed that our unknowns are actually unknowns for load flow solution are actually complex voltage V i for all i = 1, 2 to capital N where N is the number of bus in the system. Now because V i is a complex quantity so then V i can be written as magnitude V i and angle theta i. And eta also can be written as e i + jf i where e i is the real part and f i is the imaginary part.

Of course e i would be = V i cos theta i and f i would be V i sin theta i. So now what happens is that in the case of polar method our unknowns are so polar method our unknowns are V i and theta i. That is basically the magnitude and the angle. And in the rectangular method our unknowns are e i and sorry e i and f i. So then therefore so this is only difference. So then essentially in the case of the polar method we do solve for the angle directly as well as also the voltage magnitudes.

On the other hand in the rectangular method we do not solve for the voltage magnitude and the angle directly. We rather solve for the real and imaginary part of the voltages and then afterwards from this real and imaginary part of the voltages we do calculate the voltage magnitude and angles. Now from today we would be actually starting with Newton – Raphson polar method.

So we start with Newton – Raphson load flow, we call it NRLF, Newton – Raphson load flow in polar coordinate. We call it polar coordinate. Now we know that our load flow equations are load flow equations or rather power flow equations we also call load flow or power flow equations are P i = V i V k Y ik (cosine theta i – theta k – alpha ik) where k = 1 to N.

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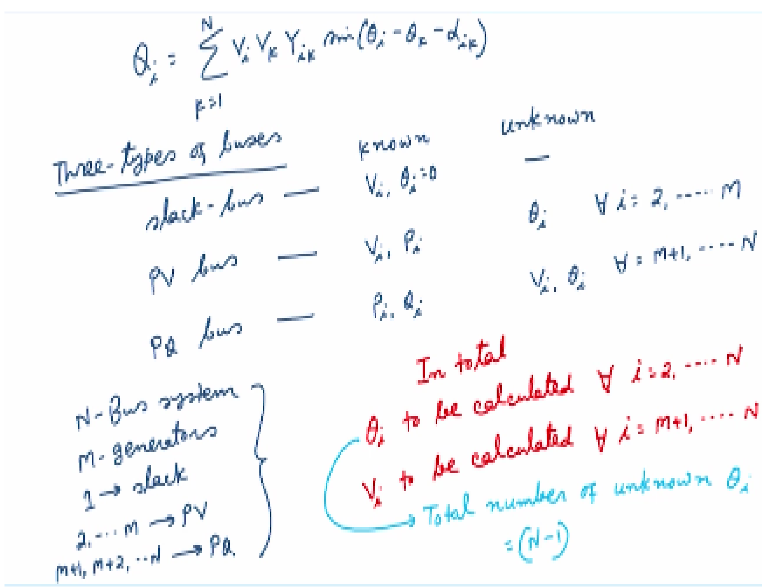

And Q i = V i V k Y ik sin (theta i – theta k – alpha i k) k = 1 to N. And we have already seen what are these quantities. For example Y ik is nothing but the magnitude of the element of the bus admittance matrix corresponding to the ith or jth column and alpha ik is the corresponding angle and P i and Q i are the injected real power and the injected reactive power at bus i. Now let us see that what is the total number of unknown.

So to essentially understand that we have to now first understand that what are the unknowns at different buses. Now we have already discussed that there are 3 types of buses. So there are 3 types of buses and these 3 types of buses, one is slack bus one is PV bus one is PQ bus. At slack bus known are V i and theta i that = 0. So then unknown are actually nothing, nil. Because we are only trying to solve for the voltage magnitude and angle at all the buses.

And after that if we know this voltage magnitude and angles at all the buses we would be able to calculate anything and everything. So then therefore here we do not take that our unknown is P i and Q i. Rather we say that at how many buses voltage magnitude or angle or both are unknown. I mean that is what we are now trying to see. At PV buses we know that our known is V i and P i. So then unknown would be theta i and at PQ buses our specified quantities are P i and Q i.

So therefore my unknown would be V i and theta i. Now if I have got as we have already said, we always say that there are N- Bus system, M – generators. That is our convention. Bus 1 is slack, 2 to M are PV and M + 1 to M + 2 to N are PQ. So this is our convention. So then therefore theta i would be known. So then therefore here this theta i would be unknown for i = 2 to M and here V i and theta i would be unknown for i = M + 1 to N.

So then in total theta i are unknown for theta i to be calculated for all i if we do combine this is equal to 2 to N and V i to be calculated for all i = M + 1 to N. So then therefore total number of unknown theta, this is equal to (N – 1) here from here.

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Total number of unknown $V_i = (N-M)$

$\Rightarrow$ Total number of unknown $= (N-1)+(N-M) = \underline{2N-M-1}$

We denote our unknown vectors as

$$\bar{V} = \left[V_{M+1},\ V_{M+2} \cdots\cdots V_N\right]^T \longrightarrow (N-M)\times 1$$

$$\bar{\theta} = \left[\theta_2\ \ \theta_3 \cdots\cdots \theta_N\right]^T \longrightarrow (N-1)\times 1$$

$$P_i = \sum_{k=1}^{N} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = P_i(\bar{\theta}, \bar{V})$$

$$Q_i = \sum_{k=1}^{N} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = Q_i(\bar{\theta}, \bar{V})$$

And similarly total number of unknown V i = it would be (N – M). So it is (N – M). So then therefore total number of unknown = (N – 1) + (N – M) = 2N – M. Now we denote our unknown vectors as V. This is an unknown vector V and this is actually the unknowns are [V M + 1, V M + 2… V N] transpose. So this would be an (N – M) * 1 vector and theta this is an unknown theta. This is theta 2, theta 3 up to theta N T transpose. So it would be a column vector.

So it would be (N – 1) * 1. Now when we look at the expression of P i again I write it is V i V k Y ik cos (theta i – theta k – alpha ik) k = 1 to N. So we can see that in this expression of P i these quantities V M + 1, V M + 2, V N are already present as well as theta 2, theta 3, theta N are also present. So then therefore any expression of P i can be written as some function we also call it I mean with an abuse of the convention or rather with an abuse of the notation we call it that P i is a function of vector theta and vector V.

Similarly, we can write down that Q i which is K = 1 to N V i V k Y ik sin (theta i – theta k – alpha ik). So this also can be written as that it is also a vector of sorry it is also an function of the unknown vectors theta and V. So then therefore each of this quantity is also a function of unknown vectors. Now let us see now what we have already said that our total number of unknown is 2N – N – 1.

So now if we wish to solve for this total 2N – N – 1 unknown quantities we also should have this many number of equations. So now let us see from where we will get this many number of equations. Now equations would be only found out only from the known quantities. Now let us see that what are the known quantities, the known quantities.

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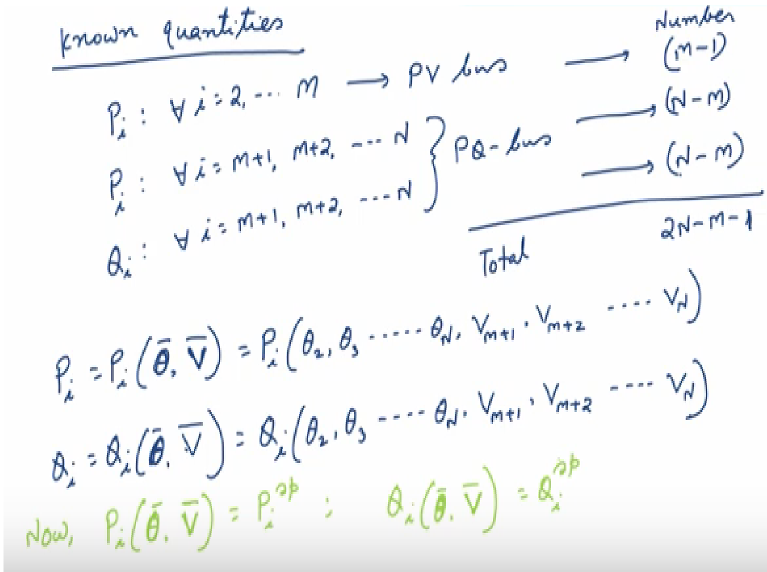

What are the known quantities we have? We have P i is known for all i = 2 to M. So this is for PV bus. And for Q bus and for PQ bus P i is known also for i = M + 1, M + 2 to N and Q i is also known for i = M + 1, M + 2 to N. So this is for PQ bus. So here the number is M – 1. Here this number is N – M. And here this number is N – M. So if I add them together, total comes out to be 2N – M – 1. So then therefore what we have got?

That we have got total number of equations that is total number of known quantities is also 2N – M – 1 and also my total number of unknown quantities is also 2N – M – 1. So then therefore this is solvable. Now the question is how do I solve it? Now we have said that P i is actually a function of vector theta and vector V. So then I can write down that P i is a function of theta 2, theta 3 up to theta N. Then V M + 1, V M + 2 to V N.

Similarly, Q i is also a function of this vector theta and vector V. So I can write down, if I expand this so this is a function of theta 2, theta 3 to theta N, V M + 1, V M + 2 to V N. So then what I have got? So I have got these are the my unknown quantities and these are my known quantities. Sorry, and basically these are my unknown quantities.

And these are my unknown quantities and my equations that is the functions are known which are basically or rather I do have some function which is a known value which is basically the function of this unknown quantity. So then therefore it fits exactly into the basic framework of the Newton – Raphson numerical method. So then what do I do? So as usual that we have to calculate the correction vector. Now in actual practice what we have?

That P i which is a function of this vector theta and V has got is equal to some value P i specified and Q i which is also a function of this theta and V, vector V is actually = Q i specified, correct? For example if I have a load bus, for example let us say bus 5 is a load bus and at that bus there is 100 MW real load and let us say 50 MVR reactive load so then if I do take our base MVA to be 100 MVA.

So then in that case what we will say that my P 5 specified would be = -1 per unit and Q 5 specified would be = -0.5 per unit. So then therefore these quantities would be known. Now, so then basically these are my equations. Now if I do expand these equations so then what do I get?

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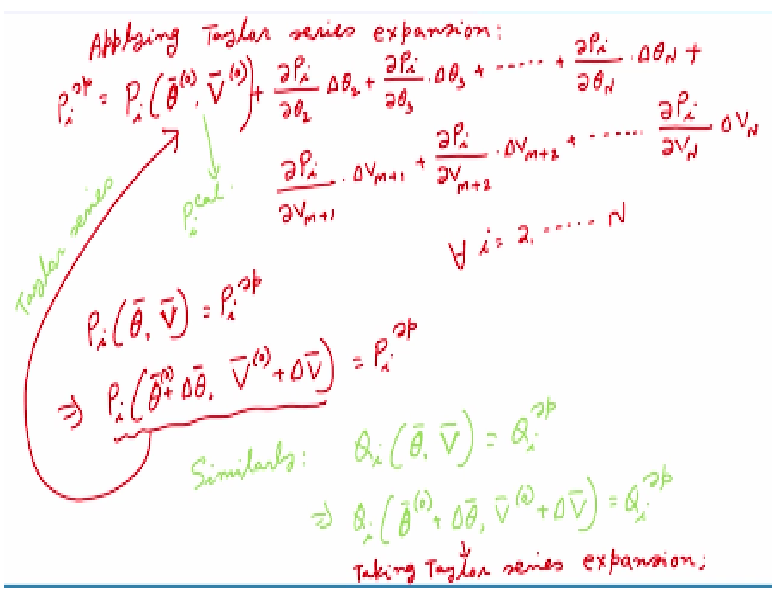

So applying Taylor series expansion, so now if I apply Taylor series expansion to all this, what I get? I get P i specified = P i(theta nought, P i V nought + del P i/del theta 2 * delta theta 2 + del P i + del theta 3 * delta theta 3… del P i/delta theta N * delta theta N + del P i/del V M + 1. This equation would be for all i = 2 to N because we can see that P i would be actually specified for 2 to N. So then if I apply this Taylor series expansion to this equation of P i we get this, right?

And theta 0 and V 0 are the initial conditions. Now basically from where this equation comes? Basically this equation comes from the fact that we have got P i theta so this is the equation which is a vector of theta and V = P i specified. So then if I wish to solve for it so then what I will get it. So then obviously I do not know theta, I do not know V but I want to solve for them. So then what I will do?

That I will assume some value of theta nought and I will assume some value of V nought and then I will actually basically calculate their correction vectors delta theta and delta V. So that should be equal to P i specified. So then therefore from here what I will get is P i theta + delta theta, so theta nought and V nought + delta V nought + delta V should be is equal to P i specified. And this term and this term has been expanded through Taylor series.

So this term has been expanded through Taylor series. So this is Taylor series, right? Similarly, we have equation is Q i which is a function of vector theta and vector V that is equal to Q i specified. So then again I take some initial guess plus some correction and I also take some initial guess here plus some correction. That would be equal to Q i specified. So now I do take, so now I again do take I mean do actually perform the first order Taylor series expansion of this expression as we have done here. So then if I do take by taking Taylor series expansion what we get?

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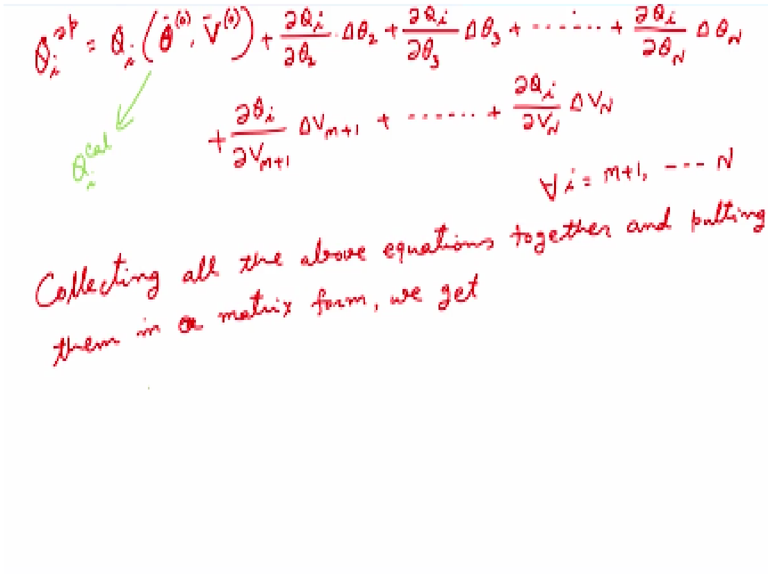

We get is that Q i specified = Q i is a function of + del Q i del theta 2 * delta theta 2 + del Q i plus this would be for i = M + 1 to N because we can see from here for Q i, i varies from M + 1 to N. So then therefore if I do collect all these equations together and do write them into a matrix form, so then what do I get. So collecting all the above putting them in a matrix form we get, what do I get? So what do I get? We get something like this.

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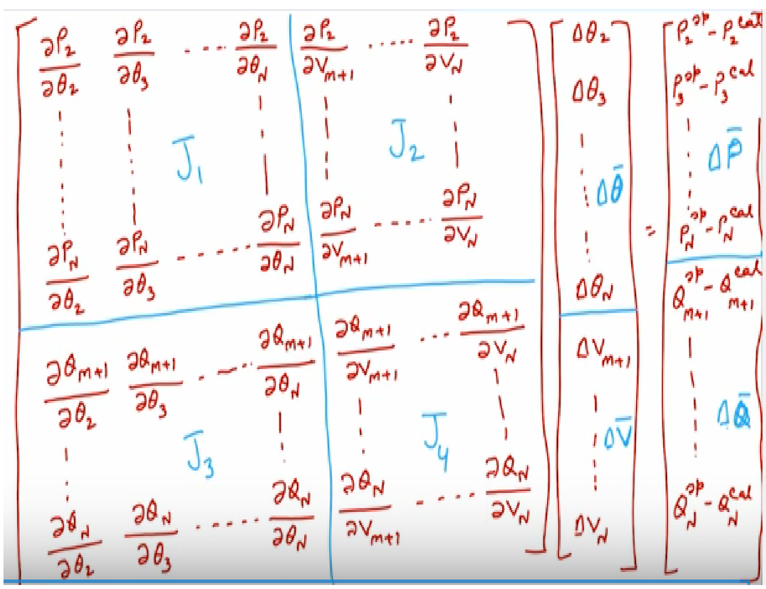

Del P 2/del theta 2, del P 2/del theta 3…del P 2/del theta N. Then del P 2/del V N + 1…del P 2/del V N. Then we do collect all this P s and we get del P N/del theta 2, del P N/del theta 3, del P N/del theta N. Then we get del P N sorry it would be del V M + 1…del P N/del V N. Then start the Q. Then del Q N/del V M + 1, del Q N/del V N. So this big matrix multiplied by delta theta 2, delta theta 3 up to delta theta N.

Then delta V M + 1, delta V N and this is P 2 specified – P 2. Now for our convenience we term this Q i calculated. Q i calculated means, what is meant by Q i calculated? Q i calculated means that I do have expression of Q i that is I mean V i V Q Y ik sin theta N – theta k – alpha i k. There in that expression I will substitute the values of theta nought and V nought. That is basically the initial guesses of all this thetas and all this V's, unknown V's.

And of course we already know the I mean what the other values of theta or V which are already known. So we do substitute these values of initial theta and initial V and then we calculate the value of Q i. That is basically to get a numerical value of Q i. So then this expression we say for the purpose of convenience as Q i calculated. Similarly, this also expression, this expression we also call as P i calculated.

Here also in that expression of P i we do substitute the values of initial values of theta and initial values of V and then after that we do get some numerical values of P i. So that is basically the calculated value of injected real and reactive power with the initial guess of angle and voltage. That is angle and voltage magnitude. So we write it as P 2 cal P 3 is P – P 3 cal. This P N specified – P N calculated. Then we have Q M + 1 specified – Q M + 1 calculated.

And then similarly Q N specified – Q N calculated. So this is what we get. So we have got a big matrix; big matrix here, big vector here, big vector here. So now let us partition them. So now what we get is we partition this big matrix like this. And we also partition this vector like this. And we also partition this vector like this. This matrix we call as del p/del theta matrix. So we call it this matrix, so this is the matrix J 1. We call it this matrix J 2. We call this matrix J 3.

We call this matrix J 4. This is a vector. We call this vector as delta theta vector. This is a vector we call it delta V vector. This is a vector. This vector we call as delta P vector. And this is a vector we call as delta Q vector Q bar vector. So then therefore with this terminology this entire matrix, big matrix can be written as more succinctly as

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$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{V} \end{bmatrix} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix}$$

[Matrix J 1, matrix J 2, matrix J 3, matrix J 4] then [delta theta vector, delta V vector] = delta P vector is equal to and delta Q vector. So in this lecture what we have done? We have first discussed what is meant by Newton – Raphson polar and rectangular method. And then after that we have specified that what are the unknown quantities for Newton – Raphson polar technique and as well as the what are the equations for this Newton – Raphson polar technique.

And subsequently we have linearized them by following the basic procedure of Newton – Raphson numerical method and by doing so and after so and after collecting all these equations together we have arrived at this succinct matrix equation. So in the next lecture and onwards we would be looking into the more details of this matrix equation. Thank you.