

**Electrical Distribution System Analysis**  
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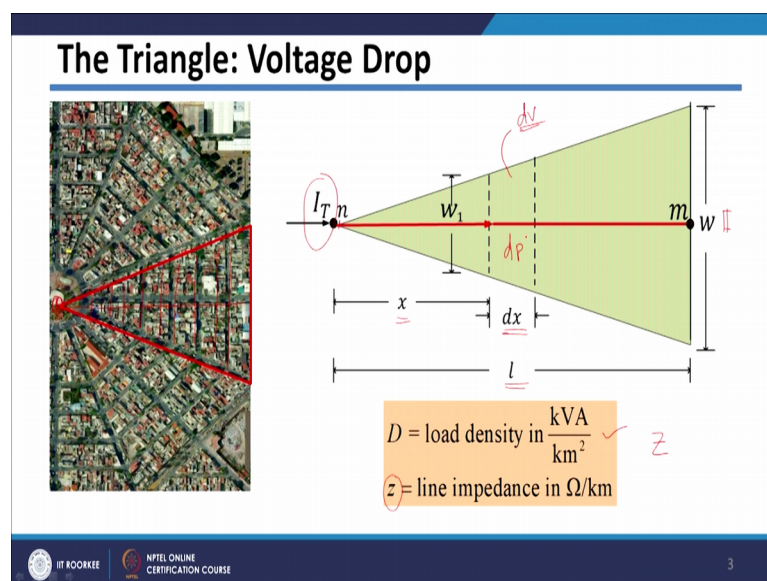
**Lecture - 09**  
**Lumping Loads in Geometric Configurations: Triangular**

Dear, students welcome to this 9th lecture of the course, Electrical Distribution System Analysis and, title of today's lecture is Lumping Loads in Geometric Configuration. And we are going to see triangular configurations configuration today.

In the last class we have seen the rectangular configurations, where we have tried to derive the voltage drop and power loss equation for rectangular configurations. And we have seen that for voltage drop calculation, we can lump the load at the middle and, for power loss calculation if we can lump the load at one third of your length of your feeder.

We also seen the application of it, where we have found out suitable voltage level for feeder. So, that drop will be within the limit of plus minus 3 percent. In today's lecture we will see triangular configuration, as we have seen in the last lecture various geometrical configurations can be used for getting approximate value of voltage drop and power loss.

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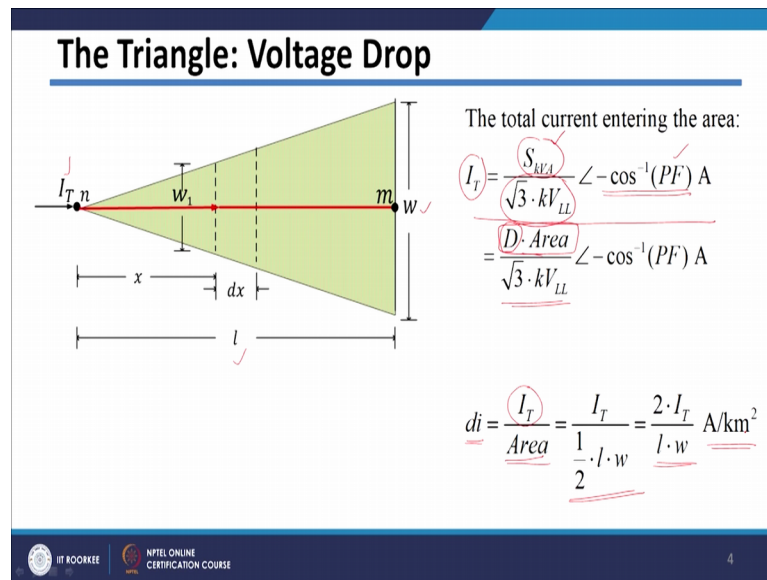
In this case consider this a triangle area here, we can assume that substation is somewhere here, and then there are actually primary feeder which is going in this area. And then there will be laterals which are connected to the primary feeder and, this laterals are supplying power to the various consumers, which are inside this particular area.

To do the approximate analysis as I explained in the last class we are assuming load density over this triangular configuration is constant. So, this constant load density as I explained in last class we can say it is kVA per kilometer square area and, impedance of this feeder is in ohms per kilometer, which I represented in I am representing by small  $z$ . And as I explained in last class there is capital  $Z$ , I am using for total impedance of the feeder.

So, small impedance small  $z$  is for impedance per kilometer and, capital  $Z$  is for total impedance of the feeder. So, in triangular configuration also, I am considering the length of the feeder is  $l$  here, and the width at the end of this triangle is  $w$ . And the total current which is going inside this triangular area is  $I_T$  and, as I told you we are interested in calculating voltage drop from this point  $n$  to  $m$ , as well as power loss from this point to  $n$  point  $l$  to  $m$ .

And as we have seen in last class for rectangular area here, also we will try to find out  $dv$ , which is voltage drop which is happening in small strip of length  $dx$  at a distance  $x$  from the source end. And then after integrating this  $dv$  for the length  $l$  we can get total voltage drop. Similarly we will calculate power loss  $dp$ , which is happening in this small strip and after integrating it from  $0$  to  $l$  we will get total power loss.

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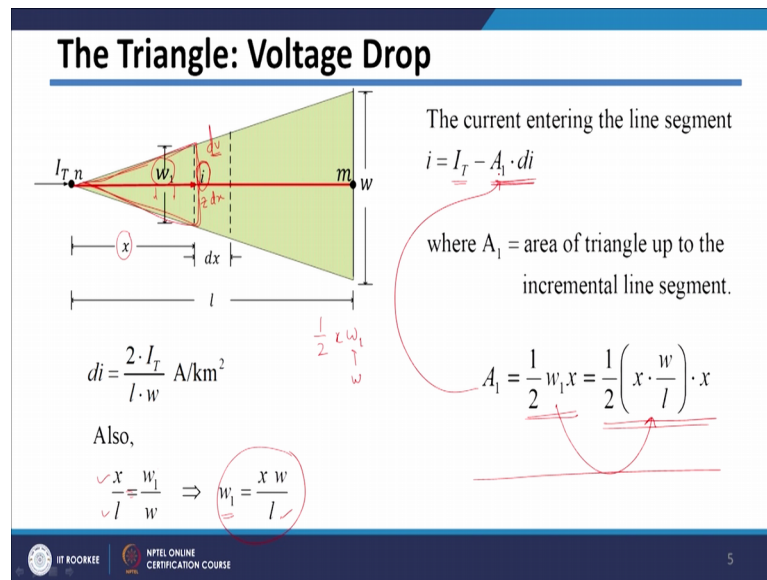


So, let see the voltage drop first. So, if you know the total kVA, we can easily get the current which is entering to this particular feeder, which is given by these equation here, which is just your kVA divided by root 3 times your line to line voltage. And then there is cos inverse of power factor where this power factor is average value of power factor over your distribution area.

In this case this total kVA supplied will be nothing but load density which is kVA per kilo meter square into area, will give me total kVA which is supplied in this particular area. So, we can get total current which is entering to this area by knowing the load density. Once I get that I can get the current density which is nothing but total current divided by area. So, in case of triangular configuration your area will be 1 half l multiplied by w, where l is length and w actually width at the end.

So, it will become twice of it divided by l multiplied by w. And, since it is current density it will be in ampere per kilometer square.

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As I told you in last class we are interest in calculating your  $dv$  and to calculate  $dv$ , we want to calculate current and then we want to calculate impedance, which is  $z$  multiplied by  $dx$  and real part of  $i$  multiplied by  $z \cdot x$  will give me voltage drop in that small strip which is  $dv$ . So, to calculate that we first need to calculate current which is entering to this small strip. And, then till this point or till this strip  $dx$  here, some current will get supplied in this area means current will go on decreasing.

So, if you calculate that current  $r$  here, so, total current minus current which is supplied in this triangular area just before your strip is  $A_1$  multiplied by your current density area, multiplied by current density will give me current which is supplied in that particular area, which is just before your strip.

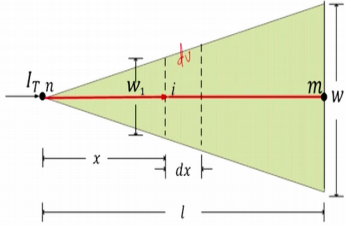
Let us see how to get this  $A_1$  here, to get  $A_1$ , we need to know  $x$  and we need to know  $w$  so, it will be just  $1/2 \cdot x \cdot w_1$ . And this  $w_1$ , we can write in terms of  $w$  for that we can just consider this here. So,  $x$  by  $l$  they will be equal to  $w_1$  by  $w$  because, they will be proportional. So, proportionality of  $x$  and  $l$  will also remain in  $w_1$  and  $w$ . So, from this I can just write  $w_1$  will be equal to  $x$  divided by  $l$  multiplied by  $w$  divided by your  $l$  here.

So, I can get  $w_1$  in terms of  $w$ . So, from this I can write equation for area. So, area will be  $1/2 \cdot w_1 \cdot x$  and  $w_1$ , we have written in terms of  $x$  and  $w$  which is write it here. And then this will be nothing but your area this will be nothing but your area of triangle

just triangular area just before your strip, from that we can actually put this  $A_1$  into this equations so, that I can get the equation of current.

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### The Triangle: Voltage Drop



$$di = \frac{I_T}{Area} = \frac{I_T}{\frac{1}{2} \cdot l \cdot w} = \frac{2 \cdot I_T}{l \cdot w} \text{ A/km}^2$$

and  $A_1 = \frac{1}{2} \left( x \cdot \frac{w}{l} \right) \cdot x$

$$i = I_T - A_1 \cdot di$$

$$= I_T - \left( \frac{1}{2} \cdot \frac{w}{l} \cdot x^2 \right) \cdot \left( \frac{2 \cdot I_T}{l \cdot w} \right)$$

$$= I_T \cdot \left( 1 - \frac{x^2}{l^2} \right)$$

Therefore,

$$dv = \text{Re}[z dx \cdot i] = \text{Re} \left[ z dx \cdot I_T \left( 1 - \frac{x^2}{l^2} \right) \right]$$

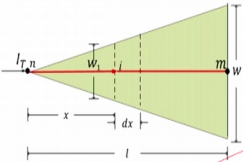
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So, has I told you this  $A_1$  triangular area I am putting into this equation here so, finally, it will be current equation will be  $I_T$  minus this is nothing but your  $A_1$  and this is nothing but your  $di$  which you have written here. So, this is your  $di$  and this is your  $A_1$ . And if you simplify it your this will become 1 square here, and this is square will remain here,  $w$   $w$  will get canceled out and, if you take it out that is 2 also will get cancel out.

If you take  $I_T$  out I will get equation for current which is this one. And then I can easily write the equation for  $dv$ , which is voltage drop which is happening in this small strip which is real part of impedance  $z dx$ , multiplied by your current. And current equation which we have got it here I can just put here so, it will be  $z dx$  multiplied by your current which you are entering into this strip.

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### The Triangle: Voltage Drop



$$dv = \operatorname{Re} \left[ z \, dx \cdot I_T \left( 1 - \frac{x^2}{l^2} \right) \right]$$

The total voltage drop from node n to node m is

$$V_{drop} = \int_0^l dv = \int_0^l \operatorname{Re} \left[ z \, dx \cdot I_T \left( 1 - \frac{x^2}{l^2} \right) \right] = \operatorname{Re} \left[ z \cdot I_T \cdot \int_0^l \left( 1 - \frac{x^2}{l^2} \right) \cdot dx \right]$$

$$= \operatorname{Re} \left[ z \cdot I_T \cdot \left( x - \frac{x^3}{3l^2} \right) \right] = \operatorname{Re} \left[ z \cdot I_T \cdot \left( l - \frac{l^3}{3l^2} \right) \right] = \operatorname{Re} \left[ \frac{2}{3} Z \cdot I_T \right]$$

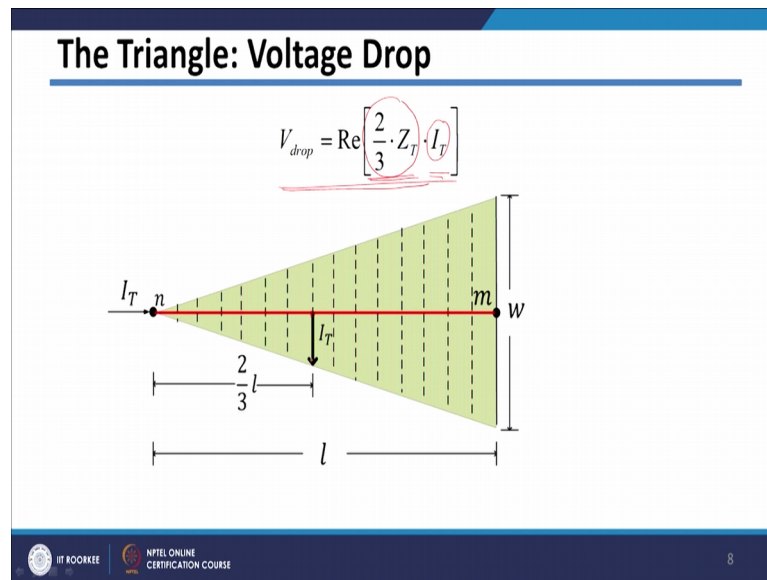
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So, to get total voltage drop across the feeder or till the end of the feeder, we need to integrate this  $dv$  from 0 to 1. So, total voltage drop across the feeder will be integration of this  $dv$  from 0 to 1, equation of  $dv$  we obtained it here I can put into this expression I will get this equation here, and in this case also your  $z$  and  $I_T$  they are constant, I can take it out remaining term they will be getting integrated.

So, they will be under integration and, if you calculate the integration of this integration of this will 1 will be  $x$ . And integration of  $x$  square by 1 square will be  $x$  cube by 3 1 square. And the limits of integration from 0 to 1, when you are putting it 0 it will be 0 only and, then when you are putting 1 it will be 1 minus 1 cube by 1 square. So, basically it will be nothing but 1 minus 1 by 3. So, it will become 2 by 3 and this there will be 1 1 here, which is multiplied by your  $z$  will give me total impedance.

So finally, this will be nothing but your total impedance. And the total voltage drop which is happening will be 2 third  $z$  multiplied by your,  $I_T$ .

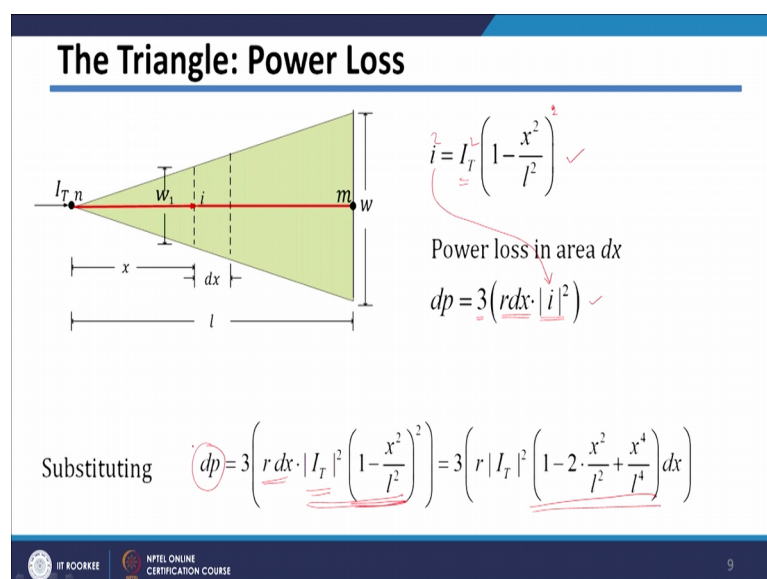
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So, from this equation we can easily find out, the lump load model of triangular configuration. So, this is your voltage drop equation and, I can see that this 2 by 3 into Z total Z multiplied by your, I T.

So, at this tells me at 2/3rd distance; I need to lump total current. So, that is why at the two third distance I lumped total current and, this will give me correct voltage drop. And, as we have seen this different load currents can be the lumped at different location like earlier cases.

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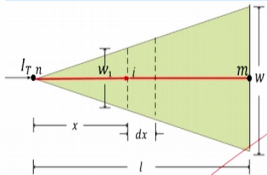
Now, let us see the power loss equation so, current equation, which we have derived which is entering into the strip, which is given by these equation we already seen it. Then power loss which is happening in this small strip will be nothing but 3 into resistance of the strip which is  $r \, dx$  and current in thing into the strip which is  $i$  square.

And we already get the current we can put into this equation, which will give me  $dp$  which will be equal to this is nothing but your resistance  $r \, dx$  and, current square so, current square means it will be  $I_T$  square multiplied by this bracket square here.

So, when you are taking  $x$  square, it will be whole square. So, it will be it square and this bracket square here. So, if you square this bracket here, I will get this term here so, this is nothing but your power loss which is happening in this is small strip area.

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### The Triangle: Power Loss



$$dp = 3 \left( r |I_T|^2 \left( 1 - 2 \cdot \frac{x^2}{l^2} + \frac{x^4}{l^4} \right) dx \right)$$

The total three-phase power loss from node n to node m becomes

$$P_{loss} = \int_0^l dp = \int_0^l 3 \left( r |I_T|^2 \left( 1 - 2 \cdot \frac{x^2}{l^2} + \frac{x^4}{l^4} \right) dx \right) = 3 \cdot r \cdot |I_T|^2 \cdot \int_0^l \left( 1 - 2 \cdot \frac{x^2}{l^2} + \frac{x^4}{l^4} \right) \cdot dx$$

$$= 3r |I_T|^2 \left( \left( x - 2 \frac{x^3}{3l^2} + \frac{x^5}{5l^4} \right) \right)_0^l = 3r |I_T|^2 \left( l - \frac{2}{3}l + \frac{l}{5} \right) = 3 \left( \frac{8}{15} R |I_T|^2 \right)$$

And as I told you to get the total power loss, you need to integrate this  $dp$  from 0 to  $l$ . So, in this case we need to integrate this  $dp$  here, so, I can put this equation of  $dp$  into this expression, I can get this expression here. And in this case also your  $r$  and  $I_T$ , they are constant I can take it out of integration. So, what will happen is we need to integrate this term here from 0 to  $l$ .

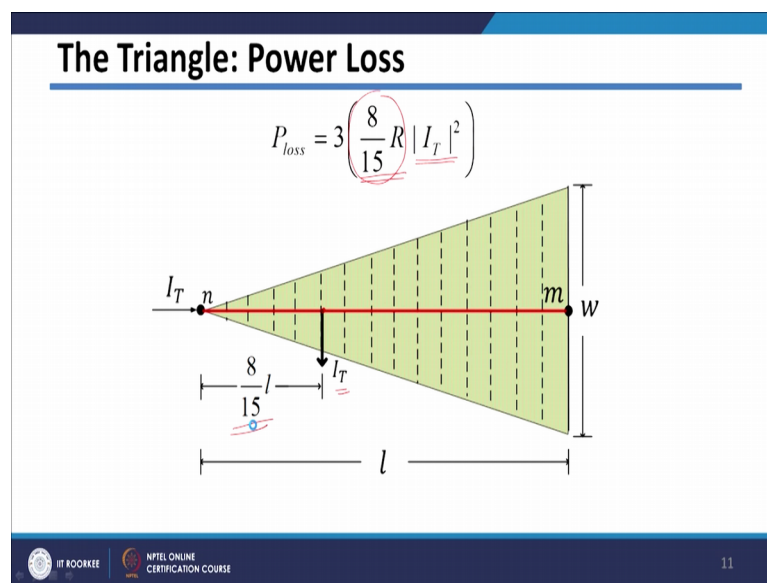
And if you take the integration of this terms integration of 1 will be  $x$ , integration of  $2x$  square by  $l$  square will be  $2x$  cube by  $3l$  square here and integration of  $x^4$  by  $l^4$  will be  $x$  raise to 5 divided by  $5l$  raise to 4.



And in this case your integration of limit is from 0 to 1 and, if you put this limits into this expression, I will get this simplified term here and, this term is nothing but your eight by 15 into 1. And this total term will be 8 by 15 into 1 and, this 1 into this resistance will be come your total resistance of the feeder which is nothing but your R here, and I T square will remain as it is.

So, overall power loss which is happening in triangular configuration is given by this particular equation.

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And from this expression, you can easily find out location of your lump load. So, from this we can see that at 8 by 15th distance, or resistance value of 8 by 15th, I can lump total current. So, total current will be lumped at 8 by 15th distance.

So, we have got actually we have got the voltage drop equation, we have got power loss equation.

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## Example

- $l = 4.5 \text{ km}$
- $w = 1.8 \text{ km}$
- $A = l \times w = \text{area} = 4.14 \text{ km}^2$
- $D = 1351.8 \text{ kVA/km}^2$ , pf 0.9 lag
- Total kVA =  $D \times A = 5596.45 \text{ kVA}$
- $K_{\text{drop}} = 0.0002191 \text{ \% drop/kVA-km}$
- $K_{\text{rise}} = 0.0002504 \text{ \% rise/kvar-km}$
- Nominal voltage =  $11 \text{ kV}$
- Find the location of 1800 kVAr capacitor bank to keep the voltage drop within  $\pm 3\%$  at the end.

The diagram illustrates a triangular load distribution on a transmission line. The load is represented by a green triangle with a base width  $w$  and a total length  $l$ . The current at the receiving end is  $I_r$  and at the sending end is  $I_s$ . The distance from the receiving end to the capacitor bank is  $\frac{2l}{3}$ , and the distance from the capacitor bank to the sending end is  $l$ . Handwritten notes indicate  $4.5 \text{ km}$  for the total length and  $1.8 \text{ km}$  for the width.

Let us see this example here. In this triangular area I have considered length of the feeder which is 4.5 kilometer. So, this is 4.5 kilometer, this is width is 1.8 kilometer. Therefore, area of the triangle will be 1 half length into width of the triangle, which will be equal to 4 by 4.14 kilometer square, which is area of the triangle. Then let us assume the load density into this triangular area is 1351.8 kVA per kilometer square with 0.9 power factor lining. Again this is average power factor over this area, then total kVA will be load density multiplied by area.

So, we have got area here, we have got this density here. So, density multiplied by area will be nothing but total kVA which are served into this particular triangular configuration. So, if you calculate that it comes around 5596.45 kVA. Let say K drop factor of this feeder is 0.0002191 percent drop per kVA per kilometer. And K rise factor is 0.0002504 percent rise per kvar per kilometer. So, K drop factors and K rise factors are given here, let us say nominal voltage of this feeder is 11 kV. And in this particular example, we are interested in finding the location of capacitor bank.

So, suppose I have capacitor bank of 1800 kvar, and I want to put this capacitor bank somewhere in this feeder, such that voltage at this end will be within the limit for given kVA, means to supply this kVA there will be a voltage drop from one end to another end across this feeder. And I want to put this capacitor bank somewhere, along the feeder. The capacitor bank which I am having is 1800 kvar.

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### Solution

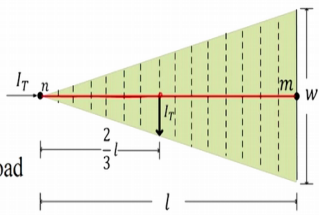
The total load of the triangular area is



$$kVA = 5596.45 \angle -25.84^\circ \text{ kVA}$$

Using the  $K_{\text{drop}}$  factor and lumping the total load

$$V_{\text{drop}} = K_{\text{drop}} \times kVA \times \left( \frac{2}{3} \times \text{dist} \right)$$

$$= 0.0002191 \times 5596.45 \times \left( \frac{2}{3} \times 4.6 \right)$$

$$= 3.76 \% \quad \checkmark \quad \pm 3\%$$




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Let say let see how we can find out this location here. So, total load of triangular area which we have calculated on the last slide is given here. Now, using K drop factor we can calculate actual drop without capacitor. So, actual drop so, K drop factor we know that kVA percentage drop per kVA per kilometer. So, if you can multiply this K drop factor by kVA and kilometer, we can get voltage drop

In this case since I am using lumped model, we see that we need to take see the total load is lumped at 2/3rd distance, we need to take distance to be 2/3rd here. So, 2/3rd of total distance is taken here. So, K drop factor is 0.0002191 which is given kVA serve, in this area which is also given. And then distance is we have seen it is 4.6 kilometer and, we have to take 2/3rd of that distance and if you can calculate, it comes around 3.76 percent. And we can see that we want this within the limit of plus minus 3 percent. And here the limit of plus minus three percent is getting violated.

Now, to avoid that we need to rise the voltages over the feeder and, those voltages can be raised by putting the capacitor along the feeder. And we were like to calculate the location of that particular capacitor.

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### Solution

$K_{rise} = 0.0002504 \text{ \% rise/kvar-km}$

$K_{rise} = \frac{V_{rise}}{dist \times kvar} \Rightarrow dist = \frac{V_{rise}}{K_{rise} \times kvar}$

The required voltage rise due to the capacitor is

$$V_{rise} = V_{drop} - 3.0 = 3.76 - 3.0 = 0.76 \%$$

The distance from node n is determined by:

$$dist = \frac{V_{rise}}{K_{rise} \cdot kvar}$$

$$= \frac{0.76}{0.0002504 \times 1800}$$

$$= 1.69 \text{ km}$$

Voltage drop due to the load is 3.76 which we have got on the last slide however, it is violating that limit. We want to keep this within the three percent limit so, 3.76 minus 3. So, we want rise of 0.76 percent which is getting by violated. For rising this voltage by 0.76 percent at the end, we need to put this capacitor. And to put this capacitor we can see we can use your K rise factor which is given here, which is 0.0002504 percent rise per kvar per kilometer and, by definition your K rise factor is V rise, in percentage divided by your distance multiplied by kvar.


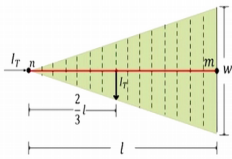
So, we can see that I can easily write distance will be equal to V rise divided by your K rise multiplied by kvar. So, by using this equation here distance can be easily recalculated so, the rise which you want by this capacitor is 0.76 percent, K rise factor is given and the capacitor which I am having is 1 1800 kvar. So, if you calculate that it gives me the distance is 1.69 kilometer means, we need to put the capacitor at a distance 1.69.

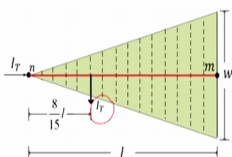
So, say let us say this is 1.69 kilometer and, and it put this 1800 kvar capacitor at the location. And because of presence of this capacitor at this place voltage at the end here, we will get rise by 0.76 percent. So, that it will be within in the limit of plus minus 3 percent. So, it will be get rise by this value and we will get actually required voltage limits without violation.

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### Summary

- Lumping Loads in Geometric Configurations
  - Triangular


$$V_{drop} = \text{Re} \left( \frac{2}{3} Z_T I_T \right)$$


$$P_{loss} = 3 \left( \frac{8}{15} R |I_T|^2 \right)$$


- Application

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So, in this particular chapter, we have seen the geometrical configurations which is triangular. So, if your area is having triangular kind of configuration, which I have shown on the right hand side figure many times this kind of triangular configuration existing your distribution system. So, in that case we have seen that we can lump this load at one location. So, in case of triangular configuration we need to lump total current at 2/3rd length for voltage drop calculation. And for calculation of power loss, we need to lump this load at 8/15th distance and, lumped load will be actually total load.

So, from seeing this equation we can easily find out these lump models, for triangular configuration. Over also seen the applications of this equations which we have derived so, that we can find out the correct location of the capacitor, if I am having some capacitor with me, I can find the correct location of that capacitor. It can be also used to find out the size of the capacitor. If you know the locations suppose I have at the end, I want to put some capacitor which will rise the voltage to the required value. Then size of the capacitor also can be calculated, because in that case distance will be known that is that up to the end of the feeder ok.

So, many ways we can use this equation to find out the capacitor sizes as well as capacitor placements.

Thank you.