Computer Aided Power System Analysis
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Lecture - 01
Modeling of Power System Components

Hello, welcome to this first lecture of the module 1. In this particular module, we would be talking about the modeling of various power system components. Essentially, what we are trying to do is as follows. Essentially, in this course, we are interested to analyze the performance of a large grid and that large grid may contain any number of buses, any number of generators, any number of transformers, any number of transmission lines, loads, etc.

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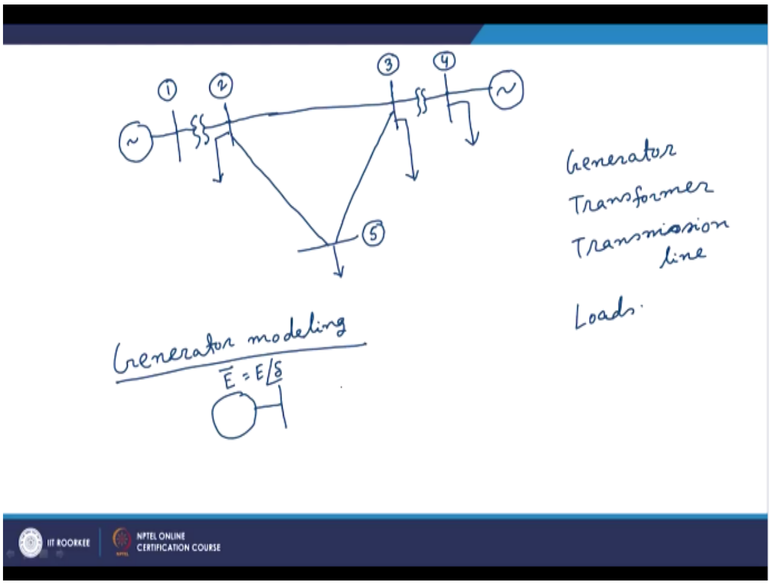

For example, if I have let say a very simple diagram, suppose there is one generator, it is connected to bus 1, then there is a transformer, then this transformer or after that there is a transmission line, then there is another transformer, then another bus, then there is load, some load and let us say this is bus 1, bus 2, bus 3, bus 4. Let us say this is bus 5 and there is also some interconnection between bus 3 and bus 2 and bus 5.

And of course there is also load at this point, there can be load at this point, there can be load at this point. So this is just a very typical instance of a general power system in which we do have one generator and also we can have say let us say we have also another generator here, we can have absolutely no problem. So in this case, we do have 2 generators, 2 transformers, 3 transmission lines, 4 loads, etc.

Now imagine the case that in our system let us say there are 100 generators, 200 transformers, 1000 lines, 300 loads, etc. So then how do you really perform the analysis? So for that, we have to actually find out the equivalent circuit of any particular power system network. Now to find out the equivalent circuit of any particular power system network, we have to first find out that what is the model of any particular component.

Now here we have got this several components, one is a generator, then we have got transformer, then we have got transmission line, loads, etc. So here we have to look at the modeling of each and every component individually until unless we do that we would be really at no position to do our power system analysis. So first we start with the modeling of a generator.

So generator modeling, now here depending upon the study we perform, any generator can be modeled in any kind of detail but then here in default the purpose of this particular course, we would be modeling a generator only by a simple voltage source. So any generator would be modeled by a simple voltage source connected at some bus and that voltage source would be given by let say something called E.

Please note that we have put an overbar on top of E denoting that it is a complex quantity that means it can have some magnitude and also can have some angle delta. Of course, depending upon other type of study, this generator can be modeled in much more detail but here in this course we are not really dealing with that, so then therefore we will not pursue the modeling of any generator to any further detail.

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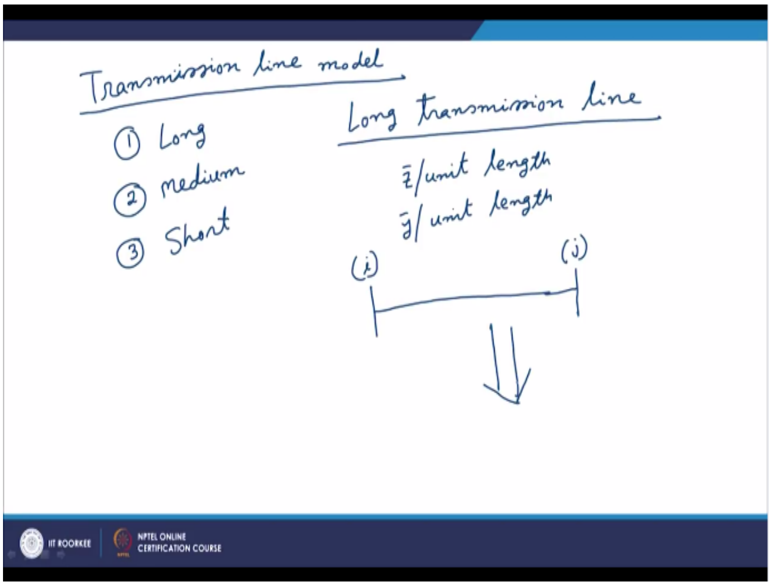

Let us now go to the transmission line. So transmission line model, now here in this course we are actually essentially interested in the analysis of a transmission grid. So then therefore depending upon the length of the transmission line, we all know that any transmission line can be classified into 3 categories; one is a long transmission line, second is medium transmission line and third is a short transmission line.

Now in the case of any high voltage transmission system, usually the lines are either of long nature or let us say they are either of medium length. Usually, short length transmission lines are not used in the high voltage transmission grid. So then therefore in this course, we would be primarily concentrating on the model of the long as well as the medium length transmission line.

But then of course we would be also covering the model of the short transmission line. So first let us talk about the long transmission line. We know that any long transmission line is actually represented by distributed line parameters. That means we say that any long transmission line have some impedance z per unit length and shunt charging susceptance y per unit length.

And this unit length can be per meter per kilometer anything but the point is that these impedance as well as the shunt charging susceptances or the admittances they are not really clubbed at one place, they are uniformly distributed over the length of the line. Now if we have to do our analysis by taking into account the uniformly distributed nature of these impedances and the admittances, then our analysis would be too much complicated.

So to avoid that what we do is that we do represent a long transmission line by its equivalent pi model.

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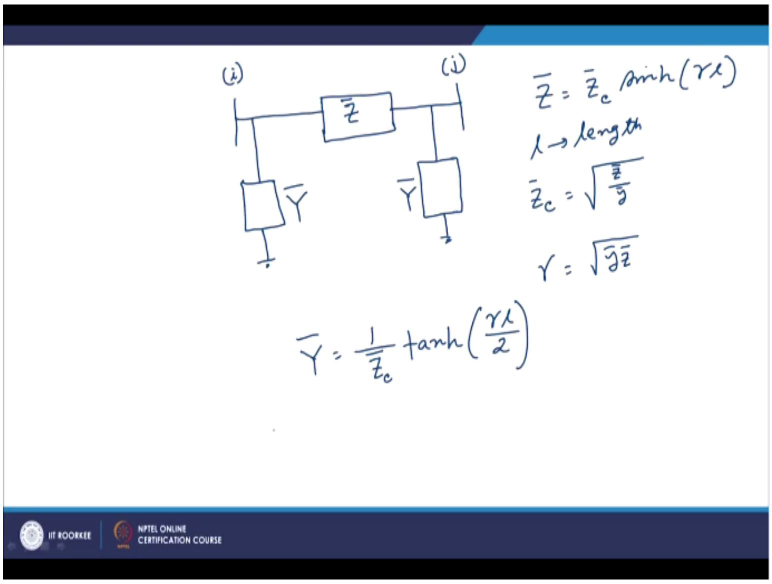

So then therefore if I have a long transmission line connected between bus i and bus j, then this long transmission line can be represented as an equivalent pi model like this. So there is one impedance which is in series between bus i and j and there is one shunt charging admittance at the two sides which are equal. Now this value z series is given by zc sin hyperbolic gamma l where l is the length.

This is the impedance, zc is root over z/y. Please note that this z is nothing but the impedance of the line per unit and this y is nothing but the admittance of the line per unit and there is gamma which is called proposition constant, this is given by root over yz and this y, this z are basically nothing but this as above. So this is the expression of the series part. What would be the expression of the shunt part?

Now when we are talking about the pi model of any transmission line, these two shunt parts are equal to each other. Now this shunt part, these are called if they say that if these are y and if this is also y, remember we are putting overbar to each and every quantity to denote that all these quantities are complex in nature. So then y is given by 1/zc. This zc is the same and this zc what we have just now defined, tan hyperbolic gamma l/2.

So then therefore you can see that if we know the per unit length impedance as well as the per unit length admittance of the long transmission line, then we can simply represent a long transmission line between bus i and j by its equivalent pi model and this equivalent pi model has got one series part and two shunt parts and these two shunt parts are equal in nature.

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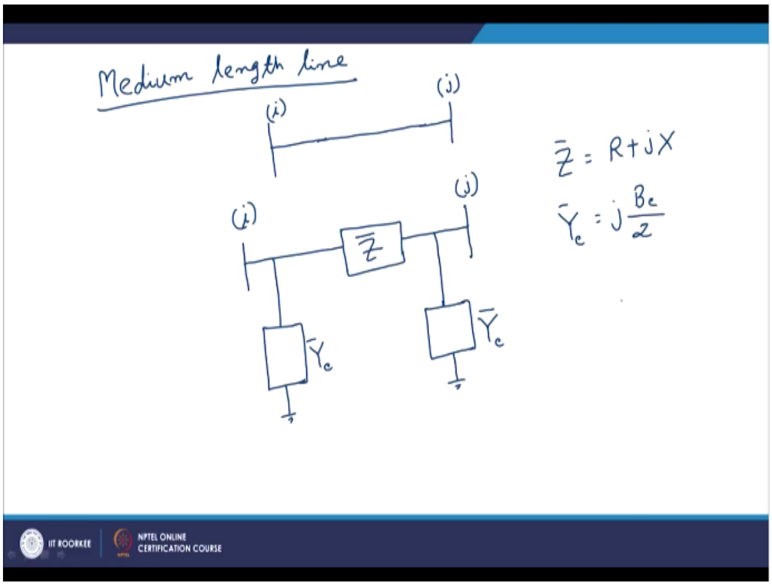

Now for the case of medium length transmission line, this model is very simple. So if I have a medium length transmission line between bus i and bus j, so this medium length transmission line also represented by a pi model like this. So this is bus i, this is bus j, this is z and this z is nothing but R that is the total resistance+jX, X is the reactance of the light and this yc, these are nothing but jBc/2 where Bc is nothing but the total shunt charging admittance of the line.

So then essentially what we are doing, we are simply dividing the total shunt charging susceptance of the line between two sides equally. So this is the very simple model. You may be wondering that why are you not really deriving this particular circuits, this is due to the fact that all this particular equivalent circuits are already covered in the very basic power system course.

So then therefore we are taking a small amount of liberty not to discuss the details of this models any further. Now let us go to the next case.

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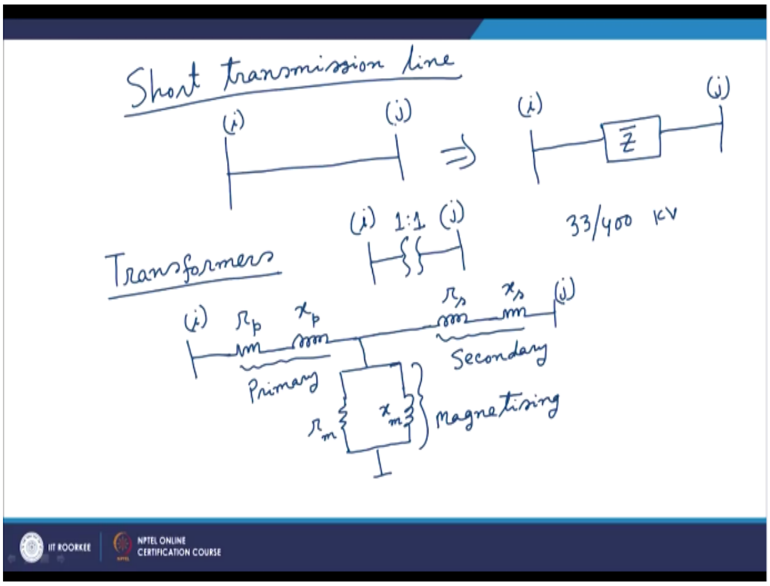

Short transmission line, this model is even simpler, so if I have a short transmission line between bus i and j, this is simply represented by only by a series impedance between bus i and j, there is no shunt charging susceptance between them because for any short transmission line, the total amount of shunt charging susceptance is pretty small. So then therefore for the purpose of our analysis, we can straight away simply neglect this which will not really cause much of an error in our calculation.

Now after we through with this model of this long, medium as well as the short transmission lines, let us go to the model of transformers. So we are talking about transformers. So then therefore what we have now for a transformer, we have say a transformer is connected between bus let us say i and j. For example, in the first slide, here we have got 1 transformer connected between bus 1 and 2.

And we have another transformer connected between bus 3 and 4, so similarly so now let us assume that I do have a transformer which is connected between bus i and j. Now from a very basic knowledge of any transformer, you know that any transformer can be represented by an equivalent circuit like this which has got something like this is the primary side resistance, primary side inductance.

Then, I do have magnetizing part, then I do have secondary side impedance. So this is bus i, this is bus j. So these are corresponding to the primary side, these are corresponding to the secondary side and these are nothing but the magnetizing. Now here we must note one thing that we are drawing this circuit in per unit because until unless we are drawing this circuit in per unit we really cannot connect this primary side as well as the secondary side together.

Because as we all know for all practical purpose if this transformer is rated let say 33/400 kV, so then therefore their impedance level or rather this actual absolute value of impedance level would be totally different, so then obviously we really cannot connect them together and also these two impedances at the primary side as well as the secondary side, they are also at the different voltage level.

So then therefore we really cannot connect them together but then when we are converting any transformer equivalent circuit into per unit so then in per unit we can draw this particular circuit. So we have got primary side, secondary side as well as the magnetizing part. Now we will know that for all practical purpose, the current as well as the power drawn by this magnetizing branch is pretty small right.

So if this current drawn by this magnetizing part is pretty small, so then therefore for the purpose of our engineering analysis we can simply neglect this part.

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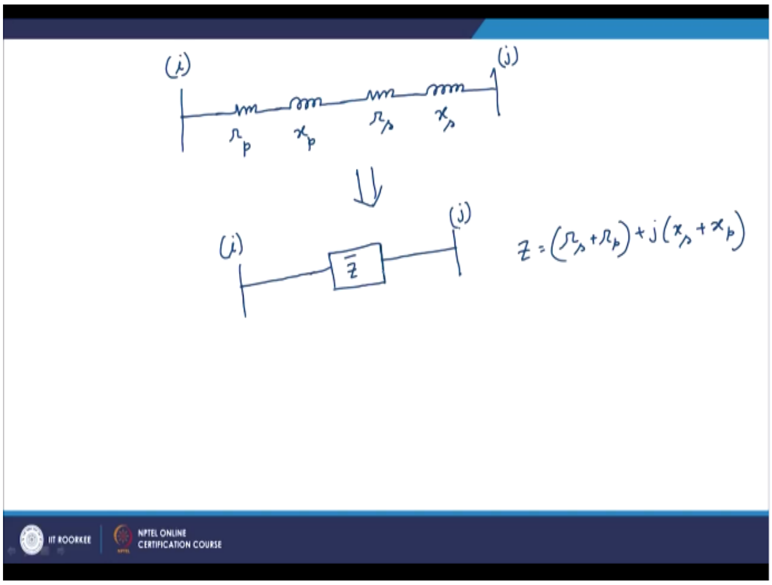

So if we do neglect this part, so then we have this equivalent circuit, so what we have is bus i and from okay here I should also let us say this is rp, r stands for resistance, p stands for primary, this is xp, x stands for reactance and this is rs, this is xs and this rm, m stands for the magnetizing and xm. Now if we do neglect rm and xm because they are drawing very little amount of current.

So my equivalent circuit becomes simple, the series connection of so it is rp, xp, rs, xs. This is bus i, bus j. This we can simply combine them together because they are in series, so we can have bus i and bus j, so I have got some transformer, reactants or other impedance z. This is bus i, bus j and this z is given by rs+rp+j xs+xp. So for all practical purpose, the transformer model is very simple.

It is just simply represented as simple impedance connected between bus i and j and this value of impedance is nothing but the total resistance of the primary+secondary side and the total reactance of the primary as the secondary side. We must note one thing that here when we are drawing this equivalent circuit of the transformer, we are drawing them in per unit and also we are drawing this circuit either reflected at the primary side or reflected at the secondary side.

However, all these things are already covered in the very basic power system course, so we are not really going into the details of this. So here we again note that rs, rp, xs, xp these are all per unit quantities. Now here when we are doing this model of this transformer, we have assumed that this transformer has got a tap ratio of 1:1.

So this particular model whatever we have got represented has got a tap ratio as 1:1. What does it mean by this tap ratio 1:1? It means that if I have a one per unit voltage at bus i, so then at bus j also we will have one per unit voltage. So this is the meaning of this tap ratio 1:1. Now in many cases or rather it is nowadays becoming more and more common to control the voltages of the transformers.

Or rather to essentially control the voltages of the transmission grid, we often put tap changers on the transformers either at the primary side or at the secondary side and to control the voltages of the transmission grid, we do adjust these taps wherever it is put such that the effective voltage at the primary side or at the secondary side, they are either increased or decreased depending upon the requirement.

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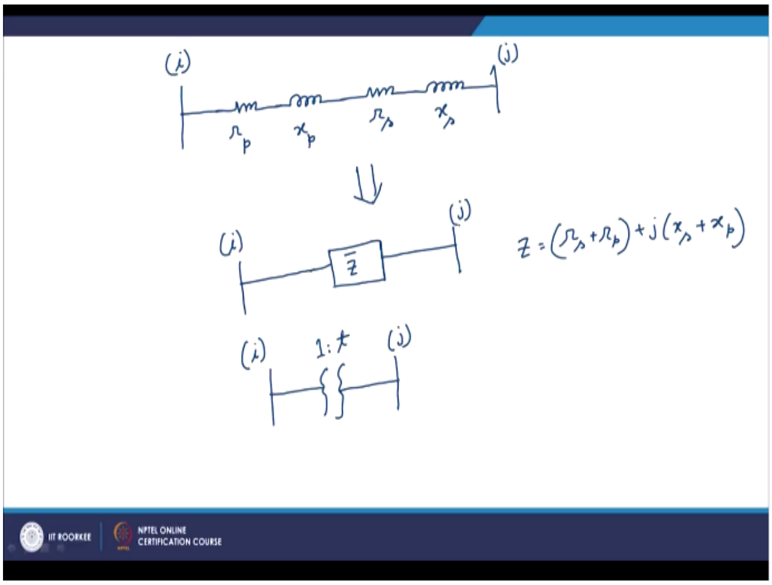

So then therefore let us take a case that I do have one transformer connected between bus i and bus j and this transformer instead of having a tap ratio of 1:1, it has got now the tap ratio of say 1:t where t is not equal to 1. So what does it mean? It means that if we put 1 per unit voltage at bus i, at bus j I will get t per unit voltage not exact equal to 1. So now we have to see that what would be the equivalent circuit of this tap changing transformer.

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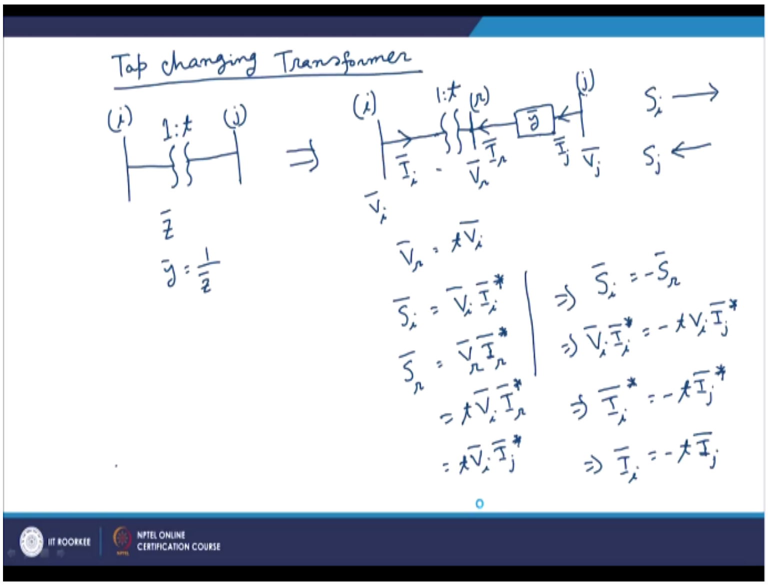

So let us do that, so we are going to discuss about this model of the tap changing transformer. So what we have as we have just now said bus i between bus i and bus j there is one transformer which has got 1:t, this is bus i, this is bus j and let us say that this transformer has got an impedance of z. So then therefore its admittance is y which is given by 1/z.

So now what you can do is so we can draw the equivalent circuit of this transformer like this, so I have got bus i, then y, this is bus j and this has got a transformer tap is 1:t. Now because this transformer has got now a tap ratio of 1:t, so then therefore if I have got a voltage Vi here, I will have a voltage here at this point let us say this point is r, let us create an artificial bus here which is called bus r.

So I have got now 3 voltages Vi, Vj and Vr, so then therefore I can say that Vr is=t*Vi because this transformer has gotten 1:t tap change new ratio. If I say that this current from bus is i is Ii and the current from bus j in this direction is Ij. So then therefore this complex power Si is Vi*Ii star and complex power S bus j is given by Vj*Ij star where Ii star and Ij star are nothing but the complex conjugate of Ii and Ij respectively.

Now here we are making a small assumption that we are making an assumption that this transformer is lossless. So if this transformer is lossless, so then therefore whatever power we are having here now and also this now here in this case because this transformer is lossless and because we have already taken its impedance outside, impedance or admittance are outside in this equivalent circuit.

So then therefore this transformer can be considered to be an ideal transformer. That means that it does not have any internal impedance. So then therefore total complex power at this point would be equal to the total complex power at bus r. So then therefore total complex power at the primary side of this transformer would be equal to the total complex power at bus r. Now if we look at the direction of Ii and Ij, we can see that that the direction of Si and is Sj are nothing but opposite to each other.

Si is actually in this direction, so Si I am just writing Si is in this direction and Sj is in this direction. So then therefore we can write down that because it is ideal so then therefore Si is=-Sj, remember Si is=-Sj. So then therefore I can write down and now here actually this should be a little there is little correction, it should be Sr. Now here we can write down Vr is=tVi. So tVi*Ir. Now tVi*Ij because you can see from this diagram that Ij and Ir if I say that this is Ir so Ij and Ir they are equal to each other.

So now from this expression if Si is=Sr so then therefore we can write down Si is Vi*Ii star is=-tVi. So then therefore from this equation Ii star is=-tIj star. Now because t is a real quantity, so then therefore I can write down that Ii is=-tIj. Now from this equation, we can find out, so now what we have to find that we have to now find out the equivalent circuit.

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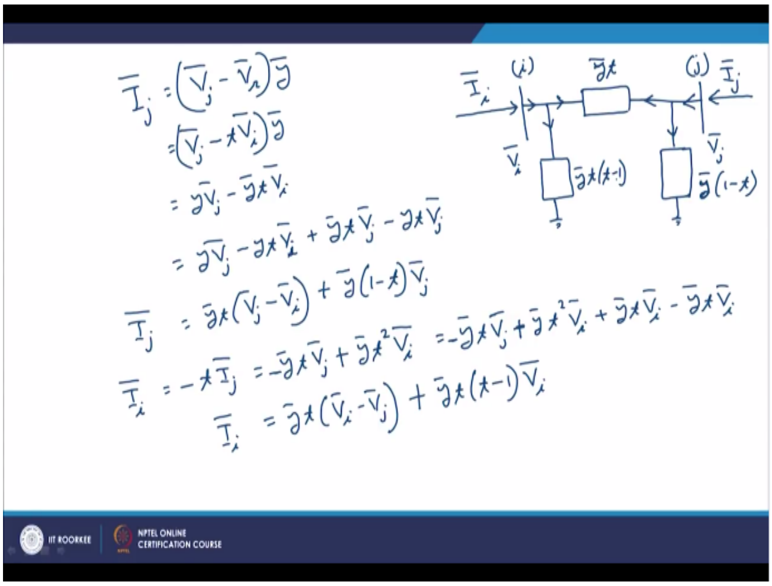

So now from here I can write down from this equation Ij is=Vj-Vr*y. So from this circuit Ij is this voltage-this voltage*this admittance. So this is Vj-Vr we have already said it is tVi, so tVi*y so it is yVj-y tVi, it is yVj, we add and subtract y tVj. So we take these two together, the sign this two together, so we get this+y*1-t*Vj. So this is the expression of Ij and Ii is= -tIj, so it is from here y tVj-y t square Vi.

We again subtract y tVi, subtract and add y tVi so y tVj-y t square Vi+y tVi-y tVi. So now here there was some mistakes, so Ii is=-tIj so it should be –yt+ so it is minus, this is plus and then we are adding and subtracting. So now it is okay, so now what we take that this and this together so what we get is take y tVi-Vj+y t*t-1*Vi. So this is the two expressions of Ij and Ii. So utilizing these two expressions, we are now in a position to find out the equivalent circuit of this tap changing transformer.

So what we have so let us do it here. So bus i, this is bus j and let say the current entering at this point is Ii and current entering this point is Ij. So this is bus j, this is bus i. Now we can see that this current entering at this point has got two parts, one part is due to yt*Vj-Vi and another part is y*1-Vj, so then therefore if I put an impedance here, sorry admittance which is equal to yt, so then therefore this current here would be equal to yt*Vj-Vi.

So this voltage has got Vi, this voltage has got Vj and the shunt part I can put this would be equal to y*1-t. So therefore this Ij will have now two parts, one part is going towards bus i which is given by Vj-Vi*yt that is this part and the another part which is going to shunt that is y into that is actually Vj*y*1-t. If we look at Ii, so for Ii, so this Ii is also going here and then I have got another shunt part at this point.

Now this current will have two path, one is going to shunt and one other is going through series towards bus j. The current which is going towards bus j is given by yt*Vi-Vj and we have already put this yt and obviously this current is nothing but yt*Vi-Vj and the shunt current would be given by Vi*yt*t-1. So then therefore this shunt admittance would be given by yt*t-1.

So this is the equivalent circuit of this tap changing transformer. So here in this lecture, so far we have covered the modeling of the generator, although this is very simple. Then, we have talked about the model of the long transmission line, medium length transmission line and the short transmission line followed by the model of the two winding transformer. First, we have covered the model of the simple, a nominal tap changing transformer and followed by you have also covered the model of an off-nominal tap changing transformer. Thank you.