Electrical Distribution System Analysis
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Lecture – 38
Direct Approach for Short-Circuit Analysis:
LL Fault and Examples

Dear students, we are learning Direct Approach for Short-Circuit Analysis. And in last two lectures, we have seen three types of faults that is LG fault line to ground fault, double line to ground fault LLG fault, and triple line to ground fault. And then we have derived various fault current equations for them, as well as post fault voltages for all these three types of faults. Today, we are going to see line to line fault.

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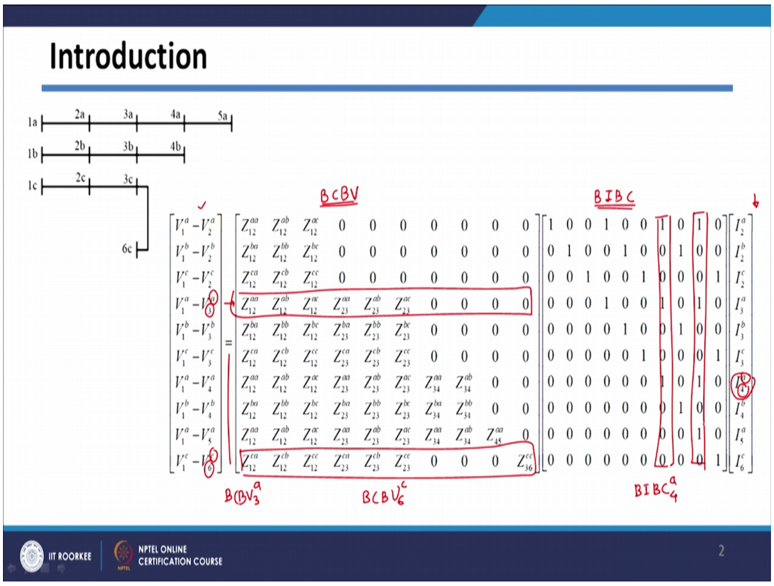

So, before going to the line to line fault, just revise what we have seen. So, we have seen that in case of line to line fault, we have divided our BCBV matrix. So, in case of direct based approach, we know that we are having this BCBV matrix, and your BIBC matrix. And here we are having voltage difference with respect to first node, and these are basically nodal current matrix.

So, and this BCBV and BIBC are calculated based on the connections of the distribution systems, and impedances of the distribution system. And in the last class, we have seen that these BCBV matrices will basically divide into rows. And we have given the names for specific rows, say if you are considering these particular rows, we have given name for this rows as BCBV, since it is corresponding to a phase. So, a and corresponding to 3rd node, so 3. So, we have call this row as BCBV 3 a.

Similarly, if you consider this row, it will be BCBV; it is corresponding to c phase. So, I will write c here, and since it is corresponding to 6 node, I will write 6 node here. Exactly similar way, we have divided our BIBC matrix, but column wise. So, if you are considering this column, since it is corresponding to this particular current that is then we can call it BIBC, since it is corresponding to node 4, so I 4, and then corresponding to a phase, so I should write a, this how you can calculate we can give the names for each of the column.

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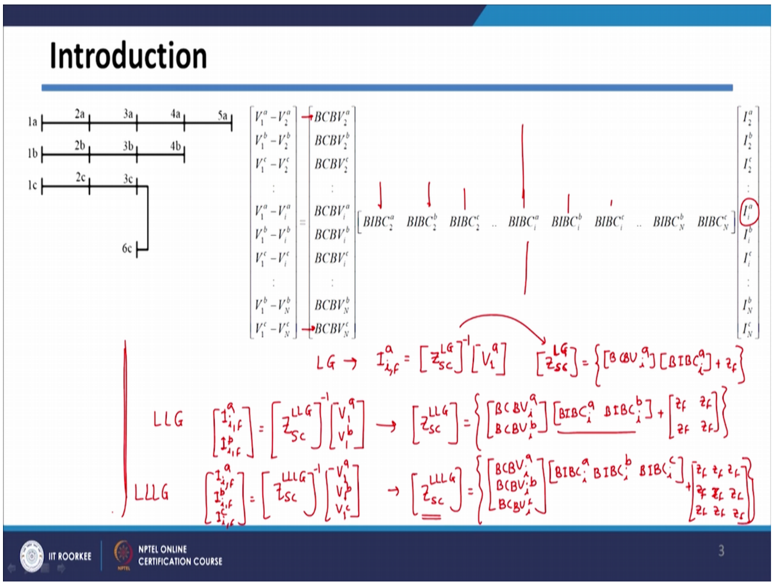

And once we do this, we will get this matrix. Where these are the rows of BCBV matrices; and these are columns of BIBC matrices. If you remember in last two classes, we have derived the various equation, short circuit current. So, we have derived that in case of LG fault that is line to ground fault your post fault current. And say if the fault is occurring on phase a of ith bus, it will be equal to your Z s c LG, and then it is inverse into voltage at 1st bus.

And then, we have seen that this Z s c LG short circuit a for LG fault will be equal to your BIBC, in corresponding to your ith column. So, this particular column, if I am considering the fault is occurring on this particular bus here, and sorry it should be

BCBV, so this is BCBV, and then you are having this BIBC corresponding to ith bus and a phase plus your Z f is your short circuit of LG fault impedance matrix. Similarly, we have derived for say LLG fault. So, in that case, there will be two currents, fault current in phase a, fault current in phase b will be equal to we have seen that here in this case, it is short circuit matrix corresponding to LLG fault, and it is inverse, and it will be get multiplied with voltages of first node of a phase and b phase.

And in this case, your Z short circuit for LLG fault is equal to you have seen, it is we need to take two columns corresponding to BCBV, corresponding to a phase BCBV matrix current pointing to b phase multiplied by you have to take column corresponding to BIBC column corresponding to a phase of ith bus. And BIBC column corresponding to b phase and ith bus, and plus Z f Z f Z f matrix of 2 by 2. So, it was actually your Z impedance matrix for LLG fault.

And in case of LLLG fault, we have seen in this case, there will be all the three currents phase, and short circuit current. So, short circuit current corresponding to phase a, short circuit current corresponding to phase b, and short circuit current corresponding to phase c, will be equal to in this case yours impedance matrix corresponding to LL triple LG fault LLLG fault, it is inverse, and in this case there will be all the three voltages V 1 a V 1 b and V 1 c.

And in this case, your impedance matrix will be this is short circuit matrix corresponding to LLLG fault will be equal to in this case, we need to take all the three rows of BCBV corresponding to that bus that is BCBV corresponding to a's phase, BCBV corresponding to b phase, BCBV corresponding to c phase, and here I we need to take columns of all the three phases of BIBC matrix BIBC of a phase, BIBC of b phase, BIBC of c phase plus there will be zth matrix of 3 by 3 size. So, this will be your short circuit matrix for LLLG fault. So, we have seen these things in last two classes. Let us go ahead with LL fault.

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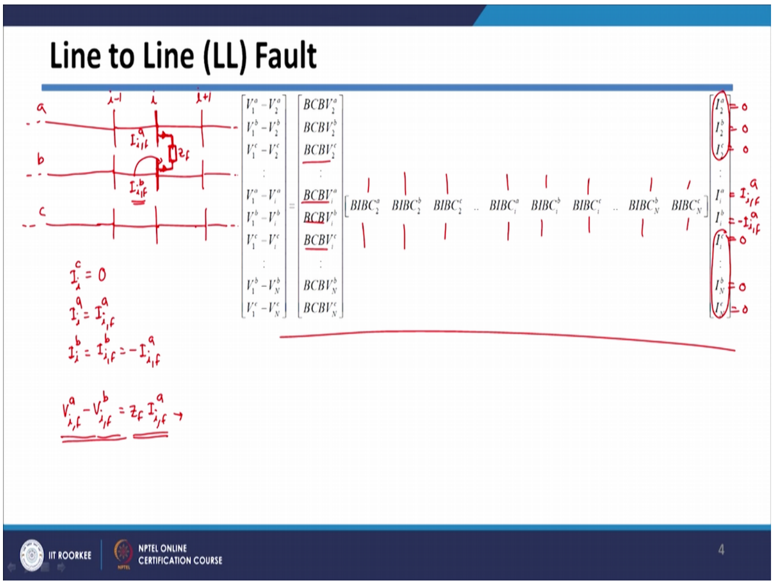

So, in case of LL fault, we will be say this is your distribution system, say this is your ith bus, this is your phase a. And then these is your phase b and say this is your phase c. This is your ith bus, they say this is the i plus 1th bus, and this is i minus 1th bus. This is your b phase, and this is your c phase and it is part of big system, so that is why I am showing dot dot here, so that it will be of part of any big system.

And let us say this is short circuited with b phase. So, phase it is getting short-circuited with b phase with say impedance Z f. So, in this case, this is current fault current in a phase, and we have seen that this comma f notation, we are using to represent post fault quantities. And this current is post fault current in b phase. So, this (Refer Time: 08:49) will be current is fault current in b phase, and this particular current is fault current in a phase. And you can see that in this case, they are opposite in direction.

So, if you consider the terminal conditions, so in this case, since c phase is not faulted. So, you can say the c phase current will be equal to 0. Then a phase current will be equal to say post fault current of a phase. And in case of b phase, it will be post fault current of b phase. However, we can see that these two currents are in opposite in direction here, so this current exists in this direction, and fault current in b phase is actually in this direction. So, I can just say, this is equal to minus of fault current in a phase.

Now if you see a voltage equation, so you can say voltage difference between phase a and phase b that will be equal to voltage across this impedance here. So, in this case, I can write post fault minus voltage at bus b post fault will be equal to Z f into your fault current in a phase. So, this voltage of a bus, and this voltage at b bus will be nothing but the impedance drop across these two phases. So, this equation is valid there.

Now, if you see this matrix, which we have got, where which we have used for load flow calculation. And where, we have written all the BCBV matrixes as row, and all the columns of BIBC matrices are shown here. So, these are nothing but columns of BIBC matrices.

Now, we can see that since the fault during the fault condition, only these two currents are very large as compared to all other load currents on the all the other buses. So, basically all other currents will be getting equal to 0, except this two current, they will be negligible though, so we are considering equal to 0. So, all these currents, they will be equal to 0, except this two current. So, this current will be equal to fault current in phase a, and this current is opposite of that, so that is why this current is fault current in phase a, but it is in a opposite direction that is why minus sign is there.

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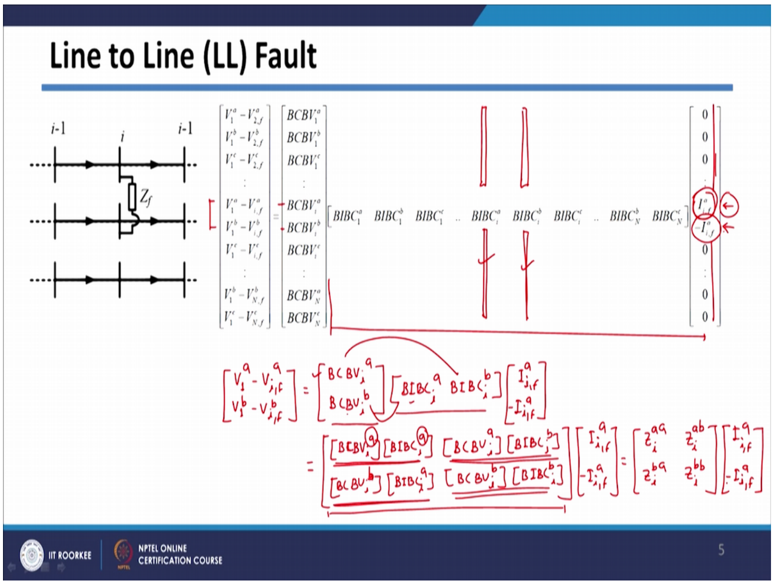

So, basically what we will get is we will get this equation here. So, as I replaced all the currents here with equal to 0, except this two current, which are fault currents. And if you see these two equations, if you write these two equation explicitly, so I am taking these two equations here. And these two equations, if I take it will be V 1 a minus post fault voltages of a bus, and V 1 b minus post fault voltages of voltage of b phase will be equal to your these two matrices will come into picture that is BCBV i a BCBV corresponding to b phase.

And if you see this part from here to here, only this column these two columns, this one, and this one, they are getting multiplied with non-zero quantity. Otherwise, all other columns are getting multiplied with respect to 0. So, only non-zero will be nothing but multiplication of this column multiplied by your this current, and this column multiplied by this current. So, basically it will be equal to BIBC corresponding to a phase, basically this column I am taking BIBC corresponding to b phase, this column I am taking, and this is getting multiplied with two currents only that is current in a phase, which is this one, and current in b phase, which is opposite of a, so that is why I can put minus sign here.

Now, if you explicitly multiply these two vectors, so I can get four parameters here that is b BCBV i a, which will get multiply with respect to this. So, I am just multiplying this vector with respect to this vector. So, there will be four terms, this is your first term, which is multiplication of this and this. Then if you multiply this with respect to this, I will get another term that is BCBV i a multiplied by BIBC i b. And if we multiply this term with this, it is BCBV corresponding to a phase, and BIBC sorry corresponding to b phase here BIBC corresponding to a phase, and fourth element is BCBV corresponding to b phase and BIBC corresponding to b phase. So, we have got these four parameters of this matrix, and these two currents are as it is.

Now, to explicit matrix in short form, I am calling this as Z i aa correspond, because BCBV also a part is coming, BIBC also a part is coming. So, this impedance part, I am calling it as Z aa i. This part these two terms, I am calling Z i ab, this part I am calling Z i ba. And this fourth part is actually Z i bb, and it is getting multiply with this fault currents a and b phase, but they are in opposite direction that is why only one current I have written with minus sign. Now let take these equations on next slide.

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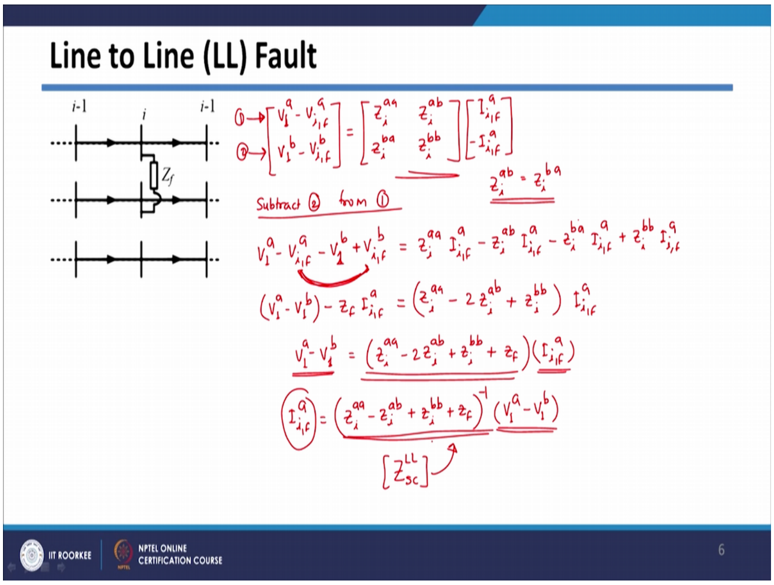

So, we are having these two equation, V 1 a minus V 1 b minus fault post fault voltage of V phase, which is equal to we have divided this matrix into four parts that is aa, Z i ab, Z i ba, Z i bb, and you have seen it is getting multiplied with respect to fault current in a phase, and fault current in b phase, which is opposite of a.

Now, let us subtract this first, or subtract second row from this first row. So, so this is first row, this is second row. So, subtract second row from first. So, if you do this, it will be V 1 a minus post fault voltage of a phase minus V 1 b minus post fault voltage of V phase, it will be plus minus minus plus it will be equal to. In this case, if you observe this matrix, your this actually symmetric, so Z i ab will be equal to Z i ba. So, in this case, if you subtract this first row from second row row, it will be Z i aa into fault current in a phase minus Z i ab into fault current in a phase minus ba into fault current in a phase plus fault current in a phase.

And then we can write this equation, it will be V 1 a minus V 1 b I am taking it together. And we have seen that this subtraction here, we have seen that this subtraction of post fault voltages of a phase, and post fault voltage of b phase is nothing but impedance multiplied by your short circuit current. So, from this, we can easily write this subtraction is nothing but minus Z f into fault current in a phase. So, this minus this will give you Z f multiplied by fault current in a phase. And in this case, we can take this fault current common out, and applying this condition, it will be Z i aa. So, minus twice of Z i ab plus Z i bb, it will be getting multiplied with your fault current in a phase.

And from this Z f we can take on right hand side, so I can write V 1 a minus V 1 b will be equal to Z i aa minus twice of Z i ab plus Z i bb plus your Z f into fault current in a phase will be this one. And we are basically interested in calculating this one. So, we can easily write fault current in a phase will be equal to inverse of this multiplied by this current your voltage vector here. So, it will be Z i a minus Z i ab plus Z i bb plus your Z f, and you need to take inverse of it multiplied by your voltage of a phase, and voltage of b phase at first bus.

And as I told you voltages of a phase and b phase at first node, they are known. These matrices are known, because they are getting evaluated from BIBC and BCBV matrices, which are actually known. So, we can get your fault current here. So, this is equation for so this we have got Z short circuit for LL fault. So, this particular matrix, I am calling Z short circuit for LL fault.

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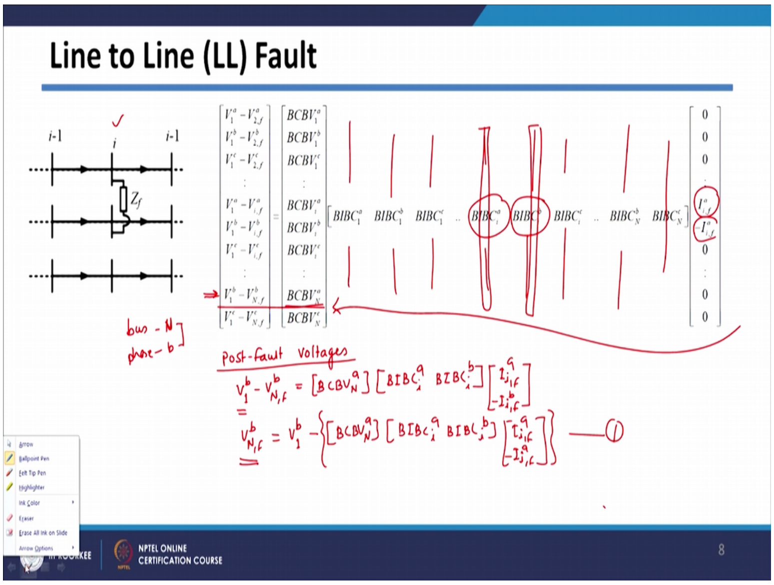

So, let us say if you want to calculate post fault voltages of any buses. Now, to calculate post-fault voltages of any buses, or other than bus i, because I have considered fault had occurred on ith bus. So, other than i, if you want to calculate, voltages post fault voltages at any buses.

Suppose you want to calculate this particular bus, so in this bus, say bus corresponding to nth bus is n, and your phase is b. And for that particular bus, you want to calculate post fault voltages. So, in this case, we need to take this equation here. So, if you take this equation, it will be V 1 b minus V N post fault of b phase, which will be equal to your BCBV matrix, this part will come. So, row corresponding to a phase, and nth bus, which is this.

And this will get multiplied with respect to this part of the matrix, and we have seen that this part of the matrix, where all these columns of BIBC, they are getting multiplied with respect to g, except these two columns, which are getting multiplied with non-zero value. So, this particular two columns, they are getting multiplied with non-zero values. So, I am taking those two columns that is BIBC corresponding to a phase, and BIBC corresponding to b phase of ith bus, and this is getting multiplied with respect to fault current in a phase minus fault current in b phase. And this is basically given fault current we have seen, we have already calculated in last slide. So, fault currents are known, when we are calculating post fault voltages.

So, we can get the post fault voltages. So, in this case, this is known this is unknown. So, I can easily write post fault voltages of b phase of nth bus will be equal to V 1 b, which is b phase voltage of 1st bus multiplied by BCBV of and then BIBC of b phase, and then a phase. So, this give with this will give you post fault voltages of any bus.

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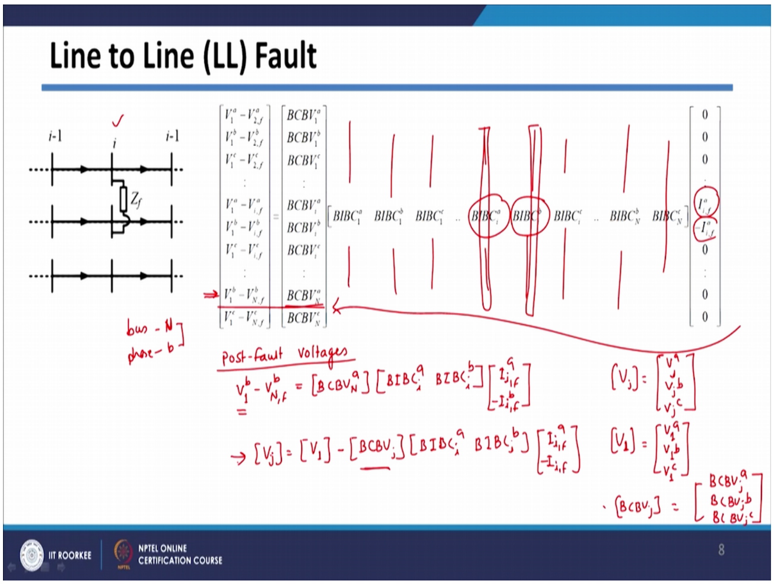

So, in collective term, I can write this equation. So, for any jth bus, I can easily write this equation. So, we are having any jth bus of all the three phases will be equal to your voltages at bus number 1 for all the three phases, which will be minus, and then BCBV matrix corresponding to j phase multiplied by this part of the matrix will remains same for all the calculation, and then it will get multiplied by with respect to this one.

Only difference here, I am considering all the three voltages here means your V j. V j matrix will be having all the three voltages, V j of ath phase, V j of b phase, and V j of c phase. Similarly, this V 1 matrix also will be having all the three voltages that is V 1 a, V 1 b, and V 1 c. And this BCBV j will be also having three terms, so BCBV j matrix will be having three rows, so it will be BCBV corresponding to j a, BCBV corresponding to j b, and BCBV corresponding to j c so all the 3 terms. So, here we are we can get for any unknown bus post fault voltages using this particular expression.

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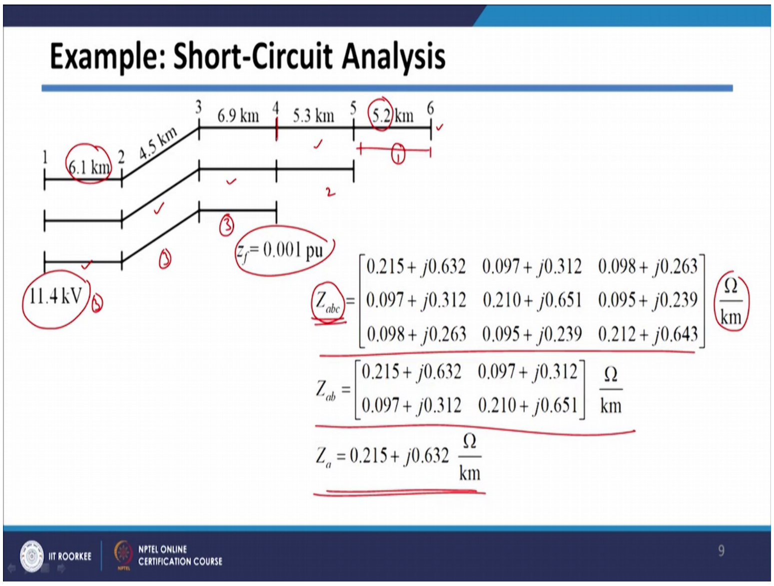

To understand it better, let us take one example, where I consider this system here, which is 6 bus system. However, in some of the sections of the line, there are only 2 lines, and in some of the line, there are only there is only one line. So, in 5 between 5 to 6; only 1 line is there, and then between 4 to 5, 2 lines are there. So, here only 1 line, 2 lines, and here all the 3 phase lines are there.

So, in this case, wherever there are three phase lines, there will be total Z a b c matrix, because all the three phases exist. So, ohm per kilometer so this matrix multiplied by whatever kilometers of that particular line will give me impedance of that particular section. So, impedance matrix of this particular section, so for first section, we can easily calculate this Z a b c multiplied by 6.1 will give me impedance matrix of first section.

And similarly, we can calculate impedance matrix for second and third section, which are three phase in nature. In this particular section between 4 to 5, only two phases are existing, in that case unique, we can use this matrix. So, this matrix multiplied by 5.3 kilometers will give me impedance matrix, which is 2 by 2 for section 4 to 5. And for section between 5 to 6 only 1 line is there, which is having in this particular impedance. So, this multiplied by 5.2 will give me impedance matrix between 5.6. So, we can get the impedance matrices for all the sections of this particular system.

And then I am considering the initial voltage of bus number 1, it will be 11.4 kV, and which is three phase voltage. And then, we are going to consider the fault impedance, which is 0.001 per unit. And in all the three cases, we are going to consider fault, which is occurring at bus 4.

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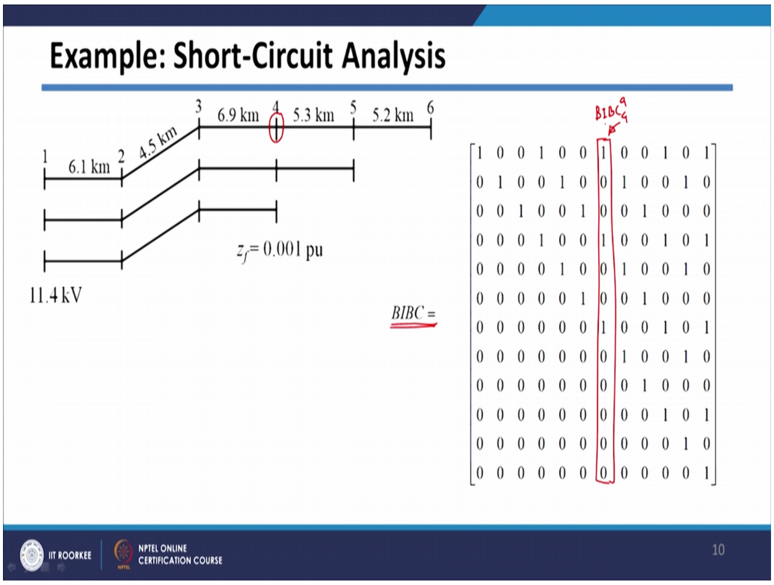

So, first step we have seen that first, we have to build or we should know, the BIBC and BCBV matrices for this particular system. So, if you calculate BIBC matrix for this, which is basically this one. And as I told you we have given the names for these columns of BIBC matrix, so any column, which I can pick will be having some name here. So, this will be say BC BIBC corresponding to it is corresponding to 4th bus, so that is why

4th and a phase. So, every column of BIBC matrix will be given names kind of kinds of this.

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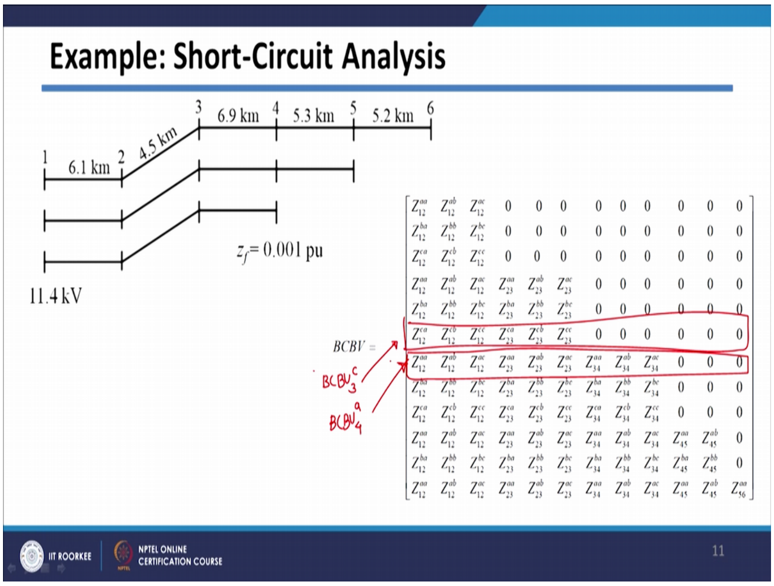

And then, we have seen BCBV matrix, which will be are having (Refer Time: 28:07) nature like this. In this case also each row of BCBV matrix will be given some name. So, since it is corresponding to bus number 3. So, we can say BCBV corresponding to 3, and since it is corresponding to c phase, I can give this name to this bus as BCBV 3 c. Like this all the rows will be given name.

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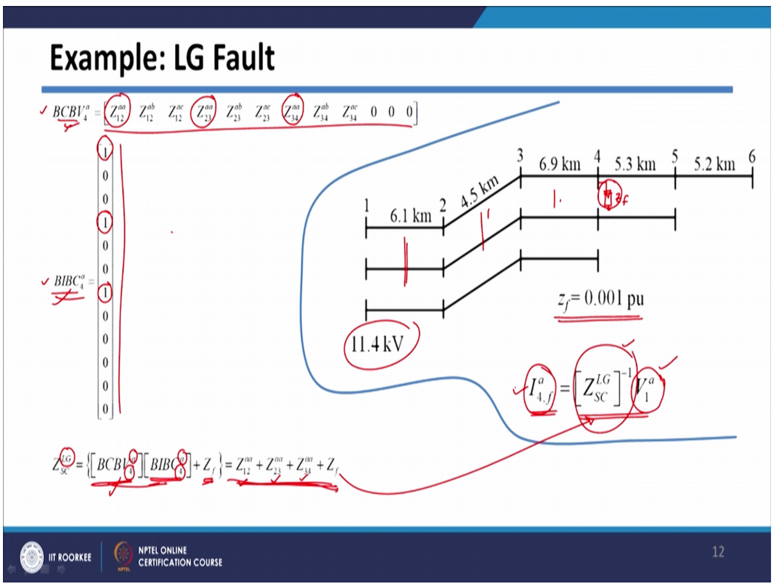

So, basically when we are considering fault single line to ground fault, which is occurring at this particular bus with fault impedance of Z f, which is 0.001 per unit. And we already know the impedances of all the sections of the line; and from that we have built your BCBV matrix and BIBC matrix. So, basically since the fault is occurring on bus number 4, I am taking and phase a. So, I am taking this BCBV matrix corresponding to bus number 4. So, this is corresponding to bus number 4 and phase a, so this particular row is BCBV corresponding to 4 and phase a.

So, I am taking this row here, similarly while take column corresponding column for BIBC matrix. So, if you take this columns and row, which are basically required for single line to ground fault analysis; So, that particular row of BCBV matrix are taking it here, and column of BIBC matrix I have taken it here. And we have seen that the fault current of bus 4 on phase a is actually given by this particular expression here. And we have seen that this fault impedance matrix is given by this particular expression here, where this is actually BCBV of 4th bus corresponding to a phase. And BIBC matrix corresponding to bus number 4, and a phase, and this is Z f.

So, basically you have got these two quantities here, and if you multiply them, so we can see that only non-zero entities will (Refer Time: 30:37). So, if you multiply this with respect to this, you will get this, which is basically this is non-zero corresponding to entry corresponding to this, this is non-zero, and entry corresponding to this, which is non-zero this one. So, it will be Z 1 2 aa plus Z 2 3 aa plus Z 3 4 aa plus this Z f is already there.

So, we can easily get this impedance matrix corresponding to your LG fault for this particular system. And if you put this fault value here, this V 1 a is already given from this 11.4 kV, you can easily get the phase a voltage of bus number 1. So, this is known, this we have calculated here, this is known, so we can calculate fault currents, fault current for phase a of bus number 4.

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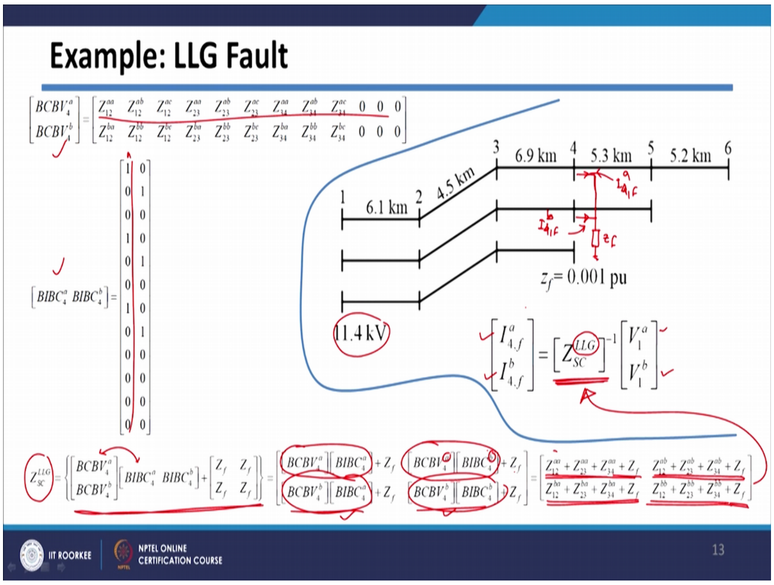

Now, let us say consider if there is LLG fault. So, in case of LLG fault, you have seen that we short circuit this two buses with impedance Z f. And we need to calculate these two currents, this current we have we have seen here, we called it as a phase current, and this current, which is flowing here, we call it as a b phase current. And the both these currents of so this actually I is 4th here, so I should write 4 here. So, bus number 4 a phase, bus number 4 b phase currents will be given by this particular matrix, which all we already derived short circuit matrix corresponding to LLG fault multiplied by voltages of bus number 1. So, these voltages are known. So, if you know this matrix here, we can easily calculate.

So, let us see how we can get this matrix. And we already derived the matrix for your LLG fault, which is given by this expression here. And if you explicitly multiply this two row, we will get this matrices here. And from this particular multiplication, you will get these terms ok. Basically, this column, and this row, this row, and this column, we are multiplying, so basically you will get this quantity here, plus Z f is still there.

Then BCBV a multiplied by BIBC corresponding to a and b phase, you will get this term here. And from this term, I will get this term here. And from this term, I will get this term here. So, this from these two matrices, I have got these four quantities into this particular matrix. And once you get this matrix, inverse of this matrix can be put it here, to get the fault currents. So, this is how fault currents can be analyzed.

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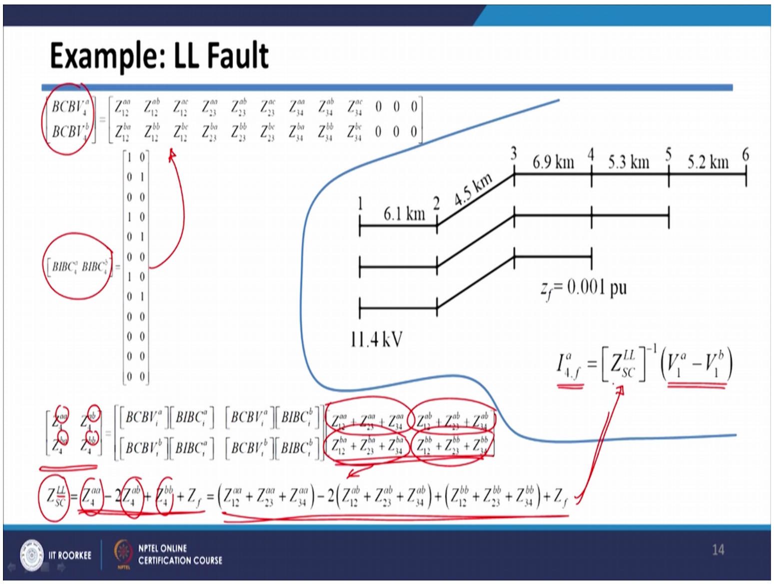

And in case of LL fault, we have just now seen, we have divided this matrix multiplication of these two columns, and these two rows into four impedance matrices, which you have seen. This is corresponding to a phase ab phase, and ba phase, and bb phase. And from this, if you multiply explicitly this matrix with respect to this matrix, I will get these 4 terms like this. And from this, we can derive the short circuit impedance matrix for LL fault, we are just now seen that is given by this particular expression here, we already derived just now. And where this is nothing but, this part is nothing but Z aa of 4th bus, this is Z ab of 4th bus, this is Z ba of 4th bus, and this is nothing but Z bb of 4th bus.

And from this, if I putting this values of Z aa, Z ab, and Z bb from this matrix into this expression, I will get this impedance matrix, and this can be easily calculated. And once you get this expression fault current, we have just seen that fault current will be inverse of this matrix multiplied by subtraction of voltages V 1 a minus V 1 b, if the fault is occurring on a and b phase. So, this is how we can calculate fault current of phase a of 4th bus.

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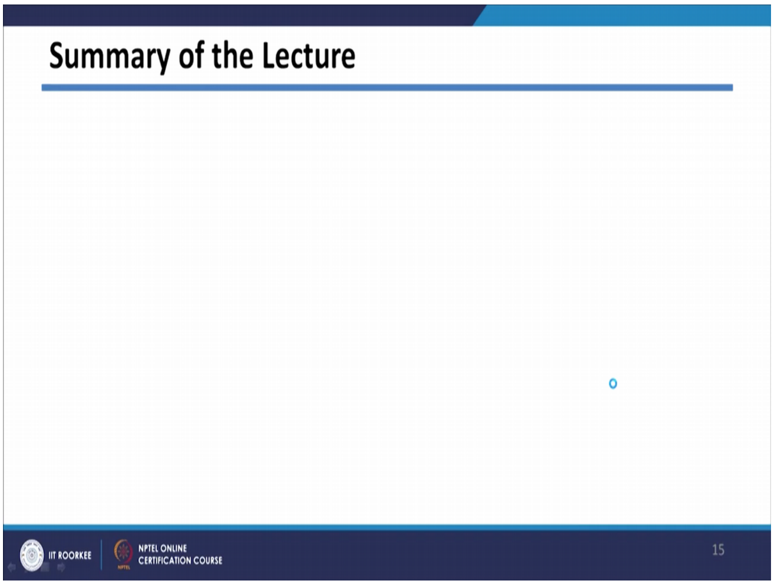

So, in summary of today's lecture, we have seen LL fault. So, for LL fault, we have derived fault current equations, and then we have seen the first fault voltages equation. And to understand it better, we have taken one example, and this example will be was solved for LG, LLG, and LL fault.

Thank you.