Electrical Distribution System Analysis
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Lecture – 39
Direct Approach for Short-Circuit Analysis:
Weakly Meshed System

We are studying Short-Circuit Analysis Based on Direct Approach. And last three lectures, we have seen four types of faults that is LG fault line to ground fault, LLG fault line to line to ground fault, triple LG fault LLLG fault, and line to line fault that is LL fault. However, we have seen these faults for only the radial system. However, we have seen that many cases, there is possibly that system is operated as weakly meshed.

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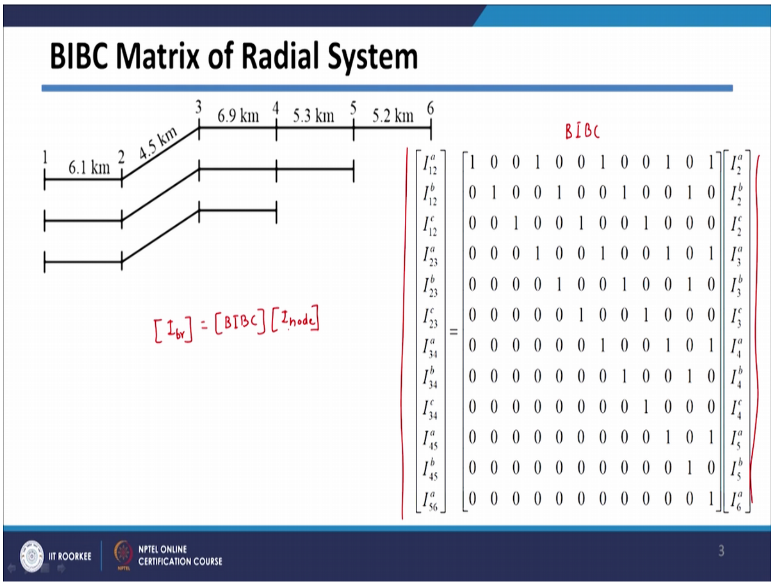

So, in today's lectures, we will see how we can do the Short-Circuit Analysis for Weakly Meshed System. So, in case of weakly meshed system, before going to the weakly meshed system, let us see the for radial system. So, in case of radial system, we have seen our branch currents into the system. So, these are basically branch current can be calculated from nodal currents, using this BIBC matrix. So, we have seen this, so we will just collectively I can write this as I branch will be equal to your BIBC matrix into your I node currents.

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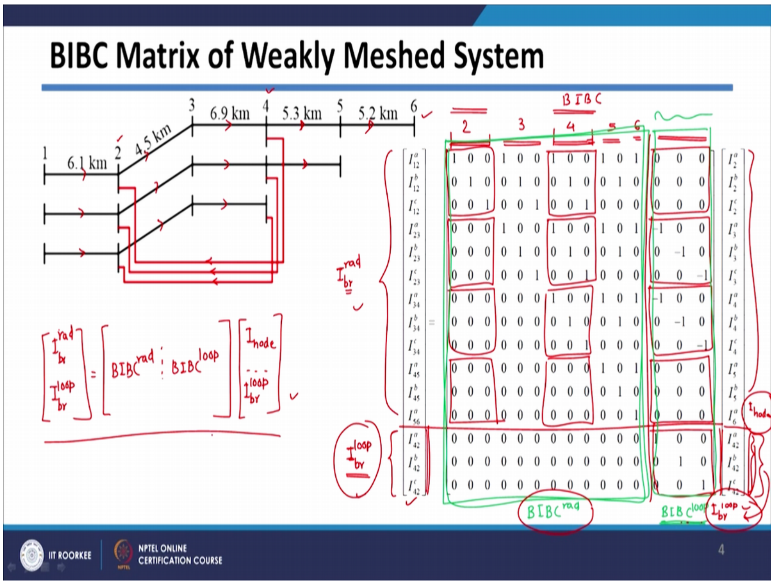

Now, in case of weakly meshed system, let us consider this system here. So, here I am considering one there is one line, which is meshed one. So, here the black one is radial system. However, there is one line, which is meshed, which is carrying current into this direction. So, here in this case currents are actually I am considering line currents, which are in this direction; And through this meshed branch, the currents in this direction.

So, for this if you get BIBC matrix, we already seen in case of load flow calculation, how to get the BIBC matrix for weakly meshed system, I am just revising it here. So, we have got this BIBC matrix here. So, before so we have got this BIBC matrix here, these three row columns are corresponding to phase bus 2, these three columns are corresponding to bus 3, these three column corresponding to bus 4, then these two columns are corresponding to bus 5. And this one column is corresponding to bus 6, because there is only one phase here in bus 6, there only two phases in bus 5 that is why only two columns are there corresponding to bus 5, and for remaining 3 buses, all the three columns are there.

And we have seen that we can easily get these branch currents. So, here if you divide this matrix into these parts, so here till now, when the system was radial, so up till this, these are actually radial branch currents. So, from here to here, there radial branch currents. This part is actually radial loop currents; sorry this is actually loop currents, this also currents, which are flowing to the loop branches. So, these are the radial branches.

So, I can call this as I b r radial, and this is nothing but I branches in loop. So, this particular part I am calling I branches, current in branches. But, from the loop part, and this is current through branches, but in radial part. This part, and this part is same, so this part will be also called as I b r loop. So, this particular part is also called as I b r loop, and this is nothing but your I nodal. So, these currents are I node.

And we have got this part of BIBC matrix by subtract, since it is connected between 4 and 2, so what we you have to do is column corresponding to 4 will be subtracted from the column corresponding to 2. So, in this case, if you subtract this block from this block, it is since it is same block we have got the block of 0 0 0s here. If you subtract this block from this block, so it will be minus 1 minus 1 minus 1, so block of minus 1 minus 1 minus 1 (Refer Time: 05:37). Similarly, if you subtract this block from this block, it will be again minus 1 minus 1 1. So, you have got this minus 1 minus 1 here. And if you subtract this block from this block, it will be 0 0 0. So, it is 0 0 here.

And then since we are actually on this side also it is same loop currents, here also it they are actually same loop currents that is why just there will be diagonal matrix of 1 1 1 to equate same quantities on this side, and same quantities on this side. So, this is basically your BIBC matrix of weakly meshed system. Now, if you consider this, this part of the matrix, since it is corresponding to radial branches I am calling it as a BIBC corresponding to radial.

And if you see, this is getting multiplied with branch current, this particular part, so I am calling it as BIBC corresponding to loop currents. So, this particular part from here to here, I am calling all the column corresponding to this block is BIBC loop, and all the column corresponding to this particular block, they are basically BIBC radial system. So, here, if you write in short form this equation, it will be equal to on the right hand side we are having currents. So, currents, it is the radial current, so branches current in radial branches, current branch current in loop branches, which will be equal to so this two terms I have taken.

And then we have seen this part of the BIBC matrix, we are calling BIBC radial part, and this part of columns we are calling BIBC, which is basically loop, and this is getting multiplied with this part we have call I node, so it will be I node here, and this part is again branch loop. So, this particular matrix BIBC matrix I can write it like this here. So, for doing the short-circuit analysis, I will directly use this matrix here.

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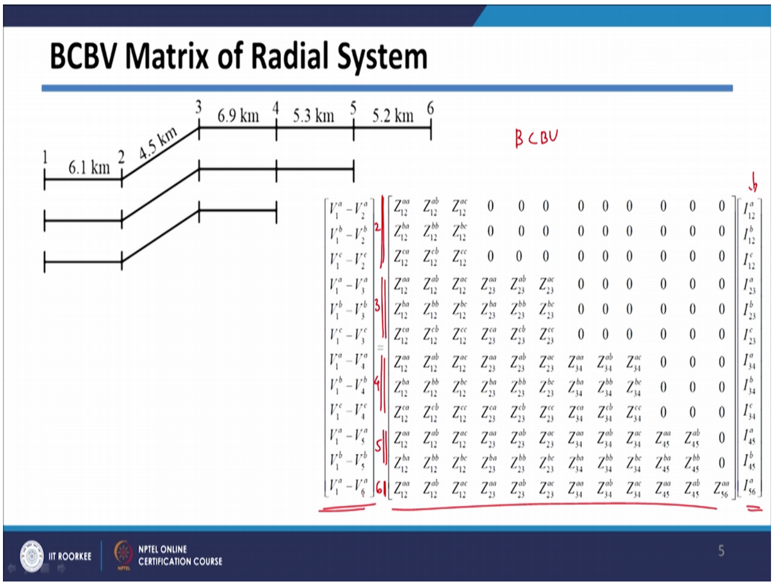

And if you see the BCBV matrix of radial system, we already seen the matrix will be something like this, there will be voltage differences on this side. And this is your BCBV matrix, and this is your branch currents matrix. So, this is branch current matrix part. This is corresponding to bus number 2, this is corresponding to bus number 3, this is corresponding to bus number 4, these two rows corresponding to bus number 5, and this one row is corresponding to bus number 6.

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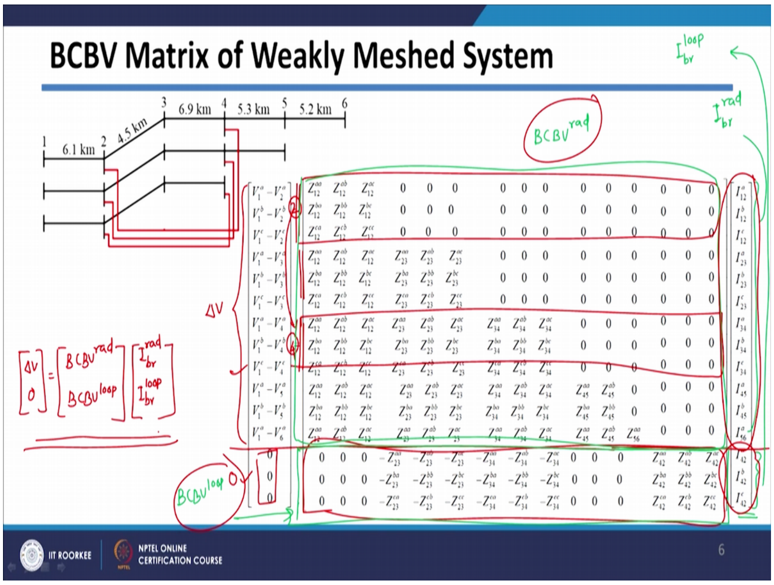

Now, if you consider that earlier weakly meshed system, this part you can see that this part is remaining same as it is. These voltage magnitudes, we have seen that we call them as delta V; this will be matrix of 0s. This part of the matrix, we are getting subtract. So, we have seen that this is corresponding to this is corresponding to bus 2, and this is corresponding to bus 4.

So, if you subtract this 4 from 2, so this particular rows minus this rows, because actually your weakly meshed system is connected between 2 and 4. So, we need to subtract this rows corresponding to 4th bus from rows corresponding to the 2nd bus. So, after doing this, we have got this part here, and already explain even during the load flow calculations.

So, in this case also, this whole matrix I am dividing into dividing into two parts. So, this part we have seen that it is corresponding to radial, so this will be called as BCBV radial part. And this bottom part is corresponding to loop forming branches. So, it will be called as, so this bottom part will be called as BCBV loop. And this again branch currents. So, this part is branch current corresponding to radial. And this part is loop current. So, this part I am just writing it here, if you take it here, it will be I b r corresponding to loop.

So, if you write this whole matrix into shortcut or short form, it will be delta V 0 on this side. So, I have taken these two quantities here, this one, and this one. Then BCBV matrix we have divided into 2, which is some part is radial, and bottom part is loop. So, this is your BCBV corresponding to radial part, and BCBV corresponding to loop part, and it is getting multiplied with respect to your branch currents. So, on the top, this one is nothing but branch current corresponding to radial branches, and this part is nothing but corresponding to loop for main branches, so I b r loop. So, this is corresponding to loop branches. So, in short form, we have got this kind of BIBC and BCBV matrices.

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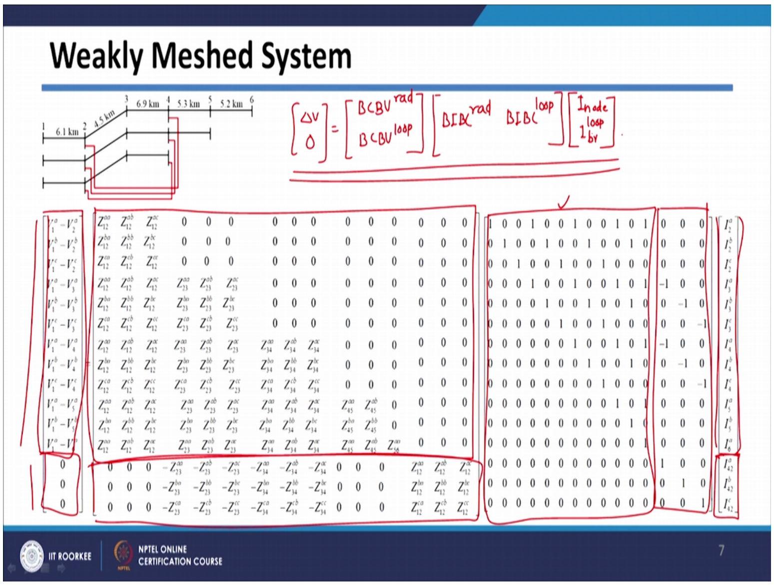

And collectively if you see, it will be like this. So, in this case also I can easily write like we have written for earlier case, this is having two parts that is delta V and 0. So, this is corresponding to 0, and this is corresponding to delta V. So, this is delta V r. And this part of BCBV matrix we have considered, it is corresponding to radial system, and this part of BCBV we have considered loop system.

Then in case of BIBC matrix, this part of the matrix is corresponding to radial system, because it will get multiplied with nodal currents. So, it will be BIBC corresponding to radial, basically this part, and this part of the matrix here is BIBC corresponding to your loop forming branches, and it will get multiplied with respect to these are the nodal currents up to this. So, it will be I node, and these are nothing but your branch currents, but loop forming branches. So, I branch loop. So, we have got this matrix here.

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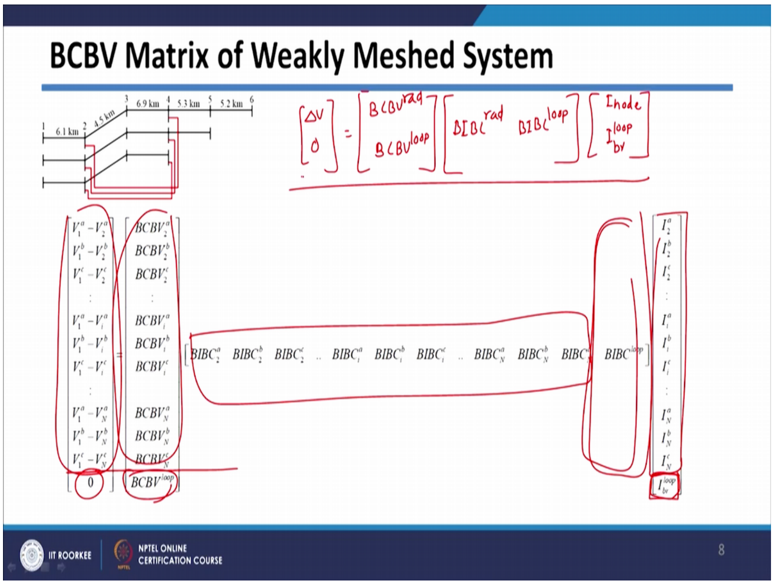

And if you write this matrix for any generalize expression, it will be like this, so ok. So, here, so this is corresponding to loop, this is corresponding to loop here, and these are corresponding to loop. So, whatever matrix we already written it here, can be written it like this here. So, this is this part is again your delta V, this part is 0 will be equal to this part is radial, this part is loop, this part of BIBC is radial, and this part of BIBC is loop, and it is getting multiplied with this part is I node nodal currents, and this part is branch current in loop forming branches. So, basically structure of your system will be something like this.

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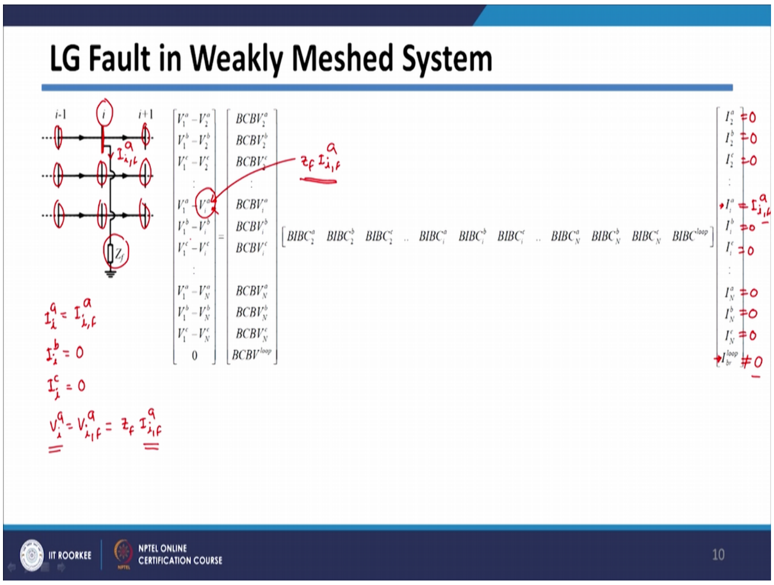

Let us see how to analyze LG fault in weakly meshed system. So, in case of weakly meshed system, suppose the LG fault has occurred at this ith bus, which is part of very large network. So, in this case, we can write the boundary conditions like this. So, current in a phase will be nothing but your fault current in a phase, and current in b phase, since is not faulted it will be 0, and current in c phase will be also 0, because it is not faulted. And one more boundary condition in terms of voltage, we can write, so voltage of bus a, which will be basically post fault voltage of bus a, which will be given by Z f multiplied by your fault current in a phase. So, this is your fault current in a phase, and multiplied by this impedance Z f will give me voltage of this ith bus, and a phase.

So, we have got these three boundary conditions here. And this is our generalize matrix, which we have discussed in earlier slide. So, if you see here, this currents accept current in phase a, means accept these current here, as well as the loop forming branch current, because this short-circuit current will try to flow from the loop branches also. So, there will be non-zero currents through loop branches. So, this will be non-zero, as well as this will be your fault current in a phase.

However, all other load currents will be 0 means negligible as compared to this fault current. So, therefore, we can put all the nodal currents to be equal to 0. So, in that case, this all these all currents, they will be equal to 0, or negligible as compared to your fault current. One more modification will happen into this matrix is this entry here, we know will get replaced by Z f multiplied by your fault current in a phase, because here we have return this voltage of a phase of ith bus will be Z f multiplied by fault current in a phase of ith bus.

So, if you replace these entries means by replacing the current with 0 except these two entries, and this voltage will get replaced with this one. And here I am representing all the voltages, which are basically post fault voltages by putting comma there.

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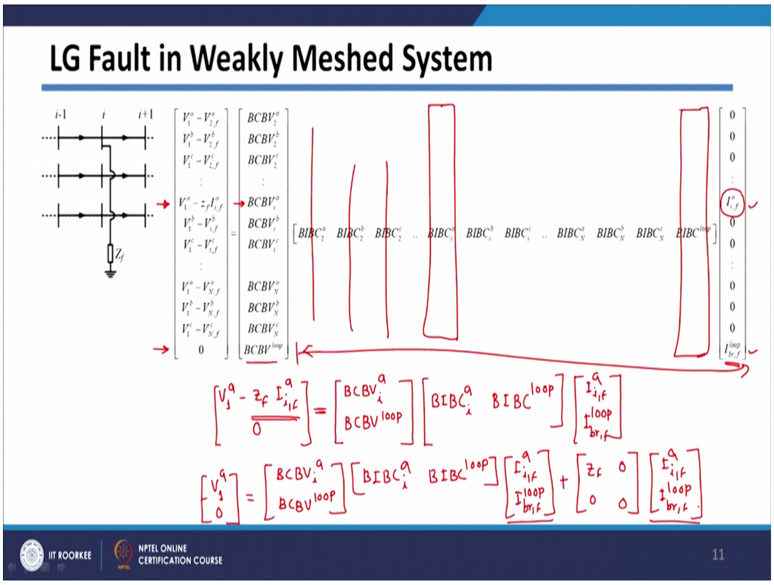

So, by replacing these entries, so we have basically replace this entry here. And these two entries, which are non-zero, other entries are 0 here. So, in this case to get the fault current value, what we can do, we can just consider this expression here, or only this row.

So, in this case, I can easily write, so V 1 a minus Z f into fault current in a phase. And we need to consider this row also, because this is getting multiplied with non-zero entry, so it will be equal to 0, and this is getting multiplied, or equal to this will be this row here, we need to consider that is BCBV corresponding to a phase ith bus, and BCBV corresponding to loop also will come into picture, because we are considering these two rows, so BCBV loop, which is get multiply.

And if you observe this part of matrix from here to here, you can see that all these BIBC columns, they are getting multiplied with respect to 0 except these column here, which is getting multiply with non-zero entry that is fault current. And this column here, which is getting multiply with your branch currents loop forming branches.

So, all other entries are 0. So, we can just take those entries here. So, this is BIBC column corresponding to a phase ith bus, and BIBC part corresponding to loop forming branches, and which will get multiply with your fault current in a phase, because this, and this is only non-zero entry here, and fault current in loop forming branches.

So, we can rewrite this equation here, so in this case, it will be V 1 a, so what I am doing and 0, I am taking this Z f multiplied by fault current in a phase on right hand side. So, here it will be V 1 a, and 0 will be equal to the same term that is BCBV corresponding to a phase, BCBV corresponding to loop forming branches, which will get multiplied with BIBC column corresponding to a phase, and BIBC columns corresponding to loop forming branches, which is getting multiply with fault current in a phase, and fault current in loop forming branches plus this term I am just writing it like this Z f 0 0 0.

And it will get multiply with fault current in a phase, and fault current in loop forming branches. So, here the same term we can just see at, because all entries are 0. So, only Z f multiplied by fault current in a phase will remain here. So, from here if you can see this term is common, I can take common out.

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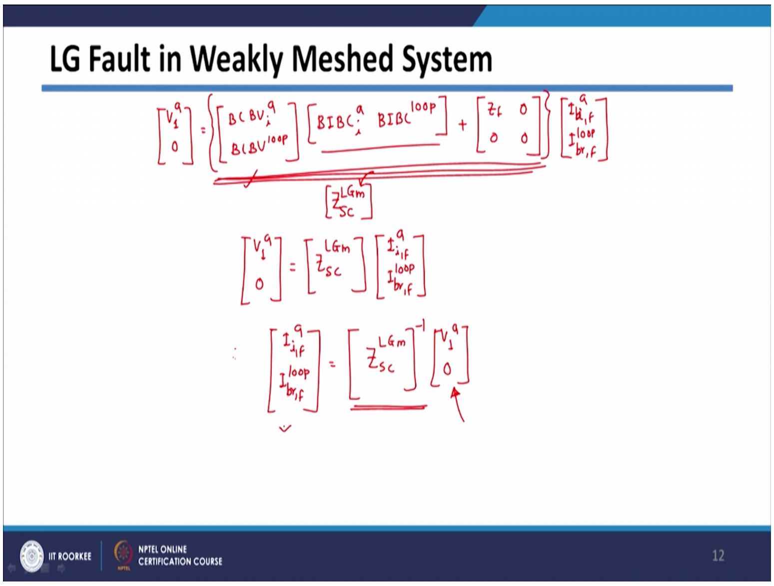

So, I can rewrite this equation like this. So, on this side it is V 1 a 0, which is equal to BCBV corresponding to loop forming branches, which is getting multiply with BIBC corresponding to a phase, and BIBC corresponding to loop forming branches plus Z f 0 0 0. And since we are taking common out from both the terms; so, it will be I fault current in a phase, and loop forming branches.

I am calling this matrix, this part of matrix has Z short-circuit corresponding to LG fault in meshed system. So, this particular part of the matrix will be called as Z LG m, because it is actually meshed network I am considering. So, I can just represent it like this. So, V

1 a 0 will be equal to your, this matrix short-circuit matrix LG m multiplied by your fault current in a phase, and loop forming branches. So, we can easily write, your fault current in a phase, and corresponding currents in loop forming branches, because this fault current may flow through some loop forming branches, so Z sc LG meshed inverse multiplied by V 1 a 0.

So, in this case, since this is voltage of first node and 0; so, this part of matrix is known, this if you observe this consists of BCBV matrix and BIBC matrix, which can be formed using your network data, as well as impedance data, and fault impedance. So, to calculate this Z sc LG for LG fault in meshed system, we need this data here, which is available and we can easily calculate it. And from that you can easily calculate your fault currents in LG for LG fault in weakly meshed system.

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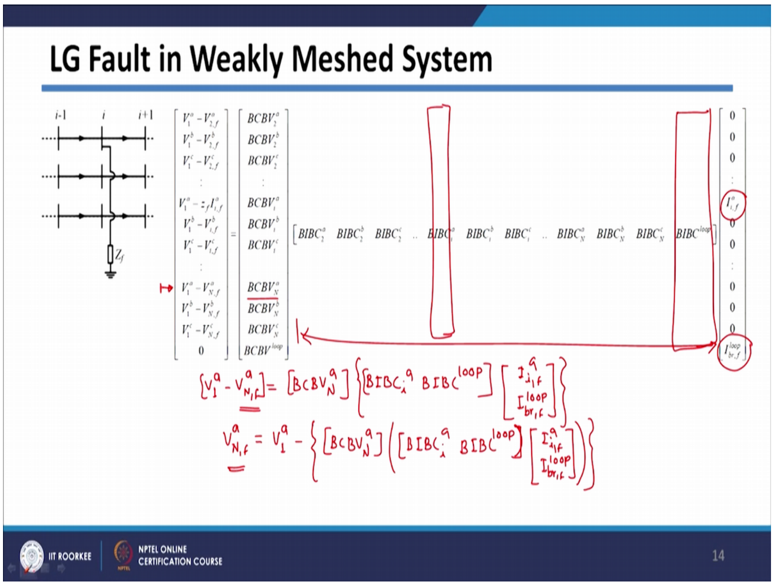

Now, let us see how we can get post fault voltages in this case. So, to calculate post fault voltages, say you want to calculate post fault voltage of say bus, or nth bus. So, in that case, I can take this expression here of this row, so it would be V 1 a minus V post fault voltage of nth bus will be equal to your BCBV. And we have seen that from this part to this part, we can represent just by since this column is getting multiply with respect to this, and these columns corresponding to loop forming branches getting multiplied with this one.

So, this can be written as BIBC a phase, and your BIBC I just erase this, so it will be BIBC corresponding to loop forming branches multiplied by your fault current in a phase, fault current in loop forming branches. So, we can easily write from this expression, your V N F a post fault voltages of nth bus will be equal to V 1 a minus this total term here, which is BCBV multiplied by your BIBC of loop forming branches, which is getting multiply with fault current in a phase, fault current in loop forming branches. So, this how we can get the voltages post fault voltages of any bus.

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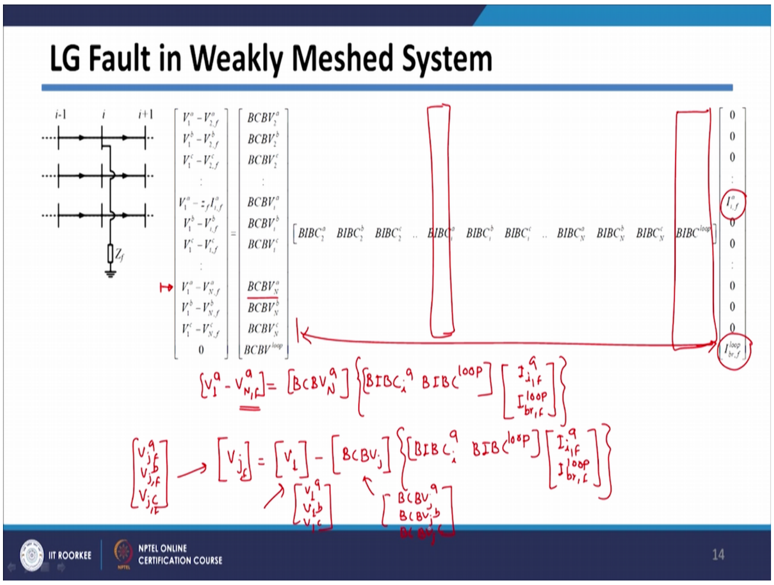

So, in generalize sense, I can easily write. So, in generalize sense, I just erase this here. So, I can write V j will be equal to V 1 minus BCBV of j multiplied by this term will remain common same, BIBC of a phase, BIBC corresponding to loop forming branches, and this will get multiply with fault current in a phase, fault current in loop forming branches.

So, here we know that this V j is nothing but V j a, V j b, and V j c to represent post fault, I can just put comma F comma F comma F here also I can put comma F, so V j F. And this we know that V 1 of a phase, V 1 of b phase, and V 1 of c phase, three phase voltages of first node. And this we know that it is BCBV of a phase, BCBV of b phase, and BCBV of c phase. So, this how we can calculate post fault voltages of any three phase buses.

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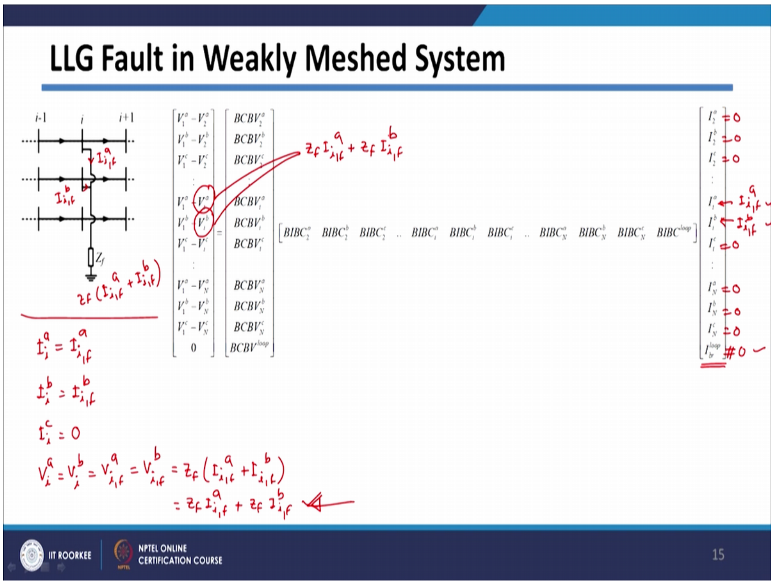

Now, let us see how we can consider LLG fault in weakly meshed system. So, in case of LLG fault, there are two buses, which will be faulted with ground. So, here I am considering phase a, and phase b, which is faulted with respect to ground. So, let us say current in a phase is post fault current in a phase, and current in b phase is post fault current in b phase.

And therefore, voltages will be Z f multiplied by post fault current in a phase plus post fault current in b phase ok. Therefore, boundary condition I can write it like this. So, first boundary condition will be corresponding to a phase current will be basically post fault current in a phase. And then corresponding to b phase, which is basically post fault current in b phase. And current in c phase will be equal to 0, because it is not faulted. And then voltages of a phase, as well as voltage of b phase will be equal to post fault voltages of a phase, and then post fault voltages of b phase, there will be equal, and that is given by Z f multiplied by your current in a phase plus your current in b phase, and this would be equal to Z f into current in a phase plus Z f multiplied by current in b phase.

So, if you observe this generalize matrix here, will easily see that the currents except these two currents, this one, and this one, as well as the currents, which are in loop forming branches will be non-zero. So, this will be non-zero here. Otherwise, all the currents will be 0, so these currents are 0 here.

And we have seen that this current is post fault current of a phase, and this current is post fault current of b phase. So, only these entries will be non-zero here. And if you see on this side left hand side, this voltage, and this voltage, we have seen that those are post fault voltages. And these post fault voltages are same, and given by this expression here, which is Z f multiplied by fault current in a phase plus Z f multiplied by fault current in b phase. So, if you replace these entries in this matrix, I will get this matrix here.

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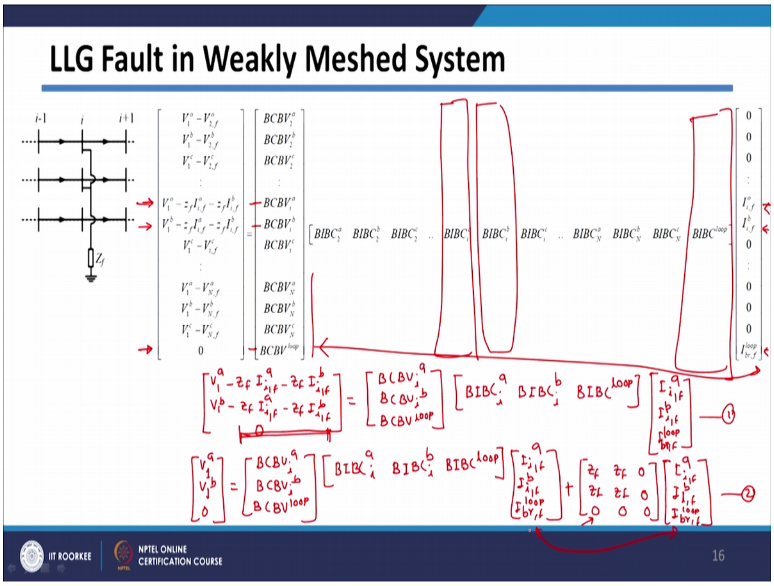

So, these corresponding entries I have just replaced in this row, and corresponding currents I replaced in this column here. So, except these two currents, i and b, and branch currents in loop forming branches, all other currents are 0. So, from this to write your fault current equations for LLG fault in weakly meshed system, we need to consider these three types of rows; So, these two rows, and rows corresponding to your loop for being branches.

So, in this case, I can write. So, I can take these rows, so V 1 a minus Z f into fault current in a phase minus Z f into fault current in b phase. Similarly, second row V 1 b minus Z f into fault current in a phase Z f into fault current in b phase, and here it will be 0, so this is your left hand side. From the right hand side, when you take this row, this row, and this row, so it will be BCBV corresponding to a phase, BCBV corresponding to b phase, and BCBV corresponding to loop forming branches, so these three rows.

And if you observe this matrix part here as we discussed here, only non nonzero multiplying columns are basically these two, and the columns corresponding to loop forming branches. So, in this case, I can just write this part of the matrix, which will be BIBC i a, BIBC b phase, and BIBC corresponding to loop forming branches, and which will get multiply with fault current in a phase, fault current in b, phase and fault current in loop forming branches. So, this is nothing but now what I am doing I am taking this part of matrix that is Z f multiplied by your fault currents on left hand right hand side.

So, in this case, it will be on left hand side it will be V 1 a V 1 b 0, which will be equal to so here this will be as it is, so BCBV corresponding a, BCBV corresponding to b, and BCBV corresponding to loop forming branches, which will get multiplied by BIBC corresponding to a phase, BIBC corresponding to b phase. And BIBC corresponding to loop forming branches, which will get multiply with this term here that is fault current in a phase, fault current in b phase. And fault current in loop forming branches plus the terms which are considered, here which I am taking on right hand side, can be written it like this, it will be Z f Z f 0 Z f Z f 0 0 0 0, and this will get multiply with fault current in a phase, fault current in b phase, and fault current in loop forming branches.

However, the multiplication with loop forming branches 0 0 0, so it will not come into picture. Similarly, the row the last row in this matrix is 0 0 0, which would not come into picture. So, it correctly represent your terms in left hand side of equation first. So, this equation 2, I will take next slide. By taking these terms, which are common fault current, and fault current into loop forming branches, which are common, I can take them out.

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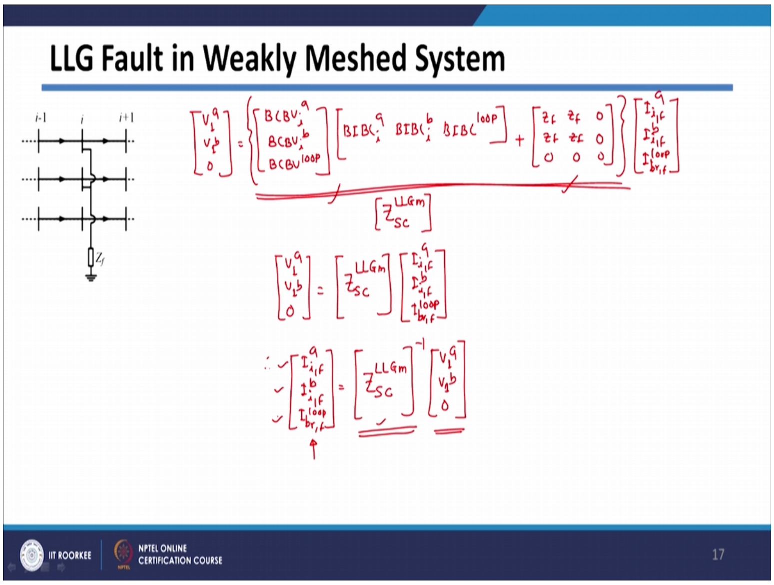

So, in this case, I can just write it like this. So, it will be V 1 a V 1 b 0 will be equal to BCBV corresponding to a, BCBV corresponding to b, BCBV corresponding to loop forming branches, which will get multiply with BIBC corresponding to a phase, BIBC corresponding to b phase. And BIBC corresponding to loop forming branches plus Z f Z f 0 Z f Z f 0 0 0 0, and the term which is taken out common is fault current in a phase, fault current in b phase, and fault current in loop forming branches.

And here this term of matrix equation, I am representing at like this Z short-circuit LLG fault, and for meshed network. So, then this system can be written as V 1 a V 1 b 0 will be equal to this matrix here, so Z short-circuit LLG meshed network multiplied by fault current in a, phase fault current in b phase, fault current in loop forming branches. So, I can say that fault current in a phase, fault current in b phase, and fault current in loop forming branches can be calculated by taking inverse of this matrix Z short-circuit for LLG fault in meshed network inverse multiplied by V 1 a V 1 b 0.

And as we discussed here this Z short-circuit matrix, we are getting it from BCBV and BIBC matrices, which basically depends on topology of system, and your fault impedances. So, once you know, this short-circuit matrix, and the voltages of first node I can calculate your short-circuit current in a phase, b phase, and all the loop forming branches.

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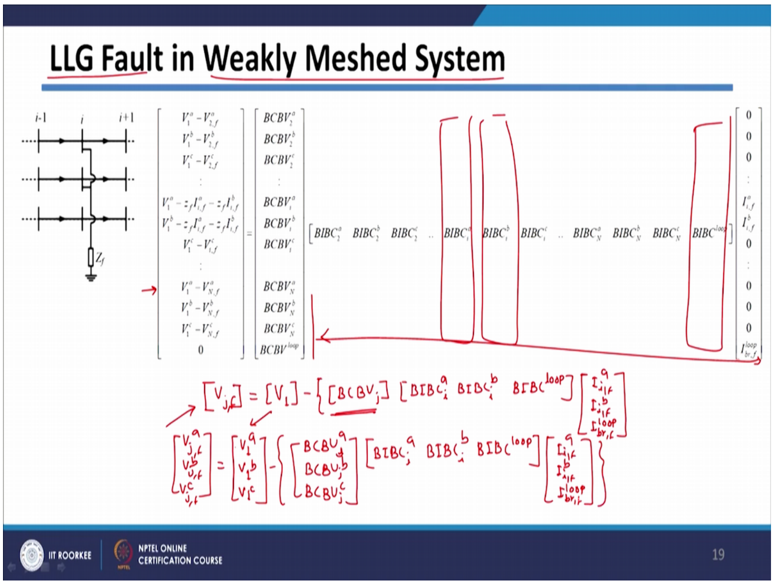

In case of post fault voltages for LLG fault in weakly meshed system, the expressions will be same. Just what you have to do, you have to take the expression here. So, it will be directly I am writing. So, it will be V j will be equal to V 1, so V j F fault minus it will be BCBV corresponding to jth bus, which will get multiply with this part of the matrix, which is BIBC, because only this three types of columns, they are getting multiplied with non-zero entries. So, BIBC a, BIBC corresponding to b phase, and BIBC corresponding to loop forming branches, and this is multiplied with fault current in a phase, fault current in b phase, and fault current in loop forming branches.

So, here this is nothing but voltages of all the three phases. So, we know that it is V j fault a, V j b, V j c. So, post fault voltages of all the three phases of jth bus will be equal to V 1 a, so this will be represent like this V 1 b, V 1 c, all the three phase voltages of first node minus in this case this will be three rows. So, BCBV j a, BCBV j b, BCBV j c, which will get multiplied this term will remain common that is BIBC a phase, BIBC corresponding to b phase, and BIBC corresponding to loop forming branches. And this will get multiply with your fault current in a phase, fault current in b phase, and fault current in loop forming branches.

So, this how we can calculate the post fault voltages of any bus in weakly meshed system for LLG fault. So, in summary of today’s lecture, we have seen how to get fault currents, in case of weakly meshed network. So, we have consider two faults, one is LG fault, in case of weakly meshed system, and LLG fault also in case of weakly meshed system. And for those faults we have calculated fault currents, and post fault voltages, we have derived the expressions for these fault currents as well as post fault voltages. In the next class, we will see the applications of these analysis methods, which are basically load flow analysis, and short-circuit analysis.

Thank you.