Electrical Distribution System Analysis
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Lecture – 08
Lumping Loads in Geometric Configurations: Rectangular

Dear students, welcome to this 8th lecture of this course Electrical Distribution System Analysis; title of today's lecture is Lumping of Loads in Geometrical Configurations and we are going to see rectangular configuration today.

We have seen in the last 2 classes the approximate methods of analysis of distribution system. We are basically stated with 2 factors which we call K factors those are K drop factor and K raise factor. K drop factor we have defied as percentage voltage drop per kVA per kilometer. So, whenever you know kVA loading and kilometer distance of your feeder you can easily calculate your voltage drop in the feeder by knowing the K drop factor.

Similarly, K raise factor can be used for placement of capacitor or to find the size of capacitor to be placed in distribution system. Particular in last lecture we have seen uniformly distributed load where the loads are distributed over the feeder.

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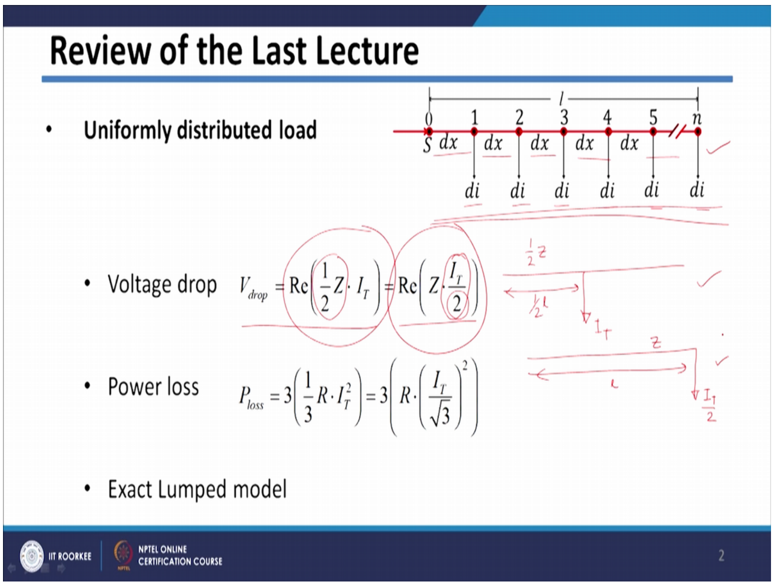

And if you see this particular figure in this slide you are having same size of the loads which are connected at distance which is same between the feeder..

This appropriation of various kinds of loads for example, we have seen that the loads which are kind of street lighting load, where different lighting loads are connected at different distance and because the poles will be at same distance are perform each other. And in the last lecture we are try to derive the voltage drop for this particular configuration and power loss equation.

And we have got these 2 equations where voltage drop is given by real part of one half Z into I T and you can write this equation as shown in other equation where just one half I have taken with respect your I T. And by seeing this equations we can lump these uniformly distributed loads at one location.

So, if you this if you see this particular equation here this load we can easily see that it can be lumped at the middle where it will be at one half Z and you need total current to be lumped at this location. So, this distance will be one half Z or one half l which is which will be having your impedance of one half Z and total current will be lumped at this middle location.

And if you see this second this tells you can lump total load at the end, but total load which is lumped at the end should be it by 2. So, if you see this configuration in that case you can lump this load at the end. So, total current which is lumped at the end will be I T by 2 and in that case total distance will be l. So, total Z will come into picture; so, total drop in that case will be real part of Z multiplied by it by 2.

However we have seen that in last class if you calculate power loss using this lumped configurations. The power loss value will be different from actual power loss that is why we try to lump these using another lumped models.

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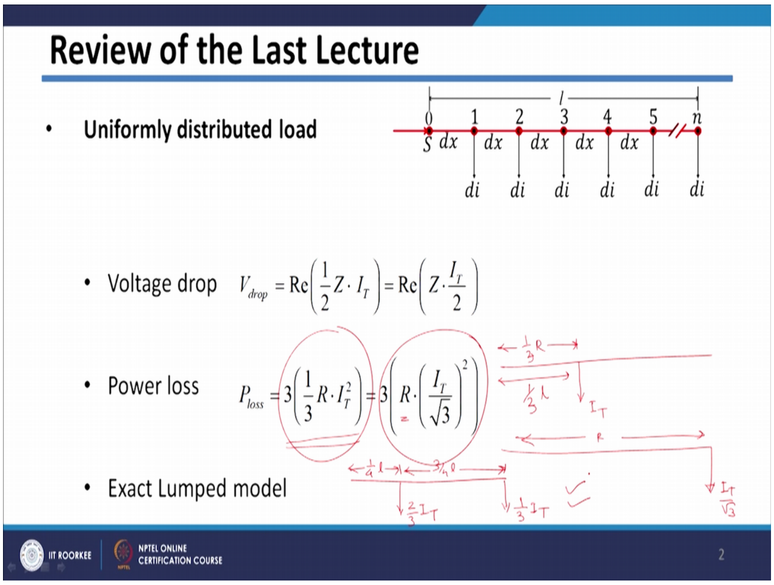

So, in case of power loss we can see that we have got this 2 equations; so in power loss is a 3 times 1 by third R into it square. So, if you see this equation we can lump total load power calculation of power loss at one third distance since it is one third R. So, it will be one third one and the resistance of this particular part of the feeder will be one third R and total load lumped at this location will be I T.

And then similarly by seeing this part of equation which is exactly same as first part; we can lump this current at the end, but the current which is lumped at end will be I T by root 3 and in that case total resistance of the feeder will come into picture which is this R here.

So, from this we can understand that for calculation of voltage drop and calculation of power loss; loads need to be lumped at different locations and then after that we have derived one more model we are lump loads can be lumped at 2 different location, but if you can calculate power loss as well as voltage drop from that model, it will come accurate and that model is exact lump model where we have seen that we can modeled the loads at 2 locations.

So, these are this 2 location and here we need to lump to the two third of I T and here remaining one third of I T and this two third of it should be lumped at one fourth of your distance of your feeder and remaining three fourth distance will be covered by this one third I T currents.

So, if you calculate power loss as well as voltage drop from this exact lump model it will come accurate. So, this was the revision of your last lecture.

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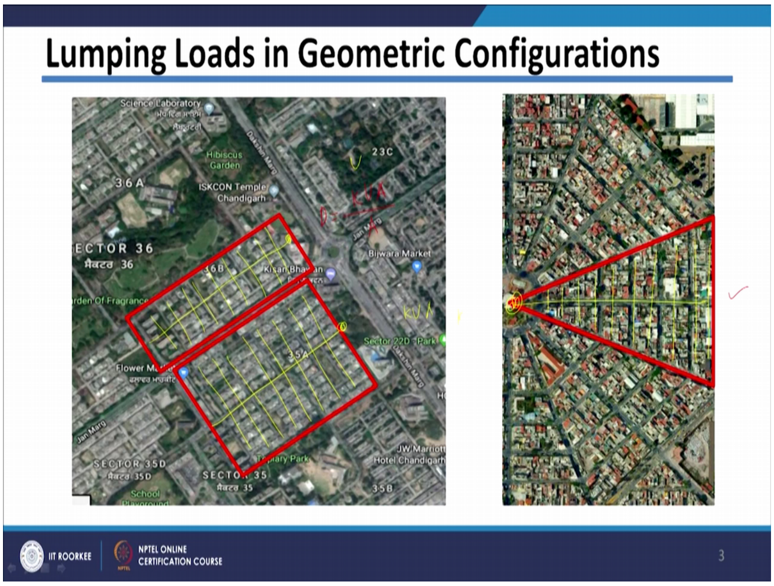

In today’s, lecture we will see how we can lumped or how we can lump the loads consisting of different geometric configurations. If you see these 2 pictures where I have shown aerial views of 2 localities and if you can see that there are homes which are placed symmetrically in this rectangular area.

So, if we can see this rectangular area; if there is primary feeder which is like this and let us say substation is here. And there are laterals which are connecting to this primary feeder and these laterals are distributing the power to these consumers which are placed in this rectangular area.

Similar configuration can be found it the in these another rectangular area where suppose the substation is here and then there will be different laterals which are coming out from primary feeder which are distributing this area. Now, this area we can assume that the load density over this area is constant.

So, it will be kVA load divided by your area of configuration. So, I am writing it as a D which is of your load density; I am considering this load density constant over this area; in actual practice it would it will not be constant, but since we are considering approximate methods. So, that we can get approximate value of voltage drops and approximate value of power losses we are assuming this load densities are constant.

Another area which I have shown it here we can see that the locality is arrange such that it is forming some kind of triangular shape; which is on the right hand side. And if there is substation which is located at this place and if the primary feeder is going out from here and then there are laterals which are connected to this, this will be come your triangular configurations.

For this kinds of geometric configurations we were we would like to get equations for voltage drops and voltage voltage equations for the power losses

Let us start with your rectangular configuration.

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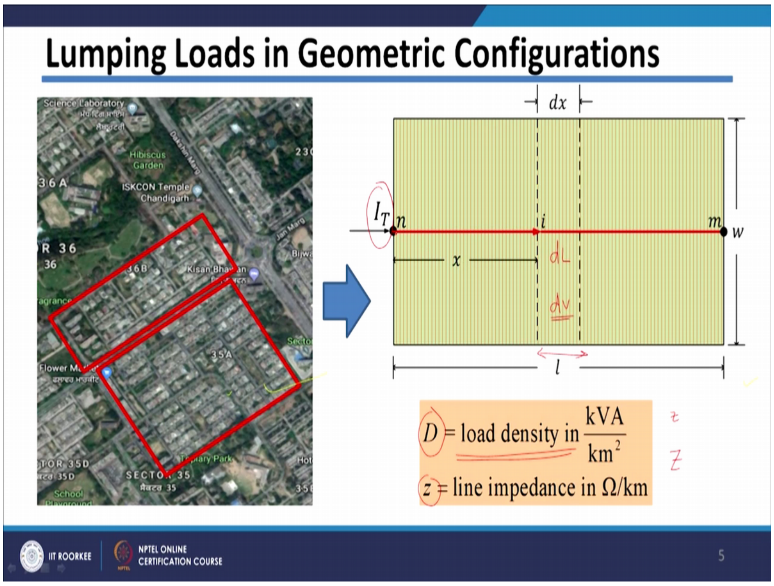

So, in rectangular configuration as I told you we can approximate your area of distribution which is shown in the picture with this kind of rectangular structure which is shown on the right hand side.

So, in this case in this case I am considering the length of this rectangular area is to be l width is w and current, which is entering total current which is entering total current which is entering to this rectangular area is say I T. And we are interested in calculating the voltage drop which is happening between point n to point m similarly power loss in this particular feeder.

For that I am assuming the load density which is in kVA per kilometer square the value is D and the line impedance in ohm per kilometer I am considering z. So, small z I am using for ohm per kilometer and capital Z will be used for total impedance of the feeder. For calculation of voltage drop or for calculation of power losses we are considering small strip here of width dx. First we will calculate voltage drop which is happening in this particular strip which is dv. And if you integrate this dv from n to m that is for length l will get total voltage drop which is happening across the feeder.

Similarly, in case of voltage in case of power loss we will calculate loss which is happening in the small strip that is say dl and after integrating this dl from n to m; we will get value of total power loss. So, let us start with voltage drop calculation.

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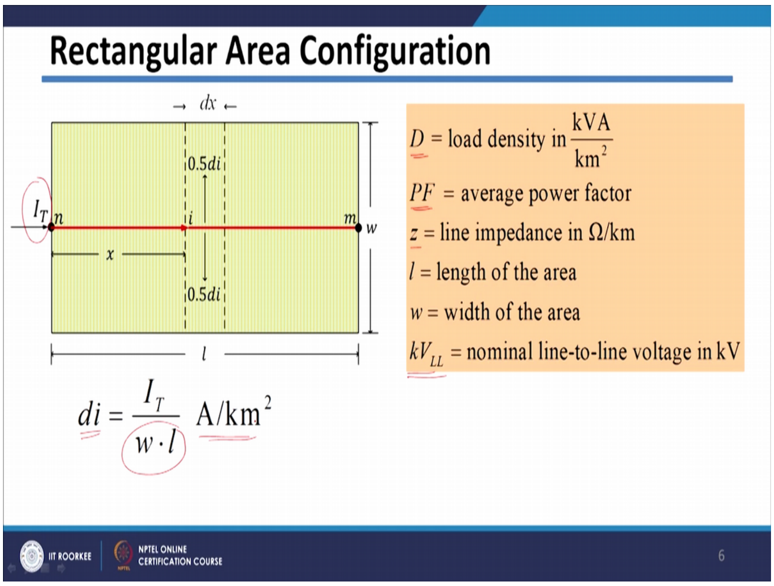

So, for voltage drop calculation as we assumed there is d amount of load density in kVA per kilometer square, power factor which is basically average power factor of the area because at different location there will be different power factors we are taking average value of power factor; z is again line impedance in ohm per kilometer, l is length of the area, w is the width of the area and nominal voltage is in kV that is K V line to line is nominal voltage of the feeder.

Now, as we assume total current which is entering this area is I T; so, di current density in the load will be it divided by area. So, area of this rectangular area is given by w multiplied by l. So, this di will be ampere per kilometer square, which is current density of this area.

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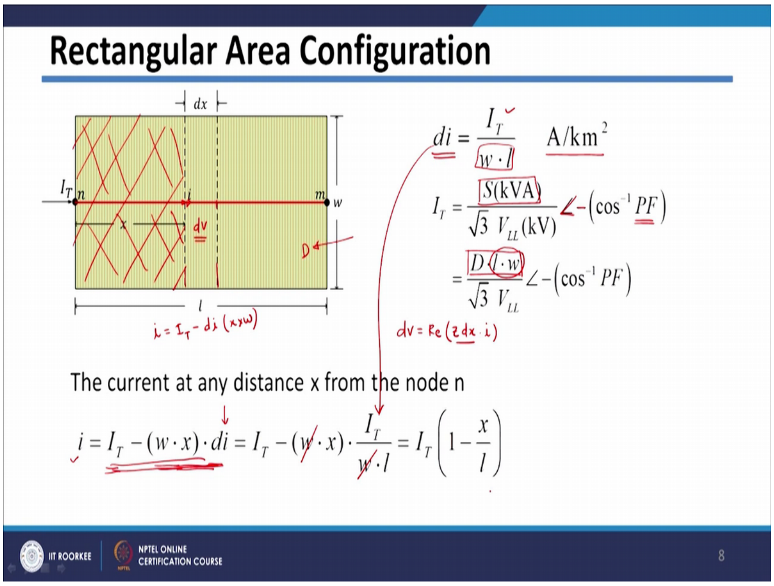

So, we can easily write di will be equal to I T divided by. So, we can easily write your di will be equal to I T divided area served which will give me current density is supplied in that particular area which will be in ampere per kilometer square, where I T can be calculated based on your load served. So, total load served is say S kVA divided by root 3 into V line to line in kV minus its angle will be minus cos inverse of power factor which is average power factor in distribution area.

So, in this case your kVA supplied in this area will be nothing, but D; D is nothing, but your load density multiplied by area which is l multiplied by w. So, total load served in this area or kVA served in this area is given by D multiplied by l multiplied by w because area is here multiplied by w here divided by root 3 into V L L and angle is minus cos inverse of power factor.

So, if you want to calculate voltage drop as we discussed we need to calculate dv means voltage drop which is happening in the small strip of length dx. So, if you want to calculate the voltage drop into this section. So, this dv will be basically equal to your impedance of this section.

So, it will be real part of impedance which will be z multiplied by dx zdx and the current which is entering to this strip i; how to get the current which is entering to this strip? your I will be nothing, but I T total current minus your current which is served in this particular area which will get subtracted and this current will be nothing, but your current density di multiplied by area; area in this case it will be x multiplied by w x multiplied by w.

So, therefore, current at any distance x from node n will be nothing, but I T minus w multiplied by x which is area here multiplied by your current density. So, this much current will get served into this area which is area before this strip remaining current will enter into this strip and that current which is entering to this strip is i which is given by this expression here and we have seen that this di is I T divided by w multiplied by l.

So, d instead of di can say I T divided by w multiplied by l. So, in this case this w and w will get cancelled out and if you take this it common out of this expression it will be it in bracket 1 minus x divided by l.

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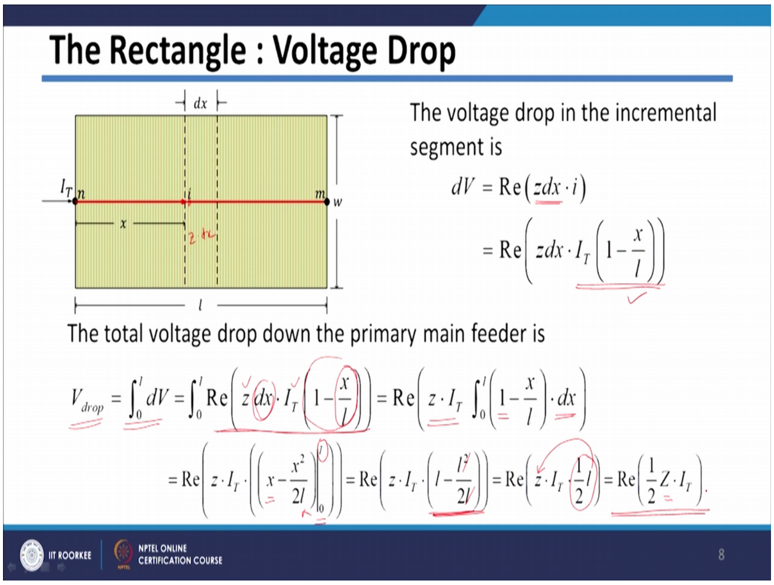

So, we can easily write the real part of impedance multiplied by your current. So, real part of impedance of the small strip will be z multiplied by dx and current we have already calculated which is I T multiplied by in bracket 1 minus x by l. So, this is nothing, but dv; so, we need to integrate this dv from 0 to l so, that we can get total voltage drop.

So, the total voltage drop down the primary main feeder from point n to point m that is from 0 to l. So, to get the total voltage drop V drop, it will be integration of dv from 0 to l and dv you already derived here which I have taken it here. And if you observe this equation there are distance related terms will get integrated other things are constant which will come out.

So, we need to integrate this part and this part and then z; I T will come out which are actually not dependent on distance which are constant. So, if you integrate this 1 minus x by l into bracket and with respect to this dx here. So, from 0 to l; so, integration of this part will be integration of 1 will be x and integration of x by l will be x square by twice of l and limits of integration are 0 to l.

So, when we are putting l into this equation; I will get this part here and when I am putting 0 into this equation it will be 0 itself. So, we are going to getting this part here after putting the limits of integration and if you see this it will be just 1 l by 2. So, l minus l by 2 will remain only l by 2; so, we are getting l by 2 here.

Now, in this case this length of the feeder multiplied by per unit impedance will give me total impedance. So, z is now the total impedance which is z multiplied by small Z multiplied by your l. So, we have got the equation of voltage drop for rectangular distribution area.

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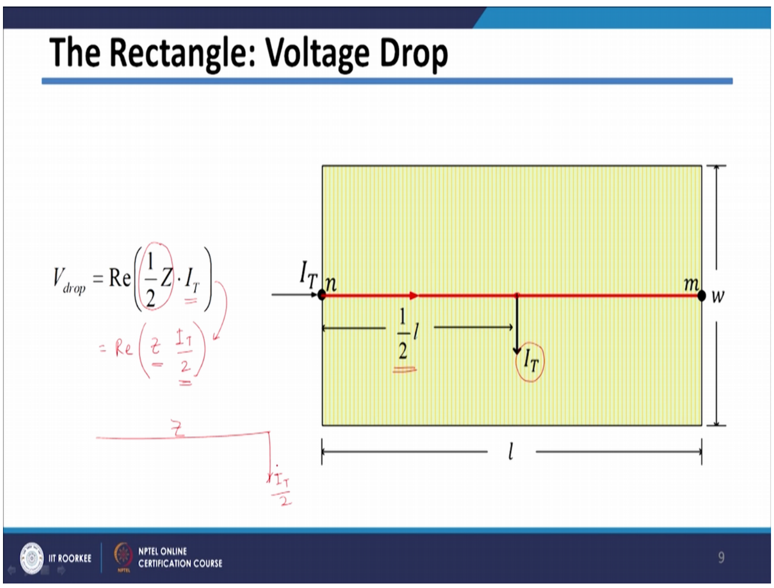

And from this we can easily see if you take one half distance that is one half Z you need to lump total current. So, this total area can be approximated by oneth lump load which is placed at the middle which is having one half length and in that case it will be total current.

And as a we have seen in case of distributed uniformly distributed loads it can be lumped at other locations also. So, in this also we can lump this loads at other locations let us see if you want to lump this load at the end. So, in that case I can just take this one half related to I T.

So, it the same equation will become like this and from this I can take if I am taking total impedance into account the half of the current should be lumped at the end. So, in that case your model will be something like this; so, you can lumped at the end total impedance will come here. And then in that case current value will be I T by 2 or total load value will be it by 2.

So, this is all about voltage drop.

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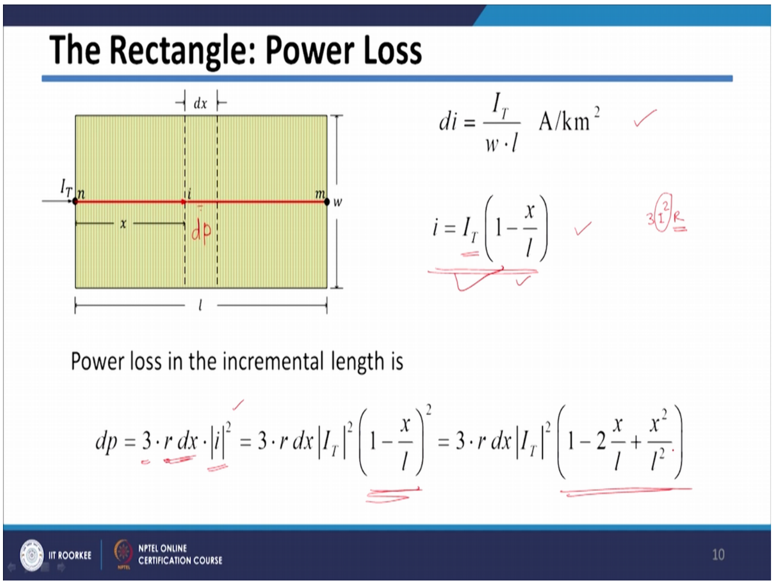

Let us see power loss; so, power loss also the current density which we have got in earlier slide I have taken it here. Similarly current which is entering into small strip, which we have derived in the earlier slides, I have taken it here.

Now, if we want to calculate power loss which is happening into this one which is nothing, but 3 I square into R; R is actually ready resistance of this small strip and then I square which is current entering into small strip. So, first try to derive the power loss dl which is power loss which is or I I am saying dp.

So, dp is actually power loss which is happening in the small strip of width dx. So, it will be 3 into resistance of the strip which is per unit resistance multiplied by distance dx. So, this will give me total the distance of strip and then current which is entering into strip we have seen it is I; so, I square. So, it is I 3 into R into I square which is nothing, but power loss which is happening in the strip.

We have got equation for I; so, I can just put this equation for I into this expression. So, it will be we have to make the square of it. So, it will become I T square here and then square of this term. So, we got this term here then we can actually square this bracket here. So, if you square it; it will be 1 minus twice of x by l plus x square by l square.

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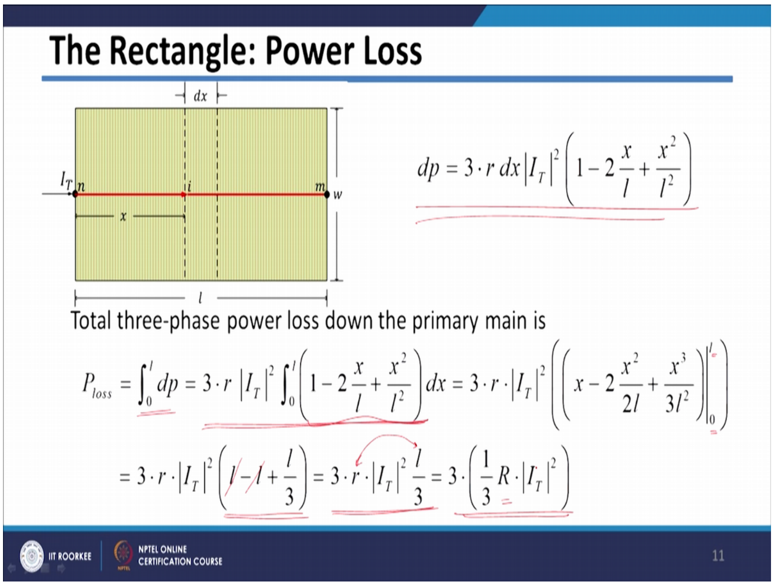

So, we have got dp as I told you then we have to integrate this dp from 0 to l to get the power loss. So, total power loss we can get from integrating this dp from 0 to l; dp we have got it here I can put into this expression. So, if I put this dp into this expression and in this case this dp since it is constant it will come out.

So, integration will just operate on this term here and if you take the integration of this term ah; it will be x minus twice of x square by 2 l plus x cube by 3 l square the limits of integration from 0 to l. So, when we are putting x is equal to l I will get this term here and when I am putting 0 it will be totally 0. So, if you can this term actually l and l will get cancelled out; so, only l 3 will remain here.

Ah. So, total power loss will be 3 into r small r into I T square into l by 3. Again in this case small r which is per unit length resistance multiplied by length will become total resistance which is R here. So, final equation to get the power loss of rectangular configuration will be 3 into one third R into I T square.

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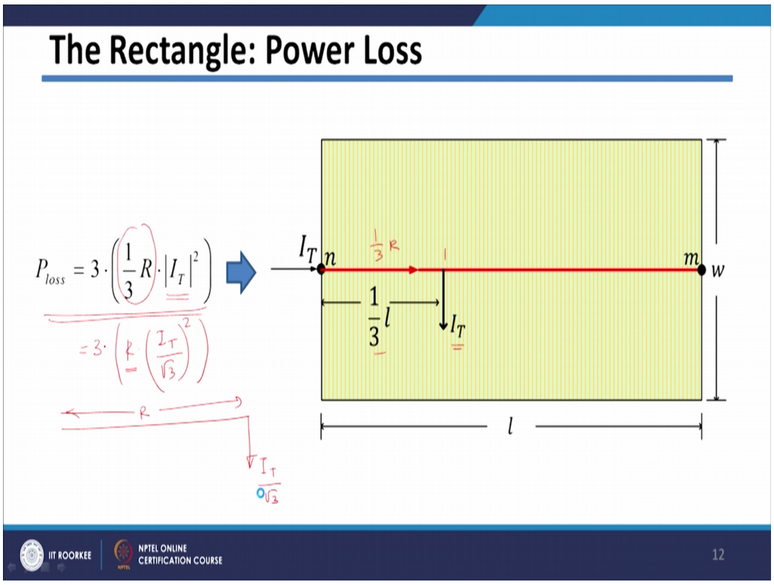

In this case also by seeing this equation here, you can easily find out oneth model of for calculation of power loss. So, we tells me at one third R you have to put whole current. So, you can see that at one third length; so, the resistance still this part will be one third R and the current will be then I T.

And as I have told you in this case also we can lump this at the end. So, I this case I can just write this top equation like this. So, it will be I T by root 3 square in that case I can easily lump. So, if I lump this load at the end in this case I am taking total decisions into account. So, it will be R resistance from start to end and then in that case our current will be it divided by root 3.

So, this is your lumped model for power loss calculation.

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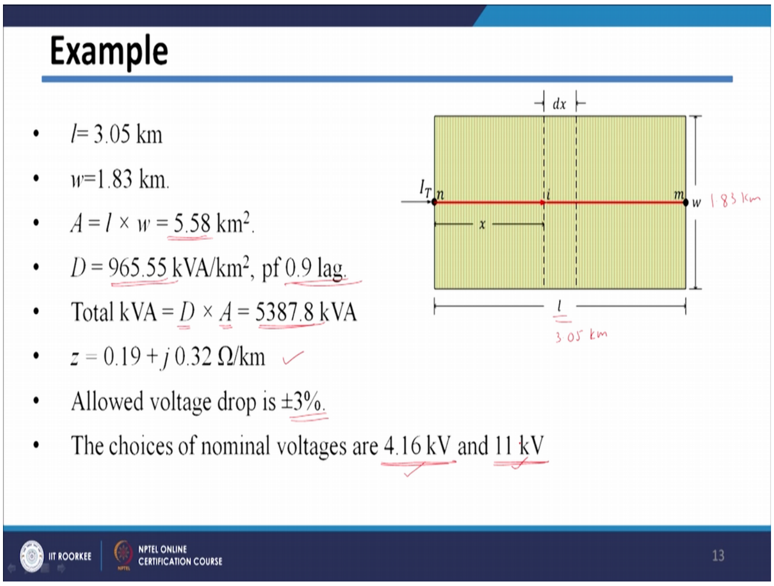

Let us see application of these equations which you have derived for calculation of power loss and voltage drop. Let us say you are having this rectangular area where this length is 3.05 kilometer, width is 1.83 kilometer, this I am taking 3.05 kilometer.

So, total area you can easily you calculate; so it will be length multiplied by width it will be 5.58 kilometer square. Let us say load density in this area is 965.55 kVA per kilometer square and power factor is 0.9 lagging which is basically average power factor over this area. So, total kVA will be this load density multiplied by area which comes out to be 5387.8 kVA.

So, this is actually total load which is served in this particular area; let us say impedance per unit length or impedance per kilometer is 0.19 plus j 0.32 ohms. And allowed voltage drop that is limits on voltage drops are actually plus minus 3 percent. So, voltage we can allow only plus minus 3 percent variations in the voltage with respect to different loading conditions.

So, for this particular load of 5387.8 kVA we want to know which voltage is suitable; either we have to use 4.16 kV as nominal voltage for the feeder or 11 kV. So, let us calculate using whatever theory we have seen till now how much will be the voltage drop; if I consider 4.16 kV as a nominal voltage and then we will see for 11 kV. So, let us calculate voltage drop for one point sorry 4.16 kV.

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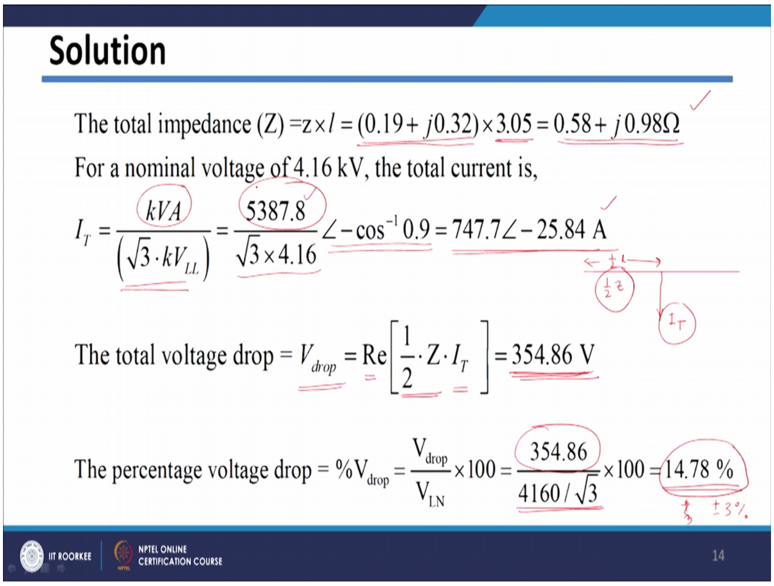

So, total impedance of full feeder will be small z multiplied by length of the feeder. So, we have got this impedance value here multiplied by length of the we have seen that it is 3.05 basically it is length of the feeder. If you calculate that total impedance which is capital Z will be 0.58 plus j 0.98 ohm.

Let us calculate current total current width entering into this rectangular area for nominal voltage of 4.16 kV. So, it is easy total kVA divided by root 3 into line to line voltage. So, total kVA we have calculate in last slide divided by root 3 into 4.16 which is voltage in kV and power factor of this current will be cons cos in also point 9 which is average power factor which we are assumed. So, it is comes out to be 747.7 and its phase angle is minus 25.84 amperes. So, this is total current is in amperes.

So, total voltage drop V drop across the full feeder length will be we have seen that in case of rectangular configuration to calculate voltage drop we can lump total load I total current or total load at one half distance one half l. So, you have to take one half z multiplied by this it will be and real part of it will be nothing, but voltage drop. So, real part of one half z because we are lumping load at the middle and then total current I T.

So, we have calculated this impedance we have calculated current. So, real if you take the real part of this impedance multiplied by current; it will come 354.86 volts. So, this is voltage drop we need to calculate percentage voltage drop.

So, in that case we need to divide this voltage drop by nominal values of voltage from line to neutral; so, in this case it will be line to neutral voltage it will be 4160 divided by root 3 and voltage drop which you are getting is 354.86.

And if you calculate it the percentage voltage drop if you are using 4.16 kV is 14.78 percent. And we have seen that our limit is actually plus minus plus minus 3 percent. So, you can see that this is if you are using 4.16 kV as a nominal voltage and you are applying this load here; you are actually voltage drop till end is 14.78 percent which is valeting valeting the limit of plus minus 3 percent.

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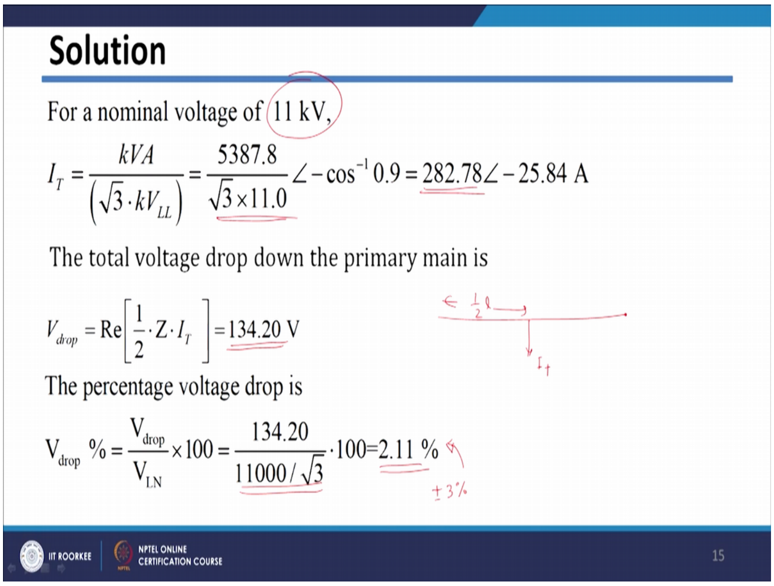

Now, check a what is happening with 11 kV. So, similarly we can calculate current for 11 kV. So, if you calculate for 11 kV the current is coming 282.78 and phase angel will remain same because we are not changing the power factor of the load.

Then similarly if you calculate voltage drop in the primary main; again here that lump same lumped model is used where total current is lumped at one half distance. And if you calculate it the voltage drop comes out to be 134.20 volts.

And if you calculate percentage voltage drop by dividing it by nominal line to neutral voltage; this is basically line to neutral voltage here, it comes out to be percentage drop 2.11.

So, we can see here when I am using 11 kV as a nominal voltage for this particular feeder, my voltage drop till the end of the feeder till this end it is coming out to be 2.11 percent which is actually within the limit because our limit is plus minus 3 percent. So, it is within limit; so for this particular configuration we can use 11 kV instead of 4.16 if it is supplying this high value of load which is 5387.8 kVA.

So, this is just application of whatever theory which we have seen.

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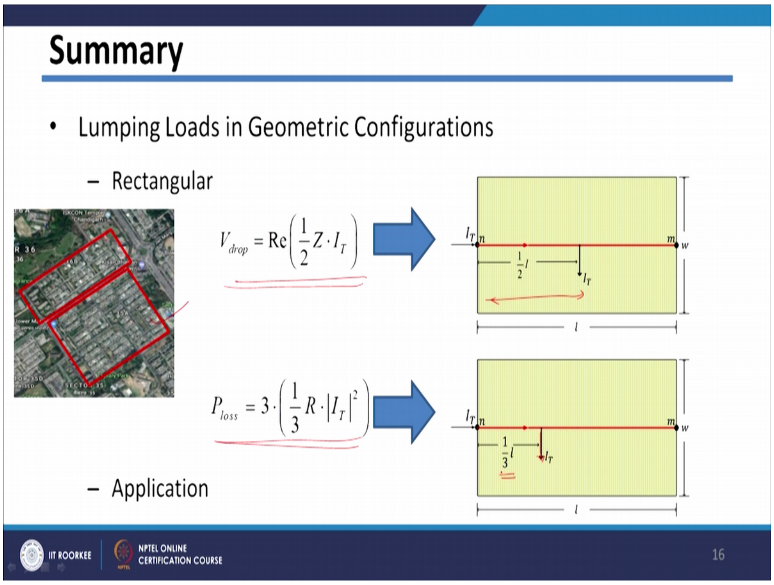

So, in summary we have seen the lumping of load for different geometric configuration; so, we have started with rectangular configuration in this lecture. And we have derived equation for voltage drop and power loss.

And first we have seen that rectangular area how we can configurate and then we have seen that when we are using voltage drop, we can lumped load biasing this equation ah; in this case we have lumped it at the middle and total current. Similarly we have got the equation for power loss which is this one and in this case we have seen that this load total load can be lumped at one third distance that is 1 1 by 3 l.

And we have also seen the application of this while selecting the voltage levels of the feeder we can approximately calculate how much voltage drop will be happening for different voltage levels. And we can suitably select the voltage level which is given giving the voltage drop within the limit.

Similar application you can use it if you are interested in calculating the cross section of conductor then also it can be used. Or if you want to calculate voltage drop for different loading conditions that also can be done by whatever methodology explained here; however, it will give little bit approximate results. In the next lecture we will try to derive the equations for triangular configurations.

Thank you.