Electrical Distribution System Analysis
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Lecture – 07
Analysis of Uniformly Distributed Loads

Dear students, welcome to this 7th lecture of the course Electrical Distribution System Analysis. The topic of today's lecture is approximate Analysis of Uniformly Distributed Load. So, topic is analysis of uniformly distributed loads. Before going to the uniformly distributed loads, we will just review what we have seen in the last lecture.

We have started the second chapter which is approximate methods of analysis in the last lecture and, we have seen that these approximate methods are very handy for the distribution engineers, we want quick and approximate answer.

So, whenever we need quick and approximate answer of your distribution system like, when you want to calculate voltage drop or power losses, you need not know all the details of the distribution system. Just by knowing the feeder loading and distances, we can easily approximately calculate them.

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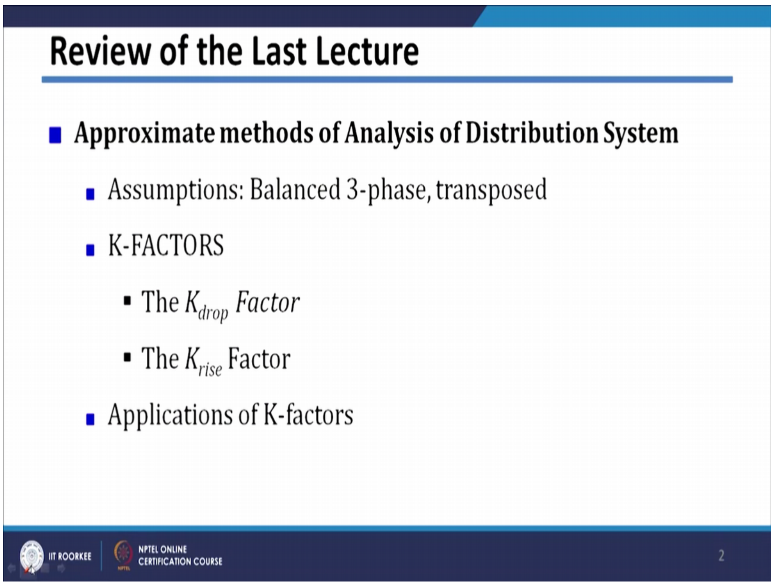

So, we have started with some approximate methods for that in the last class we have seen two types of factors, one is called as K drop factor which is percent voltage drop per

KVR per kilometer. And, then one more factor which you are seen which is K rise factor which is per percent voltage rise per KVR per kilometer, voltage drop factor is associated with loads and, this K rise factor is associated with capacitor banks. And we also seen various example which demonstrates the applications of these K factors.

Now, we will go for uniformly distributed loads, many times or you can say actual loads in the distribution system they are not uniformly distributed. The loads which are connected to distribution system at different points, they will be having different magnitudes.

However, while analyzing the distribution system, you will see that it is very difficult to model each and every load, we need to lump these loads at a particular point and do the analysis. Because there will be thousands of customers which will be connected to the distribution system and, if we model all those thousand customers it will take very long time. So, to do the lump modeling we need to do some assumptions, like many times we model this uniformly distributed load feeder, like many times.

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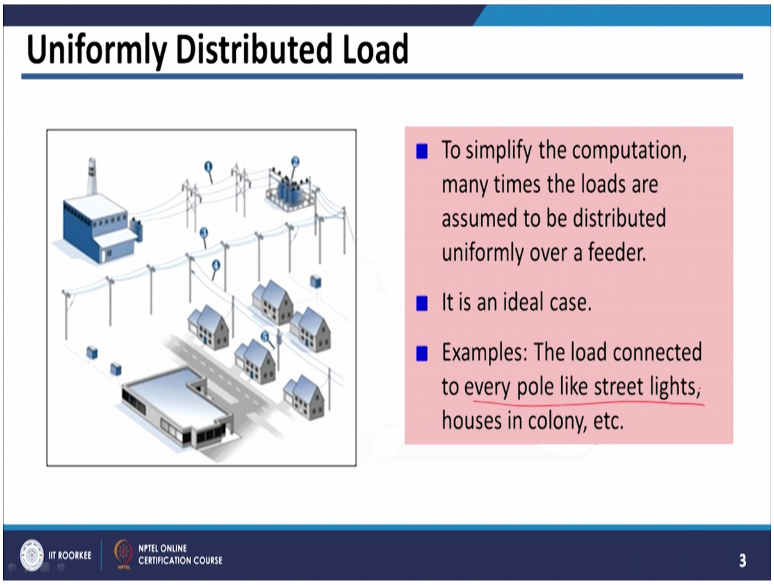

We know that there are loads which are say loads like street lights, which are connected at every poles. This kinds of load will be taking same amount of power at every poles. So, we can see that these loads will be called as uniform kind of loads.

Also like I shown in this particular picture, there are many houses which will be connected to the distribution feeder and, the economic background of all those people who are living in the particular colony may be approximately equal and, in that case also we can model these houses, or model this feeder as a uniformly distributed load.

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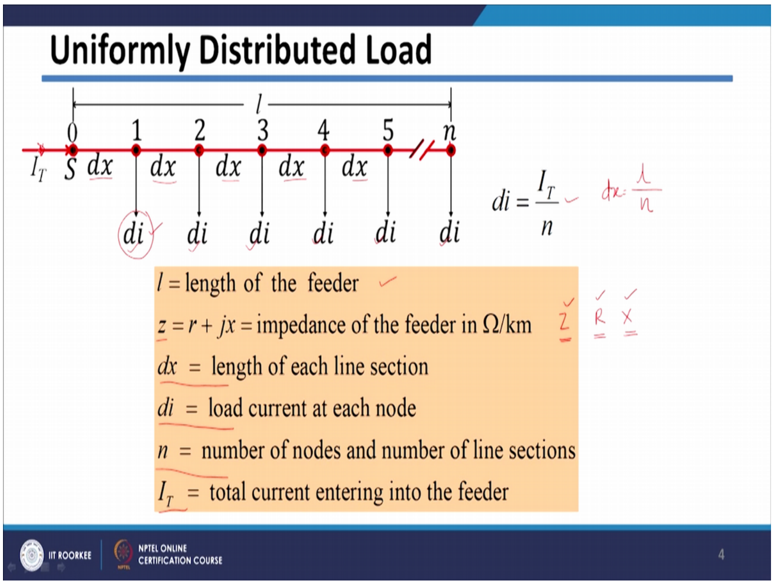

Let us see how we can approximate these uniformly distributed loads. Now, let us say you are having feeder which is shown in this figure, which is ranging from node number 0 to node number of n. And there are uniformly distributed load each taking the current of say di. So, current taken by each of the consumer connected to the distribution system is a di.

So, in this case we can say that there are uniformly distributed load at equal distances dx dx dx ok. Therefore, we can say that if there are n section so, this di current in one section will be equal to it divided by n. So, it in this case is total current input to this feeder. So, it divided by n will give me di, which is load at 1 node. We can also see that if the distance of feeder is l. So, in that case your dx will be equal to your l by n number of sections ok.

So, in this case what I am considering length of the feeder is l, the impedance of the feeder per kilometer is small z, which is r plus jx small r plus small jx the capital Z value and capital R and x denotes the total impedance total resistance and total reactance of the feeder, dx is the length of one section, di is load connected at each node, n is number of nodes into the system and it is total current input to this particular feeder.

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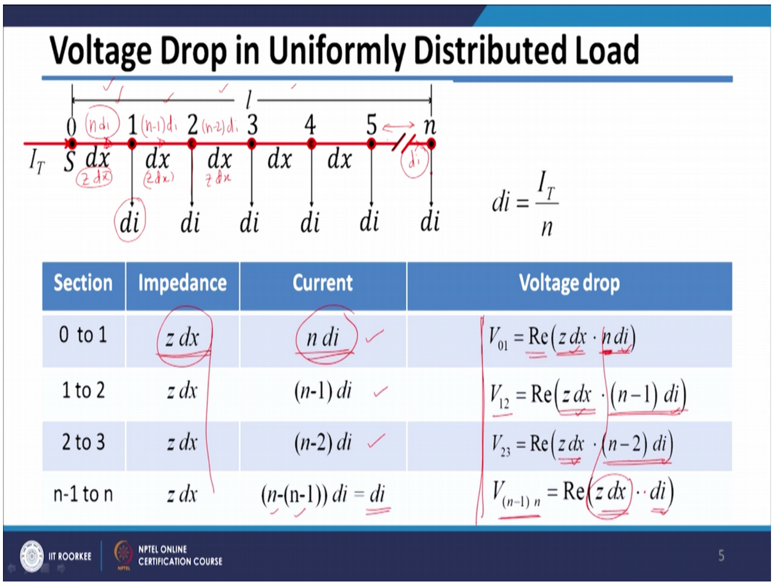

So, first we will see how we can approximately estimate the value of voltage drops, we can easily find out in the first section the whole it current will be flowing. So, total it current will be flowing into one section and, then in second section so, total current which is flowing in this section will be n minus di sorry n multiplied by di.

The current which is flowing through this section one current will get dropped here. So, current which is flowing in the second section will be n minus 1 into di. Similarly one more di will get dropped here. So, in third section it will be 1 minus 2 into di.

Similarly, if you go down so in the last section current will be just di which is equal to load at the last end ok. And the impedance of each section will be just z per unit impedance multiplied by distance of this section. So, impedance of this section will be z multiplied by dx, similarly for all the sections it will be z multiplied by dx.

Therefore, impedances I have written here impedances of the all the section they will be z multiplied by dx. And the current flowing in first section as we have seen it is the n di, in second section it is the n minus 1 di, in third section it is n minus 2 di like this. In the last section n minus n minus 1 di, which is basically equal to just di which is in the last section.

Then voltage drop we have seen it is a real part of z multiplied by current. So, if you calculate real part of z multiplied by current. So, this is your impedance and this is your current. So, it will be real part of z dx multiplied by n di similarly in second section that is V 1 2 voltage drop in section 2, or impedance is z x, but current is n minus 1 di like this.

If you do in third section impedance is z dx and current will be n minus 1 di like this, if you go to the last section the impedance drop in the last section, it will be real part of z dx dot di because impedance is z dx and your current is di.

And then total voltage drop across this feeder will be just addition of voltage drops which are happening in each of the section means, voltage drop which is happening from 0 to 1 1 to 2 2 to 3 3 to 4. So, if you add all these voltage drops, we can get total voltage drop across the feeder.

So, while adding all these voltage drop you can easily see that this z dx term is common, as well as your this di term is common. Means when we add them we just add we can take this z dx and multiplied by di common and, there will be series of addition where n plus n minus 1 plus n minus 2 plus up to you can go up to 1.

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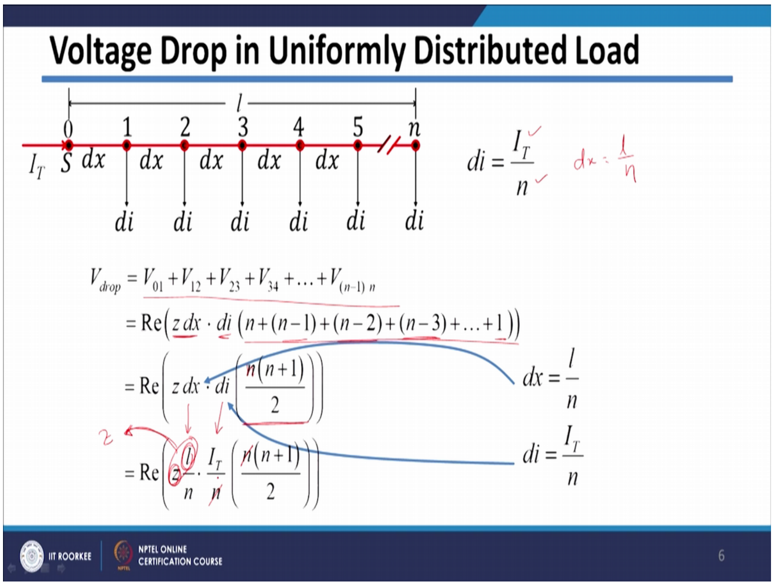

Therefore if you add these voltages as I told your z dx which is coming common out and, di is also coming out and then there will be actually series which is n plus n minus 1 plus n minus 2 plus n minus 3 up to 1. And we know that total sum of this series can be given by this formula, where this series can be given by n into n plus 1 divided by 2.

And we know that this dx is nothing, but total length divided by number of section. So, dx I can write total length divided by number of section and, your current di is nothing, but total current divided by number of sections.

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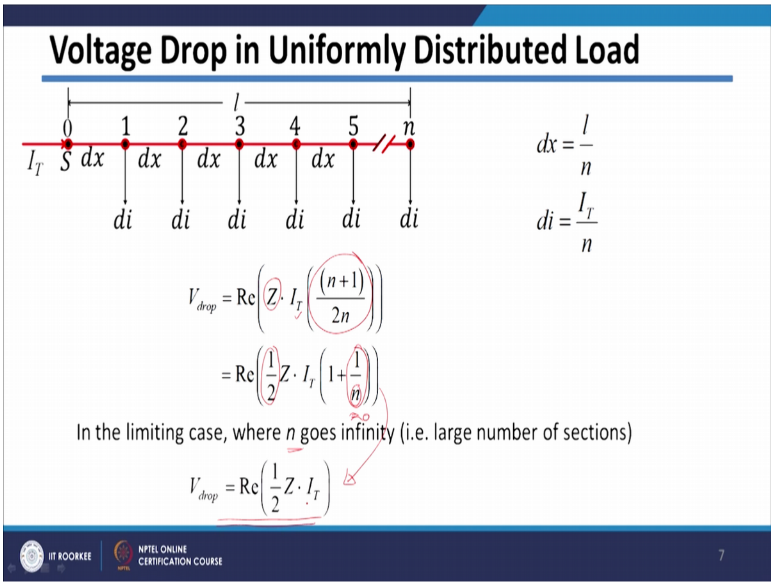

So, therefore so, in this case this becomes equal to total impedance of the feeder. So, z multiplied by small i z multiplied by l this length. So, per unit length per unit length impedance multiplied by length will give me total impedance z and, then this n and this n will get cancelled out. So, it is z multiplied by I T by n into n plus 1 divided by 2. So, if you take on this side so, there will be total Z which is total impedance of the feeder I T total current into the feeder and, you are having this term here which is depend on your number of sections.

Now, if you take this one 1 by 2 common out, we will having this term here and in the limiting case, where you take large number of sections or n goes infinity so, if the n is large this value will be very very small. So, in that case this will be approximately equal to 0. So, if you the do that you will get this equation so, voltage drop will be a real part of one-half Z multiplied by your I T total current; so, this is nothing, but your total voltage drop.

Now, main aim of this exercise is to lump the load at say one location a into the feeder, because as I told you since the load is distributed you cannot model each and every load. So, we need to lump this load at some location to so, to lump this load we can how many chances.

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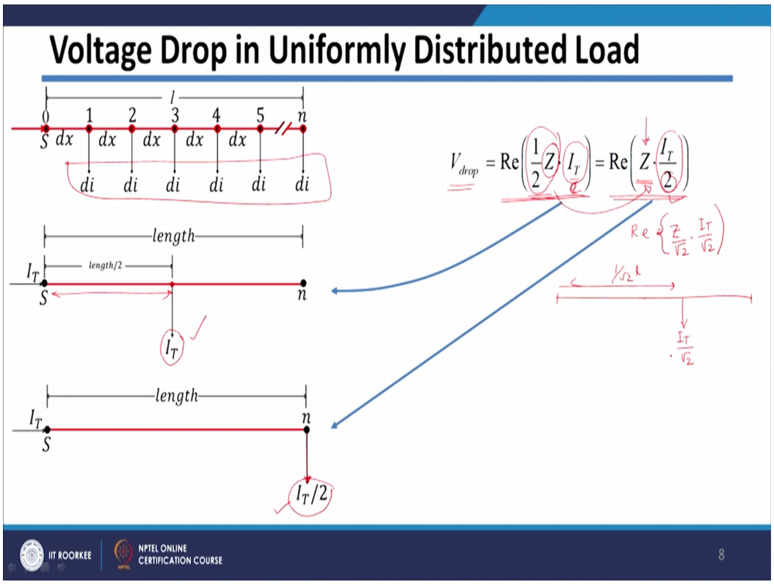

If you see this voltage drop which we have got real part of one-half Z multiplied by I T, or same equation I can just of taking this 2 divided by I T here, I can just write same equation Z dot I T by 2.

Now, if you lump this load total uniformly distributed load which is basically this, at one location by seeing this equation we can easily see that, I can lump this load at one-half the distance means, half of the impedance and in that case I need to take the total load into account. So, at one-half distance if you model this or lump this load; So, this is your one-half distance here till this point and, total current which we are on to lump is I T so, total current is lumped.

However from this we can from this formula, you can see that whenever there is total Z I can use I T by 2, means whenever there is total feeder, I am consisting means when I am lumping the load at the end of the feeder I need to use I T by 2 it, means when I am lumping the load at end of the feeder your current should be I T by 2. So, I can lump load at the middle, or I can lump load at the end.

So, when I am lumping at the middle the total current I need to model total load, I need to model. But when I am lumping the load at the end I need to model half of the load which is connected half of the total distributed load addition of total distributed load.

And you can see that these are not only two ways, in which I can model these loads in slumping can be possible at many different locations means, if you see I can write this same equation a real part of Z by root 2 multiplied by I T by root 2, which is exactly same like this. So, I just divided 2 2 into 2 part that is root 2 multiplied by root 2. And in this case I can easily tell that, I can lump this load at 1 by root 2 location so, this actually 1 by a 1 1 by root 2 distance root 2 l distance and the load total load, which is lumped at this location is it by root 2.

So, you can see that I have shown you three ways, in which I can lump load; however, there are many ways by just we are jesting this equation we can lump this load. Now, see if you lump this load at this location, how much power loss we get. Let us see if I lump this load at using this particular figure.

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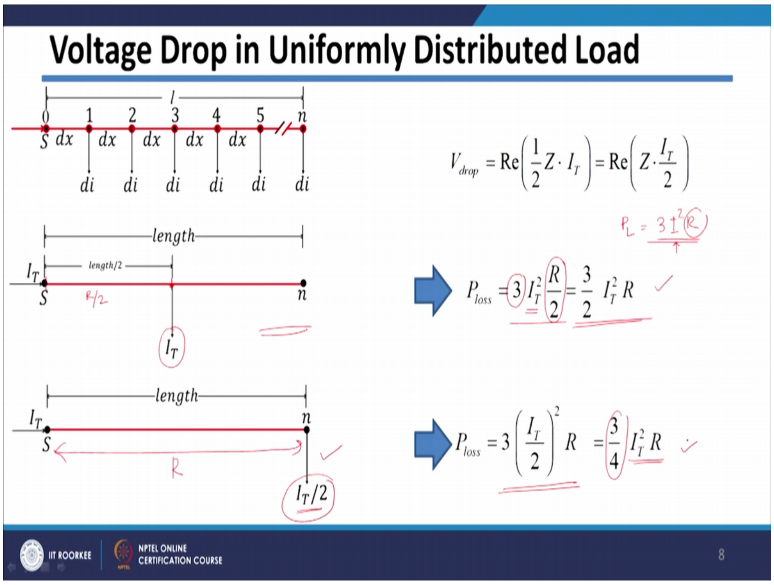

Then you can see that power loss into this feeder will be this 3 will because of 3 phases. So, we know that actually your power loss is nothing, but I square R, since if there are 3 phases and it use 3 I square r which is power loss.

So, in this case my current as I told you current in this case and it coming total I T current. So, it will be I T square and your resistance in this case will be half of the feeder resistance that is resistance will be R by 2 up to half distance. So, in that case your power loss will be 3 I T square and resistance is coming half which is R by 2. So, therefore, total power loss will be 3 by 2 into I T square R. However, if you see this figure 2 and, if you calculate power loss using this bottom figure where we lumped load at the end of the feeder.

So, in this case if you want to calculate 3 I square R current, in this case is I T divided by 2 so, current will be I T divided by 2 square, but since we are considering total feeder here resistance of the feeder will be R. So, 3 I T square R will give for formula in that case you are getting your power loss into the feeder, which is 3 by 4 into I T square R. However, you can see that when we lump at middle we are getting this equation and, when we are lumping at the end we are getting this equation.

However both of these equations are not correct. So, if they are not giving correct power loss means, if you lump the load based on your voltage drops, you will not get the power losses correct. So, if you want power losses into the system correct you need to lump this load at different locations. So, we will first see what are these locations where we can lump the loads. So, first we will calculate the total power loss and, then we will see in case of power loss, where the lumping up load is possible.

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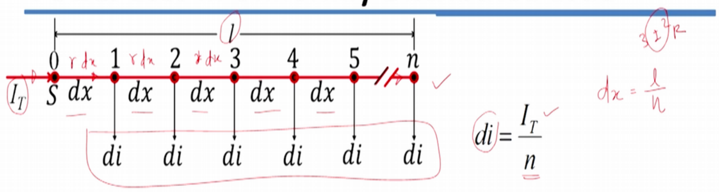
Power Loss in Uniformly Distributed Load



Section	Resistance	Current	Power Loss
0 to 1	$r\,dx$	$n\,di$	$PL_{01} = 3\cdot\left(r\,dx \cdot n^2\, di^2\right)$
1 to 2	$r\,dx$	$(n\text{-}1)\,di$	$PL_{12} = 3\cdot\left(r\,dx \cdot (n-1)^2\, di^2\right)$
2 to 3	$r\,dx$	$(n\text{-}2)\,di$	$PL_{23} = 3\cdot\left(r\,dx \cdot (n-2)^2\, di^2\right)$
n-1 to n	$r\,dx$	$(n\text{-}(n\text{-}1))\,di = di$	$PL_{n-1\,n} = 3\cdot\left(r\,dx \cdot di^2\right)$

So, let us see this figure here, it is same you are having this it current which is entering to this feeder and, it is uniformly distributed load with di magnitude and at same sections distance that is dx total length l and n ok. And we know that di will be nothing, but total current flow which is period at feeder divided by your number of sections. Similarly we know that your dx this terms will be n l divided by n total length divided by number of sections.

Now, let us calculate power loss so, similar to your impedance you can see that resistance of the each section will be also equal. So, each section will be r dx resistance and, then we are seen the currents which are flowing in the different sections will be given by so, in this first section from 0 to 1, your current will be n di and, then 1 2 to 1 current will get dropped that is n minus 1 di n minus 2 di up to this up to last section it will be just di current.

And we know that power loss is nothing, but 3 into I square R. So, power loss which is happening in section number 1 will be 3 multiplied by resistance in this case r dx and current is n di. So, we want I square term. So, in that case it will be n square into di square.

In second section resistance is same r dx, but current is decreasing by di amount that is why it will be n minus 1 square multiplied by di square. In third case current resistance is same current is decreasing by 1 di so, the n minus 1 n minus 2 square multiplied by your di square. And in the last section only di current is there so, it will be di square so, r dx resistance multiplied by di square.

So, these are nothing, but the power losses which are happening in each of the section. So, total power loss which is happening over this whole feeder will be addition of all these power losses.

So, to do the addition of these power losses you need to add them so, in this case if you observe at in the each term you are having this r dx term common, as well as your di term common. So, when you add it you need to add just the series consisting of n squared terms. So, there will be n square plus n minus 1 square plus n minus 2 square plus and so, on up to 1.

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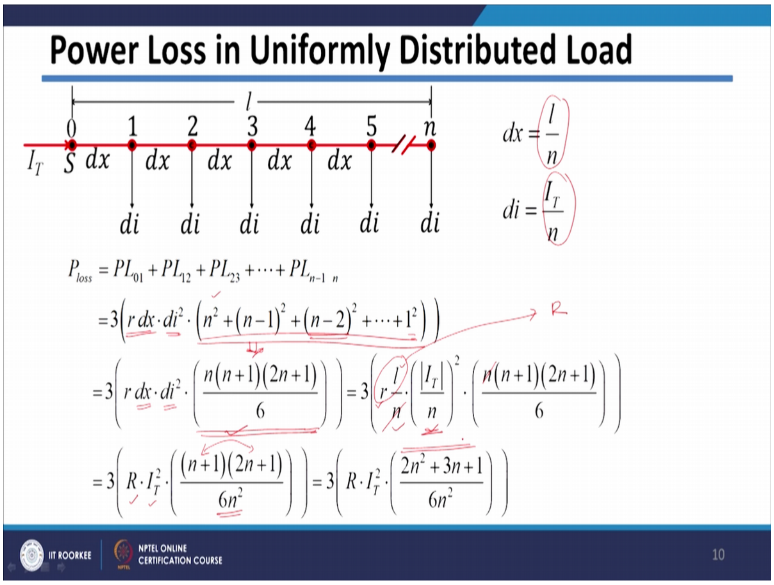

So, if you explicitly write it by taking r dx and di common from those earlier figures, you are getting this series here which is series of squares that is n square plus n minus 1 square plus n minus 2 square up to 1. And if you find out equivalent of this series, we know that this series can be summed up summed using this particular formula here. So, this series will turn out to be this where this dx we can put your total length by a number of section l by n and di.

We can put total current by a number of section that is I T by n so, here if we put this you will get dx is equal to l by n and, I T will be di will be I T divided by n square. In this case your r multiplied by l r is per unit length resistance multiplied by length will give me total resistance R, this n and this n will get cancelled out here, we are getting one n square term.

So, if you simplify it I will get this term r multiplied by l is getting this capital R here, total resistance I T square as it is and this is became 6 n square. And then if you do the multiplication of these two terms, I will get this term here that is 2 n square plus 3 n plus 1.

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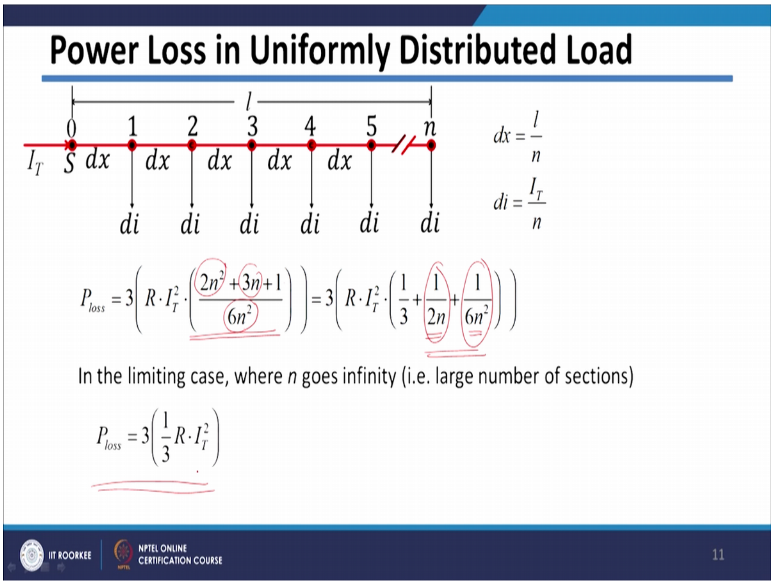

We further simplify it we further simplify this term here by dividing each component by 6 n square. So, if you divide 6 n square if we divide 2 n square by 6 n square it is 1 by 3 3 n by 6 n square, it is 1 by n and 1 by 6 n square as it is.

Similar to voltage drop calculation where we assumed large number of sections and, if you are the similar thing here, assuming large number of sections that is large n value, if the n value is large this term and this term will become negligible negligibly small. So, in that case if this these two terms are negligibly small, I will get power loss equation which is this.

Now, as I told you here also you want to lump, the load at 1 or 2 locations such that I will get this particular power loss. So, let us say we want to lump the load at only 1 location.

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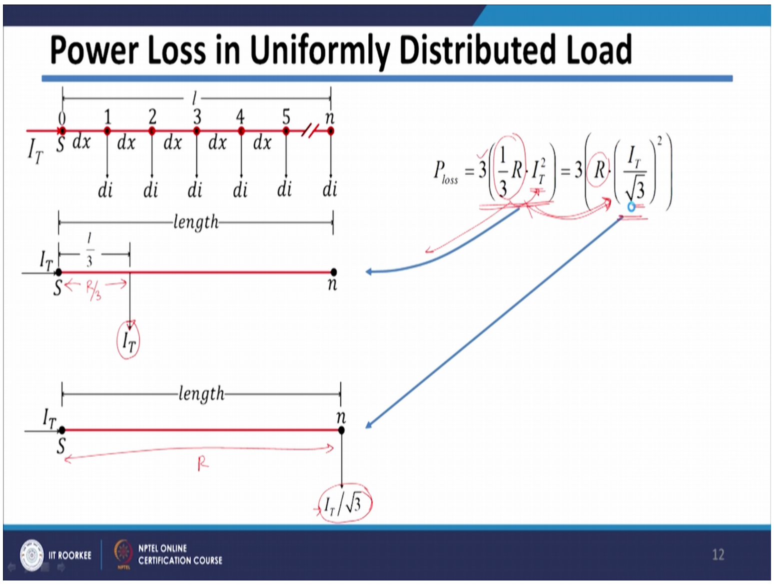

So, after seeing this equation this same equation, I can write by taking this 3 into divided by I T square. So, in that case it will become root 3 here so, the both these terms are same here. Now, when we lump the load using this equation, I will get this figure.

So, this equation tells me when I am taking 1 by 3 R, I can take total current into account means total current I can lump at 1 by 3 distance. So, this particular feeder section will be having R by 3 resistance. So, in that case it will be I T square multiplied by your R by 3 resistance and 3 times is actually for 3 pages.

Now, the same equation I have written in another form and, if you plot the figure or if you lump this load using equation which is shown on right hand side. You can see that here, I am taking total resistance into account means I have to take full length of the feeder into account. So, it will be having total R resistance and in that case current will be it by root 3 ok. So, when I am lumping the current at the end it will be I T by root 3. So, total load I can use total load divided by root 3 value, current which is taken by load will be I T by root 3.

And similar to your voltage drop calculation, in this case also these are not only two ways, in which I can the lump I can lump the loads. However, there are many ways I can lump load, or till now we have seen for calculation of voltage drop. We are lumped load at different locations, or when we lump those load at those location we are not getting power loss correct. So, for power losses we need to lump load at different locations and, to calculate voltage drop we need to lump a loads at different loads a different locations.

So, if you lump the load at some location for voltage drop calculation, they will not give your power loss accurately, or if you lump the load for power loss calculation, they will not give your voltage drop accurately. So, to get both these values accurately we need to lump these loads at two different different locations.

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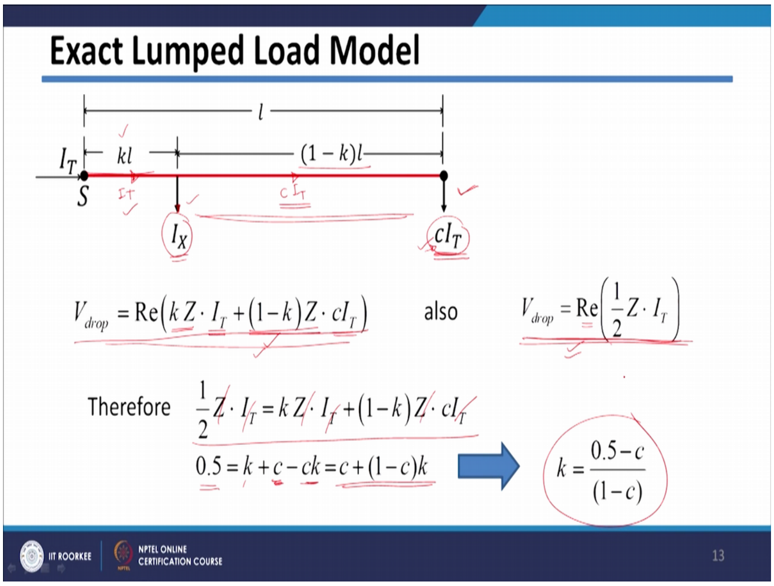

So, in this particular figure I have shown these lumping at two different locations, but in this case I want to lump the load such that, if I calculate voltage drop. I should get true voltage drops also when I am calculating power loss, I should get correct power loss or true power loss ok.

So, let us say these lumps loads are lumped at these two locations. The distance of first load from substation is k fraction of l and so, there will be remaining distance which is 1 minus k multiplied by l, then I assume some load is since some load is connected at first location remaining load, which is c fraction of I T will be lumped at the end.

So, some part I am lumping at the end and, remaining part I am lumping at some location around the feeder and, I want to calculate what is this location and, what should be my lumped at lump load at these locations. So, basically I want to calculate what is value of k and what is value of c ok. Let us write voltage drop equation for this particular configuration.

So, we can see that to calculate voltage drop, we need to know on the feeder currents. So, if we see this in the first section the feeder current will be total I T and, then the is current will be dropped in first section. So, here the is current will get subtracted so, remaining current will be flowing from here. So, I can saying this is c I T which is going to this feeder. So, first feeder section this feeder section will be having I T current and, the remaining section will be having c times I T current and c and c will be less than one fraction of I T.

So, if you calculate voltage drop. So, it will be real part of voltage drop happening in first section plus real part of the voltage drop which is happening in second section. So, real voltage drop which is happening first section will be impedance, which is k multiplied by Z and, current in first section we have seen I T. In second section the impedance will be 1 minus k multiplied by Z and your current we have seen this c I T.

So, in this case your voltage drop will be given by this 1; however, we have seen that true voltage drop value in the last part of the lecture, it is real part of one-half Z dot I T. So, this is true value and this is lumping at two location we have got this value and to get true value, or correct locations of these lumping of the load, we need to get this drop should be equal to this drop.

So, if you equate this these two drops so, this will be equal to this will be equal to this, I just written them here it way after equating and, in this case your Z I T, Z I T will get cancelled out and what will remain is here, it is 0.5 here, it is k and in this it is c and here it is minus k into c. And if you simplify this you will get c plus 1 minus c into k. So, about one equation another equation we can get it from power loss calculation and equating them.

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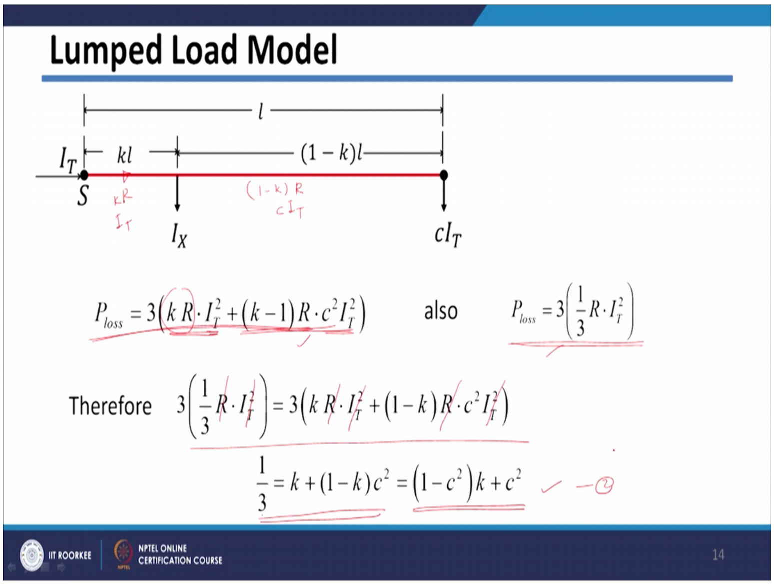

So, finally, you got k is equal to this value. Now, another equation we can get it from power loss calculation. So, in this case power loss so, to calculate the power loss into this section the resistance of this section will be k into R. And the current which is flowing through this section will be total current that is I T, resistance of this section will be 1 minus k into R and, current which is flowing through this section will be c multiplied by I T.

So, I can easily write the power loss equation. So, power loss in this case will be power loss which is happening in the first section, where resistance is k multiplied by R and current is I T and, resistance in the second section is 1 minus k into R and, current into second section is c I T so, that is why c square I T square. So, this will be actually power loss by lumping at load at two location and, the true value of power loss which you have got is given by this expression. And as I told you these both these power loss should be equal.

So, if you equate them I will get this equation means this will be equal to this, in that case if you observe this equation here R I T square, R I T square, R I T square which is basically common, which will get cancelled out. And, then I just written them remaining quantities into this equation and, if you simplify this equation I will get this equation here.

Now, I have two equations and two unknowns so, one equation we have got on earlier slide and, this is equation number two and after solving these two equations like this.

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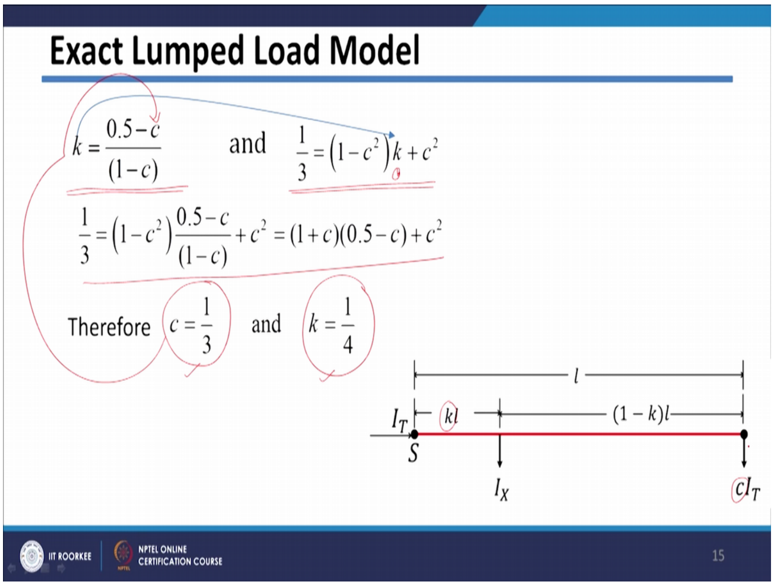

So, from power loss equation or equate after equating power loss we are got this equation, this equation and after equating your voltage drop we are got this equation, this equation. Now, if you put value of k into this particular equation I will get this and, this equation is basically just in terms of c and, if you solve this equation I will get c is equal to 1 by 3 and k is equal to 1 by 4. So, after solving c I can just put value of c into this equation to get the value of k.

Now, as I told you for lumping these loads at two locations. So, that I will get true power loss and 2 voltage drop true voltage drop, we have got this constant c and k.

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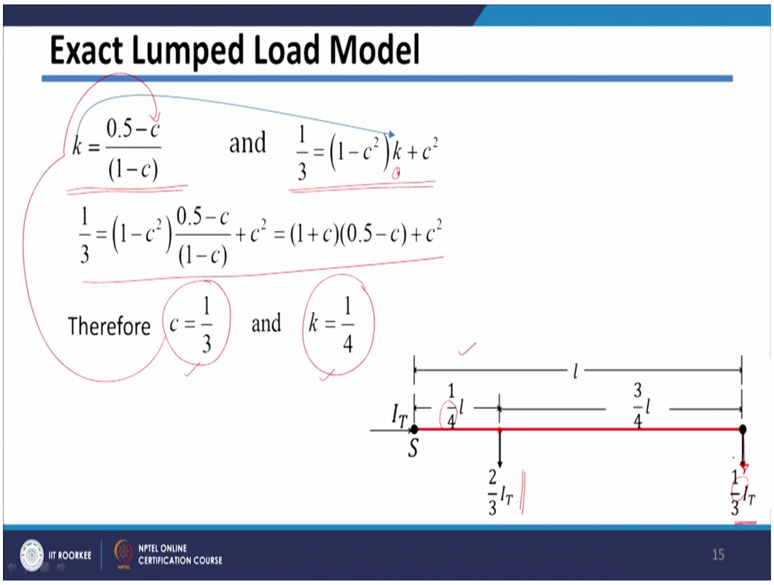

So, in this particular figure we have got this k here and c here and if you put them, I will get k is equal to 1 by 4 and your c is equal to 1 by 3. So, 1 by third I T is flowing through or 1 by 3 I T is lumped at and means remaining 2 by 3 I T should be lumped at this location.

And distance of this location is given by this that is one-fourth of the length, we have to lump this two-third load and, then at the end we have to lump this remaining one-third load. And if you calculate the voltage drop and, the power loss using these two lump loads it will come accurate, it will not be like lumping at one location, where we are getting different location for true power loss calculation and, different locations for voltage drop calculation.

So, in the summary of this lecture we are seen uniformly distributed load, many times they are observed or you need to approximate because, we cannot model all the loads in the distribution system. And we have seen that if there are such uniformly distributed loads in the distribution system. We can lump them at one location, but we have seen that when we are lumping the loads are uniformly distributed loads at one location.

The calculated voltage drop and calculated power loss, they are not true means for voltage drop calculation. We need to lump this load at some location that will not give correct power loss; however, whenever you are calculating location for power loss we will not get current voltage correct voltage drop. So, we have seen that to get both these values correct. So, we need to lump these loads at two locations.

Thank you.