Electrical Distribution System Analysis
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Lecture – 37
Direct Approach for Short-Circuit Analysis:
LLG and LLLG Fault

Dear students, we are studying Short-Circuit Analysis. And in the last class, we have started with Direct Approach for Short-Circuit Analysis. In direct approach, we have seen that we have we can use direct approach, which you have used for load flow calculations, where you already studied though two matrices, which are called as BCBV, and BIBC. And in this particular approach, we can use it for short-circuit analysis also.

In the last class, we have seen that how we can actually divide your BIBC matrix, and BCBV matrix, so that we can use it for short circuit analysis. Similarly, we have seen short-circuit analysis for a single line to ground fault. And in this particular lecture, we will see few more faults using direct approach based method.

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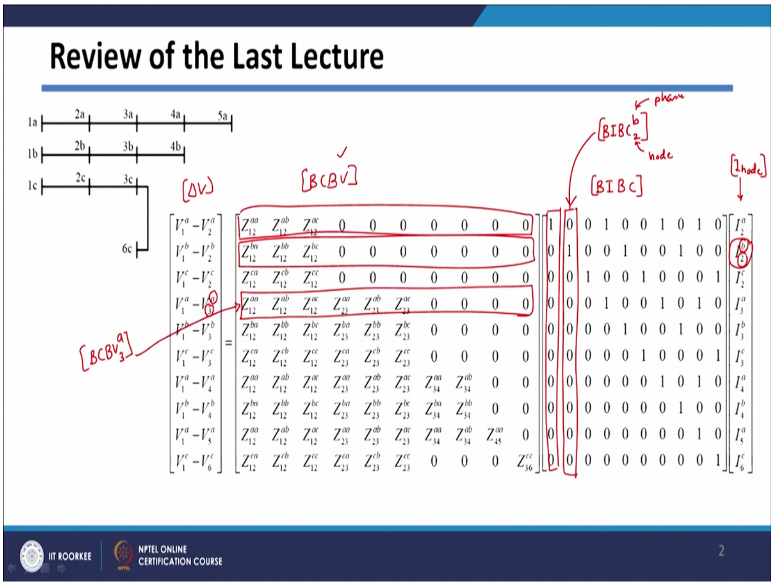

So, in particular in the last class, we have seen that using your direct approach based method. We can write this equation, which is basically delta V, which is equal to your BCBV matrix, which is having structure like this contains impedances. And then this is

BIBC matrix, which depending upon a topology consists of 1s and 0s. And these are actually I node matrix, so these are basically nodal currents.

To use these matrices for short-circuit analysis, what we did in last class, we have divided this BCBV matrices into different rows. So, we have considered each rows this of this BCBV matrix, and given different names like. Suppose you want to consider this row, so name of this row will be since it is corresponding to node number 3, and phase a, it will be called as BCBV, and superscript subscript will be 3, because it is node number, and phase is a. So, this particular row of this matrix will be called as BCBV 3 a. So, we have divided whole BCBV matrices into number of rows, and we have given names for each of the rows.

And then we have divided your BIBC matrix into number of columns considering each column, and we have given names of this column. So, consider if you are considering this column, this column will be called as BIBC. Since, it is corresponding to it is getting multiplied to this particular entry every time of this matrix. So, since it is corresponding to node number 2 and b phase, it will be called as node 2 and b. So, it will BIBC 2 b. So, see this is corresponding to your node number, and this is corresponding to your phase number. So, this BIBC matrix will also be converted into number of columns.

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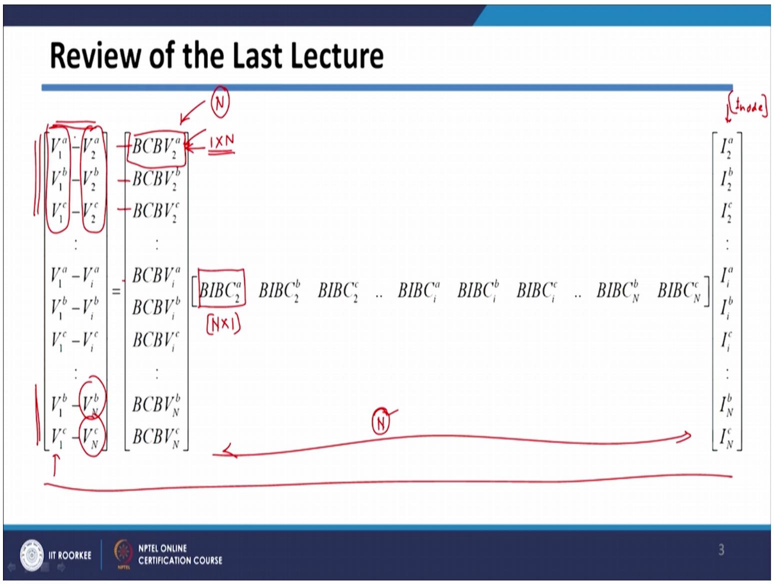

And if you see, if you generalize structure of your BIBC and BCBV base based load flow analysis; it will be something like this. It will be having number of rows, if there are

N numbers of unknown nodes into your system, all the three phase nodes consisting together. Then this number of rows of BCBV will be there will be N number of rows. And the size of this BCBV one row, it will be 1 by N. So, there will be 1 row and N columns, in there will be N columns into this particular row.

And if you see observe this BIBC, it will be one column. So, it will be size would be N rows, and it is column matrix, so it will be 1. So, then this BIBC matrix we also be consisting of N columns, if there are N three phase unknown nodes into your system. So, unknown nodes if you are considering all the nodes, except source node as a unknown, then this three phase nodes will be considered here.

And these are I nodes generalized way; it will be basically load currents. And these are voltage difference with respect to source node. So, every we have taking the voltage difference of every node with respect to source three phase voltages. So, this is actually source three these are basically source node three phase voltages, and these are actually unknown nodes voltages. This is structure of your direct approach based short-circuit analysis matrices.

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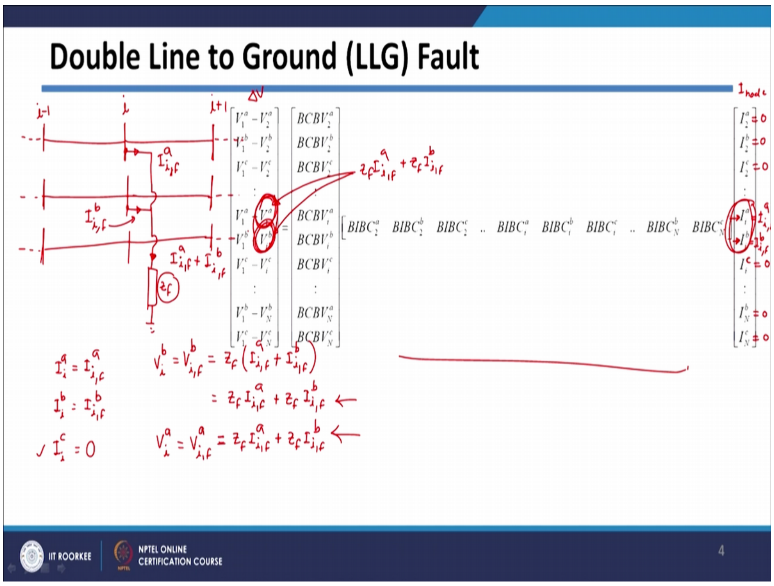

And we are going to use them for short-circuit analysis. And as I told you in last class, we have studied single line to ground. Let us start with now double line to ground fault here. So, in this case, again consider any general bus, say this is ith, bus this is i plus 1, and say i minus 1 bus. And since it is part of large system, I can just show it is other system is just dotted, because it is part of large system. Then this is your b phase, again this is part of large system. Then this is your c phase, which is again part of large system. And since, we are considering double line to ground LLG fault here. This I am considering fault on a and b phase, double line to ground fault, so this a and b phase is shorted, and neatly shorted with respect to ground with Z f is equal to some value. There will be for this is fault impedance of this one.

Now, what will happen when this short-circuit happens, there will be some current, which is flowing from a phase. So, this current will be post fault current as I told you comma f I am giving to represent the quantity, which is post fault quantity and a face. So, this is corresponding to ith bus, ath phase, and it is post fault. Similar way current, which is coming from b phase will be. So, this current will be given as I the bus current post fault and corresponding to b phase. Therefore, total fault current will be equal to addition of these two. So, total fault current will be I i f a phase plus your b phase fault current.

Therefore, if you write the nodal conditions, so I can write here, the current of a phase will be basically post fault current of a phase. Current in b phase will be basically post fault current of b phase. And current in c phase is basically c phase is not faulted, so it might be serving a load current, which will be negligible as compared to your fault current, so we can consider that as a 0. Similarly, all other load currents at all other nodes, we can consider and them as 0.

Now, if you see the voltages of these two buses, so voltage of b phase say post fault voltage. So, post fault voltage at b phase will be equal to it will be Z f multiplied by addition of both the currents, because both the currents are flowing. So, this current multiplied by this impedance will be nothing but voltage at this point. And you can see that same voltages coming at bus b also. So, at both the places, the voltage is same, so plus b. So, this will be equal to your Z f plus Z f into fault current in b phase.

And as you if we have seen that the same voltages coming at a phase also; So, for a phase also I can write this same condition that at a phase your fault current is exactly same, because these both the terms voltage at both the terminals, it is coming same. So, it will be also equal to b, b phase fault current. Now, let us see our take our generalized equation here. So, generalized equation, which we are described earlier, where we have divided whole BCBV matrix into number of rows, and whole BIBC matrix into number of columns. And this is nothing but your delta V matrix, and these are I nodes.

And as we have seen I node currents are almost 0 or negligible as compare to fault currents. So, therefore, all other currents we can consider them to be 0. Like we considered for single to line ground fault here, so c phase current will be also equal to 0. However, these two currents will be present. So, this current is a phase; and this is fault current of b phase. So, only these two entries will be existing there, which will be replaced by fault currents in a phase. So, this will be fault current in a phase, this will be fault current in b phase, all other currents will be replaced by 0.

Also on this side, if you see delta V side, these two voltages, we are calculated post fault. So, post fault these two voltages will be this voltage, we already calculated. So, this voltage post fault, this voltage will become Z f plus Z f b. Similarly, this voltage is also same. So, this also entry will also get changed by the same amount. So, here you just change these two entries with these two entries, all other quantity will remain same. So, if you modify this particular matrix by changing these entries, means making these currents 0 the accept.

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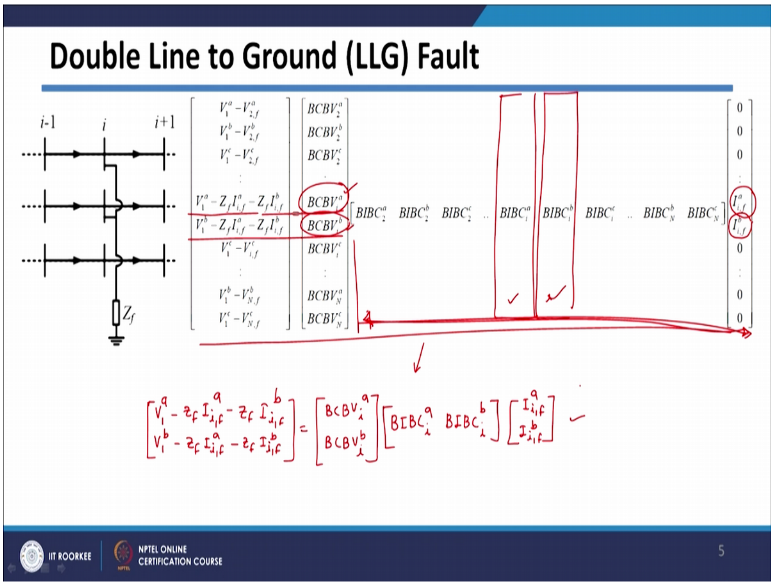

These two currents, and modifying these two entries to post fault voltages, we will basically get this matrix. So, as explained in earlier slide, I just modified these two entries here, and these two entries here, and these entries I just post fault voltages I have written. So, here I just added your comma f terms, so that we can represent them as a post fault voltages.

So, in case of double line to ground fault, your structure of BIBC and BCBV matrix will be something like this. And from this, we can easily see for any ith bus using these two equation, I can easily write. So, it will be, so if you consider these two equations, this is 1 equation, and this is two equation 2. In this case, these two entries will come into picture. So, I can just write it like these only two rows, row two equations I am taking Z f into fault current in a phase, so it is Z f multiplied by fault current in b phase on right hand side. Similarly, V 1 b minus a phase, and fault current in b phase of ith bus. And this will be equal to here this two rows will come into picture; So, BCBV ith row and of ath phase, BCBV of ith bus and bth phase.

And if you see this matrix part here, from here to end only there are two non-zero currents, this, and this, because of that only column corresponding to these two currents. So, this column is corresponding to fault current in a phase, and this column is corresponding to fault current in b phase. All other columns if you observe, they are getting multiplied with respect to 0. So, the equivalent matrix of structure, or equivalent matrix of this system of to this, it is actually BIBC a phase, just I have to take this column, and I have to take this column here. So, it is and it will be getting multiplied by this two entries. All other columns actually getting multiplied by 0 so, we are not considered, while writing this particular equation here.

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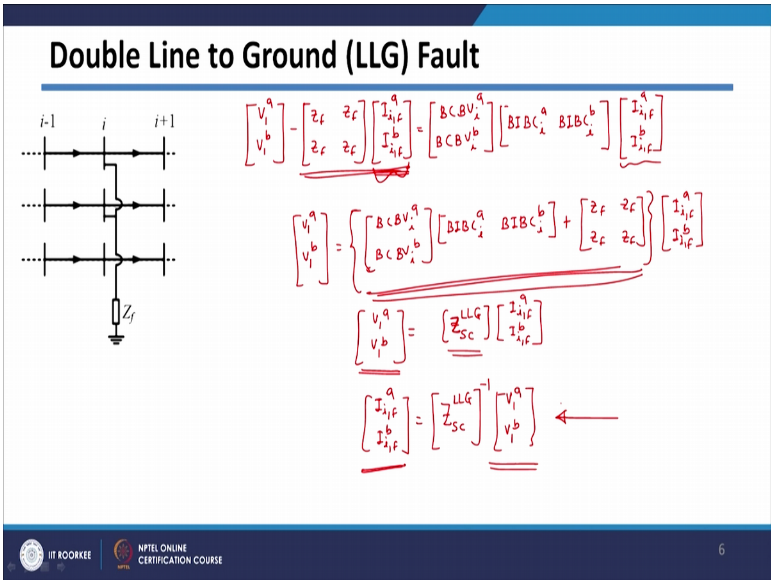

Now, if you write this equation in simplified form. So, if you just simplify it, I will get this equation here. So, V 1 a V 1 b minus Z f Z f Z f Z f into fault current in a phase fault current in b phase is equal to BCBV matrix rows corresponding to a phase and b phase of ith bus, and then column of BIBC corresponding to ith bus and a phase, and BIBC column corresponding to i and b ith bus and bth phase, and this is again we have seen that it is getting multiplied with your fault currents. So, basically we are getting this equation here.

And in this particular equation, we are interested in getting fault currents from a and b phase. So, we can just simplify this equation, so that we can get writing them in terms of fault currents. So, I can just write it like this. So, V 1 I can just take this term on of right hand side, and these currents are actually common, we can take it out. So, if I simplify it, I can just write it like this, so this matrix is BCBV of b phase, and then columns of BIBCs corresponding to a phase, and b phase of ith bus plus you are getting this Z f matrix on right hand side, and this is getting multiplied with your fault currents.

And if you this observe, I am calling this particular part as Z short-circuit matrix, in case of LLG fault. And so, in this particular equation will become equal to V 1 a V 1 b will be equal to. So, this part I am calling short-circuit matrix of LLG fault, and here it will be fault current in a phase and fault current in b phase. And then we can actually calculate fault currents, because this part is known, because these are source voltages at bus number 1, which are basically known. And the fault currents in a and b phase will be equal to the inverse of this matrix, which will basically 2 by 2 matrix LLG, and we need to take inverse of it. And this voltages basically known, we can I calculate fault currents. So, this is how using this equation, we can calculate fault currents in double line to ground faults.

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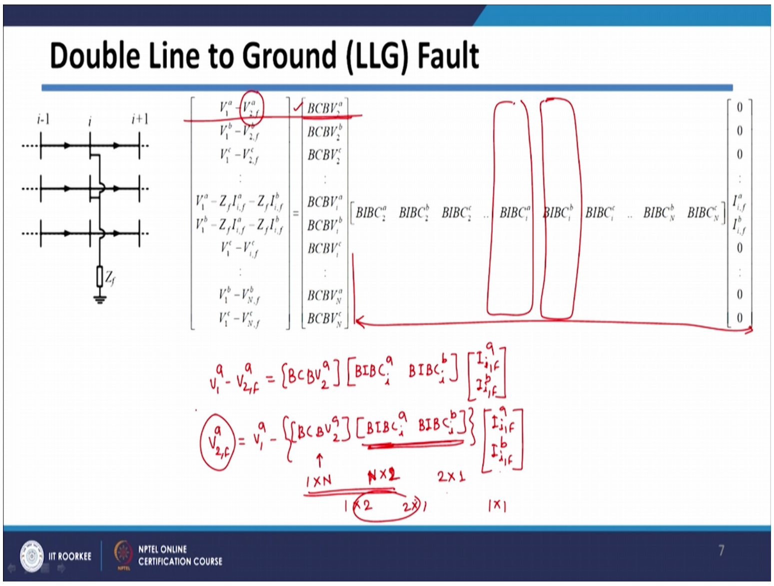

The post fault voltages, in this case can be calculated similar way. Like we have calculated in case of LG fault, so that same equation, which will be of taken it here, I am taking it again it here. And if you see this, if you calculate, if you want to calculate this particular post fault voltage, then we need to write the equation for this. So, if you write the equation for this, it will be V 1 a minus V 2 post fault a will be equal to this particular matrix part that is BCBV 2 of a phase bus number 2 and a phase. So, we have to consider this row corresponding this.

And we have seen that the equivalent of this part of matrix equation is having only these two columns are getting multiplied with non-zero current, otherwise all other getting multiplied with 0 currents. So, we can write them like this, and the fault currents we already calculated, and those fault currents are fault currents in a phase, and fault currents in b phase, we have seen how to calculate in last slide. And therefore, we can write post fault voltages of a phase of bus 2 will be equal to V 1 a minus this matrix part here that is BCBV 2 a of b phase.

This row will be 1 row multiplied by, so this actually row, so that there will be N columns interact. So, 1 multiplied by N. Size of this you observe, it will be actually 1 column N rows. So, N multiplied by 1, so total size will be N multiplied by 2 here. So, size of this part will be actually there are 2 columns, and N rows. So, that is why N rows and 2 columns. And size of this current part is actually 2 rows, and 1 column. So, overall if you see, so from this will get 1 multiplied by 2 and from this you are getting 2 multiplied by 1. So, finally, we will get 1 by 1 size matrix, which is actually basically we will get voltage here.

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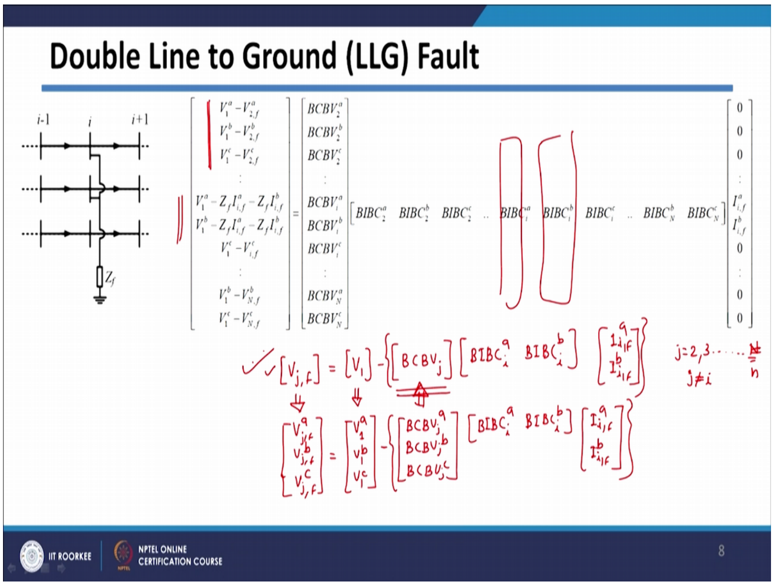

Similar way, generalize way if you want write the voltage equation using whatever we have done in case of LG fault, I will just write them here. So, it will be V j post fault will be equal to V 1 minus BCBV corresponding to any jth bus, where we want to calculate post fault voltages, and this will get multiplied with respect to BIBC. Either shown you only these two columns are getting multiplied with respect to fault currents, and multiplied by your fault currents sorry this is fault current.

So, this is the generalized equations to get the post fault voltages of any jth bus in this case j might be changing from 2, 3 up to N n 3 base buses. So, instead of that actually I will just write small n, so N small n three phase buses; where I do not want to take i or j not equal to i. So, i bus ith bus you already fault is occurred. So, except ith bus, we can calculate voltages of any jth 3 bus three phase bus here.

So, basically this will be this matrix, if you are expand, it will be V j a V j b V j c post fault. So, all the post fault voltages of jth bus, all the three phases will be equal to this particular matrix will be V 1 a V 1 b V 1 c for all the three phases minus, this particular matrix will be having all the three rows. So, BCBV j a BCBV j b BCBV j c so, collectively I can write it like this.

However, this part will remain same. So, here this BIBC we already considering the faulted phases here a and b, and this is getting multiplied with fault currents, which are already we have calculated before calculating post fault voltages. So, fault currents are known, before calculating post fault voltages. So, by using this particular equation, you can get post fault voltages at any bus.

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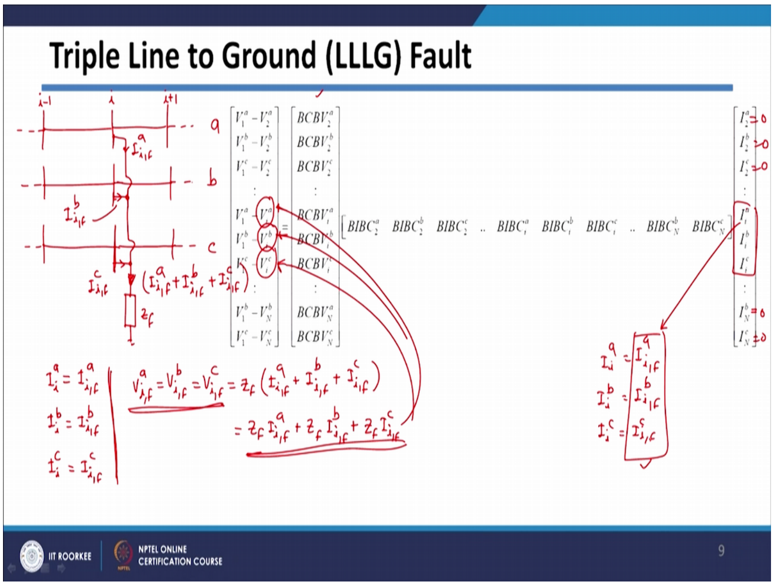

Now, let us see a triple line to ground fault. And in case of tipple line to ground fault, the ground fault occurring three phase to line three phase to ground. So, let us say, in this case also let us considered any ith bus, so this is will i plus 1th bus, this is say i minus 1 bus. So, this is say corresponding to a phase, then corresponding to your b phase, and this is corresponding to c phase.

And in this case, triple line to ground fault is occurring means, all the three phases, they are getting short-circuited with impedance of Z f. So, fault impedance is Z f. So, this phase also getting short-circuited here, this phase you also getting short-circuited. So, all the three phases will carry short-circuit current.

So, terminal condition I can easily write. So, in this case, I can write the current of a phase will be basically fault current of a phase, current of b phase will be basically fault current of b phase, and current of c phase, because all the three phases are shorted current of c phase. Now, if you see the, so this is actually your current with fault current in a phase, this is your fault current in b phase, and this is your fault current in c phase. So, here there will be total current, so the current flowing to the fault, it will be addition of all the three currents.

Now, post fault if we observe, the voltages of these three versus a b and c, they will be same because these are this terminal voltages will be same which will be just. So, I can just directly write V post fault voltages of a phase, post fault voltages of b phase, and post fault voltage of c phase will be equal, and that will be equal to your Z f multiplied by all the three currents, addition of all the three currents, current in b phase fault current in c phase. So, that will be equal to Z f into fault current in a phase plus Z f into fault current in b phase plus Z f into fault current in c phase. So, we have got the terminal conditions of our boundary conditions for this particular fault.

Now, we will try to feed this quantities into our general matrix of direct approach. So, general matrix of direct approach we already seen, it is rows of BCBVs, and columns of BIBCs. And what we did earlier is we just replace to your current values on right hand side, and fault voltages values of faulted versus on left hand side. So, basically we have change this, and we have changed this here.

So, let us see what change will happen. In this case, all the three phases are faulted. So, as compared to fault currents in ith bus, all other bus currents will be negligible, because they are just serving the load currents. So, all these currents, except these currents will be except these currents, they will be 0. And we have seen that this current I i a is actually fault current of a, so I can just replace this by this one. And in b phase, there will be fault current in b phase also. And then c phase, there will be fault current. So, these three entries will be get replaced by fault currents of these three entries, otherwise all entries will be 0.

Similarly, post fault voltages on left hand side, so this three entries, so V i a, V i b, and V i c will be replaced by post fault voltages at that particular bus, and all they are same. And these will be getting replaced by this particular term here. So, if you see this term, if you put this term in all the three locations and all the replace these currents with fault currents you will get, this equation here.

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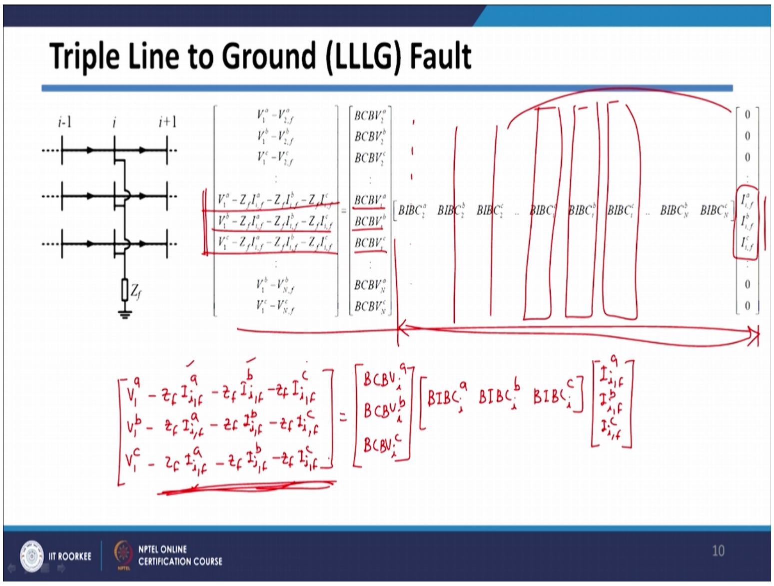

So, here I just replaced post fault voltages of all the three phases. And here I just replaced by faulted currents. So, during the fault condition, your structure of your direct approach based matrices will be something like this. Now, in this case also just first take these three rows corresponding to voltage bus. So, if you take these three rows corresponding to faulted bus, and write them separately the equations will be something like this. So, it will be Z f into minus Z f into b phase fault current minus Z f into c phase fault current.

Similarly, I can write for b phase, it will be Z f multiplied by a phase fault current Z f multiplied by b phase fault current c phase fault current. And then c also we are getting similar terms, so it will be minus here Z f multiplied by a phase fault current Z f multiplied by b phase called fault current Z f multiplied by c phase fault current which will be equal to you are having this three rows, which will come into picture. So, this is BCBV row corresponding to a phase, BCBV row corresponding to b phase, BCBV corresponding to c phase of ith bus.

And again if you observe this matrix part here from here to here, these columns are getting multiplied with respect to 0, except this three columns here now. Now, these three columns are getting multiplied with respect to these non-zero fault currents; otherwise all other currents are 0. So, therefore, simplified form I can write it like this, BIBC column corresponding to b phase, BIBC column corresponding to c phase of ith bus, and this is getting multiplied with the fault current in a phase, fault currents in b phase, fault current in c phase.

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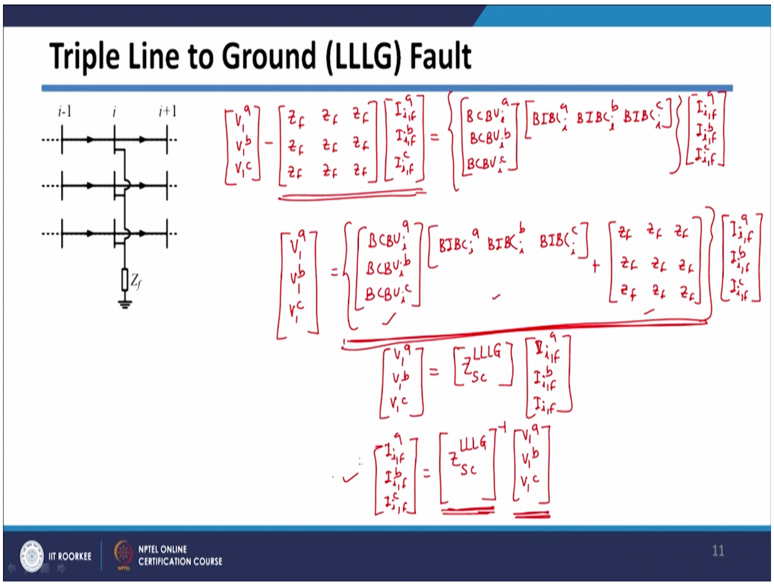

Now, we are interested in getting these values of fault current. So, we can simplify it to get that. So, if you can simplify this equation here, you will get V 1 a V 1 b V 1 c will be equal to, or we can say minus here, you are getting Z f matrix 3 by 3, and this is getting multiplied with fault currents of a phase, fault currents of b phase, and fault currents of c phase.

This is basically I am writing from this part, if you see this Z f is getting multiplied with respect to various currents here, and this part I am just writing in matrix form, which will basically come like this here, which will be equal to your BCBV matrix. And as I have seen here, there are three rows of BCBV matrices. So, corresponding to a, BCBV row corresponding to b, BCBV row corresponding to c of ith bus, and BIBC corresponding to a, column BIBC column corresponding to b, BIBC column corresponding to c of ith bus, and this is getting multiplied by your currents c.

Now, we can take this term on right hand side, and take fault currents common out. So, we can get this matrix here, BCBV row corresponding to a, BCBV row corresponding to b, c, and then BIBC column corresponding to a, b, and c, and plus your Z matrix we have taking on right hand side, so Z f Z f Z f, and this is getting multiplied with respect to fault current in a phase, fault current in b phase, fault current in c phase.

And this particular matrix from here to here, I am calling it as Z short-circuit matrix for triple LG fault. So, this will be equal to then on this side, it will be V 1 a V 1 b V 1 c, and here it will all the three phase fault current that is fault current in a phase, fault current in b phase, and fault current in c phase. So, I can easily write, all the three fault currents in a phase, and b phase, and in c phase, all the three fault currents in all the three phases will be equal to the inverse of this short-circuit matrix of all the three phases, you have to take inverse of it, and then V 1 a V 1 b and V 1 c.

And as I told you the voltages of first node, they are known to us. And then this matrix will be also known, because it is just depend on your BIBC and BCBV matrices, and fault impedance. So, this is also known. And if you get this matrix and voltages of first node, you will get the fault current into the system automatically.

So, this is how we calculate the fault currents in triple line to ground fault case. The calculation of post fault voltages will be exactly similar, like we have considered in case of single line to ground fault, as well as double line to ground fault. In this case also you can use the same equation, which you have used.

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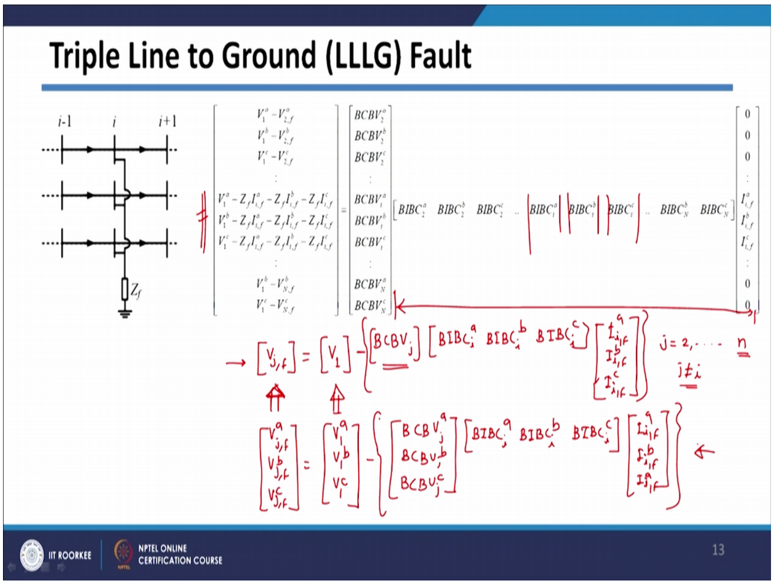

So, here we have this equation, I am directly writing it. So, V j we have seen that it will be equal to V 1 minus we have this BCBV matrices corresponding to j, and then in this case you are having this three columns of the simplification of this part of the matrix, you have seen it is BIBC, you are having three columns, because only these three columns are getting multiplied with non-zero currents. And these currents are currents in a phase sorry current in a phase, current in b phase, and in fault current in c phase of ith bus. So, this particular equation so, in this case j is changing from node number 2 up to small n 3 phase nodes. Where j is not equal to i, because you all you are not calculating the post fault voltages for faulted bus, remaining bus you are calculating.

So, this is how we can calculate post fault voltages. So, as I explained in last part, this will be your three phase matrix, which will be consisting of 3 bus voltages, which are post fault voltages of a phase, b phase, and c phase. So, the expanded form of this particular matrix will be something like this. Expanded form of this particular matrix will be V 1 a V 1 b V 1 c minus expanded form of this particular part of matrix will be BCBV j corresponding to a phase, BCBV j corresponding to b phase, BCBV of jth bus corresponding to c phase.

So, basically we are taking all the three rows corresponding to jth bus, we are going to calculate post fault voltages. And this part will remain same BIBC column corresponding to b phase, BIBC column corresponding to c phase, and this will get multiplied with fault currents, which are already calculated. So, when you are calculating post fault voltages, post fault currents are already calculated. So, using this equation, you can get post fault voltages of various buses, or various non faulted buses.

So, in this particular lecture, we have seen two faults, those are double line to ground fault, and triple line to ground fault. So, we have derived the matrices, or equations for fault currents. In case of double line to ground fault, there will be two fault currents in two phases. In case of triple line to ground faulted, there will be three all the three phase currents, which are faulted currents. So, we have derived the equations for them. And then we also derived the equations for post fault voltages. Basically, equations for the post fault voltages will be same for almost same for all the types of faults.

So, here we will stop today, and next class we will see few more faults.

Thank you.