Electrical Distribution System Analysis
Dr. Ganesh Kumbhar
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 35
Thevenin's Equivalent and Phase Variable Based Short Circuit Analysis

In a last lecture, we have started short circuit analysis of distribution system. And in the last lecture we have started with symmetrical component based analysis because you already know it we have just revised it. However, there are many limitations of this particular sequence component based theory.

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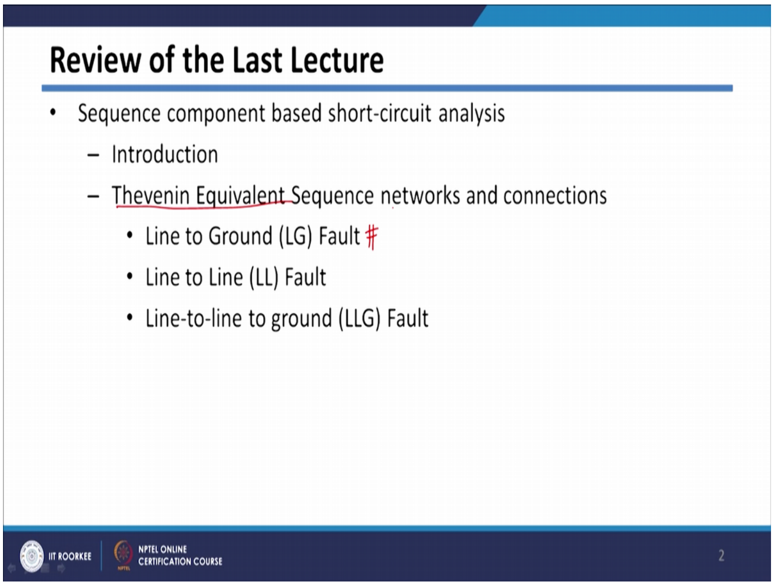

So, in the particularly last lecture we have seen simple sequence based analysis where if there is any network we have seen how to get the Thevenin's equivalent sequence networks till your fault point and then we have seen how to connect this particular sequence network if there are particular types of fault. So, you have seen there are three types of fault which can be analyzed using sequence networks the connections that is line to ground fault, line to line fault, and line to line to ground fault.

Now let us see; what are the limitations of sequence component based analysis. And why it cannot be used in distribution system it can be used. However, there it will introduce error and will see what are the sources of errors in distribution sequence component based analysis.

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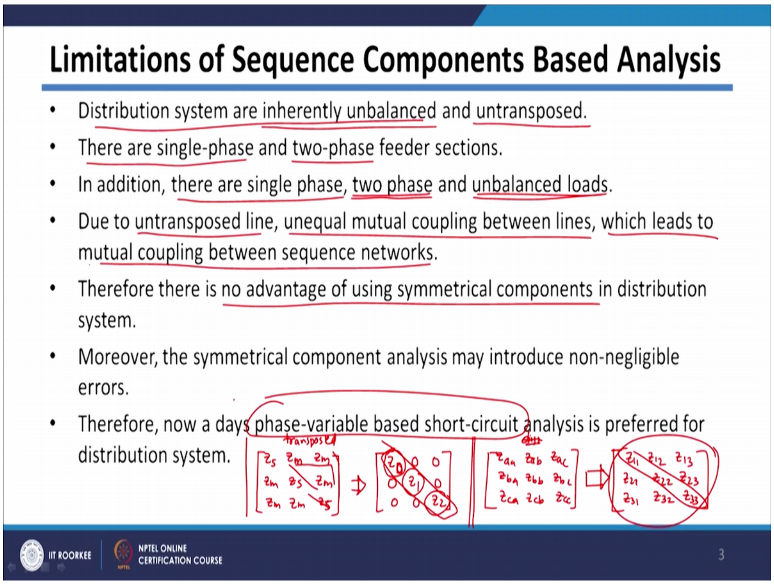

So, we have seen that we distribution system is inherently unbalanced and untransposed. And as we have seen that in case of sequence component theory one basic assumption is that system is balanced before fault and only positive sequence quantities exist in the network before the fault. So, that condition will not be get will not get satisfied in case of distribution system because distribution systems are very much unbalanced before fault also.

Also there are many sections of the line which will be having single-phase laterals or two-phase laterals or feeder section will be single-phase feeders or two-phase feeder. In a in addition, there will be some loads which are single-phase load, some load will where two-phase loads; and they are basically very much unbalanced that is another problem. Due to a untransposed line, there will be unequal mutual coupling between the lines, which leads mutual coupling between the sequence network.

So, you have seen that if the line is transposed in that case your impedance matrix will be having nature something like this z s z m z m z m z s z m z m z m and z s. So, all the three of diagonal entries are equal as well as non diagonal equal entry that is z m they are equal. And if you take if you convert them into sequence domain we have seen that we get z 0 0 0 z 0 z 1 and then your z 2. And in this case we can see that your z 0 and z 1 they are uncoupled. Similarly, z 1 and z 2 they are uncoupled. This happening because this is we are considering for transposed line.

However, if there is line is untranspose these z ms are not equal to each other; similarly these are not equal. So, in that case your matrix will be something like this. So, it will be z aa z ab z ac z ba z bb z bc z ca z cb and z cc. And all of them are different from each other. In that case, if you get sequence matrix sequence impedances, it will not be only the regular integer exist there; in that case it will be something like this z 12 z 12 z 13 z 21 z 22 z 23 z 31 z 32 z 33.

So, we can see here these sequence networks are not exactly decoupled. So, there is coupling between six sequence network itself. And basically this will whatever advantages we are getting in case of transmission system that systems were decoupled and you can easily connect them to fault current, because there is no mutual coupling between impedances of this network.

However, you can see that in case of distribution systems which are basically un transposed one these impedances in sequence network they are coupled one coupled with each other. Therefore, we are not getting exactly any advantage of using syncs symmetrical components in distribution system. There are studies which have carried out by many people and which also tells us that symmetrical component will give non-negligible errors in distribution system short circuit analysis. Therefore, nowadays phase variable base short circuit analysis is more preferred as compared to sequence impedance base method.

So, in this case, we are not transferring our system into sequence domains basically whatever analysis we want to do, you we are going to do in phase variable domain itself. So, initially to start with let say one simple method we can consider which is given in Book of Cresting.

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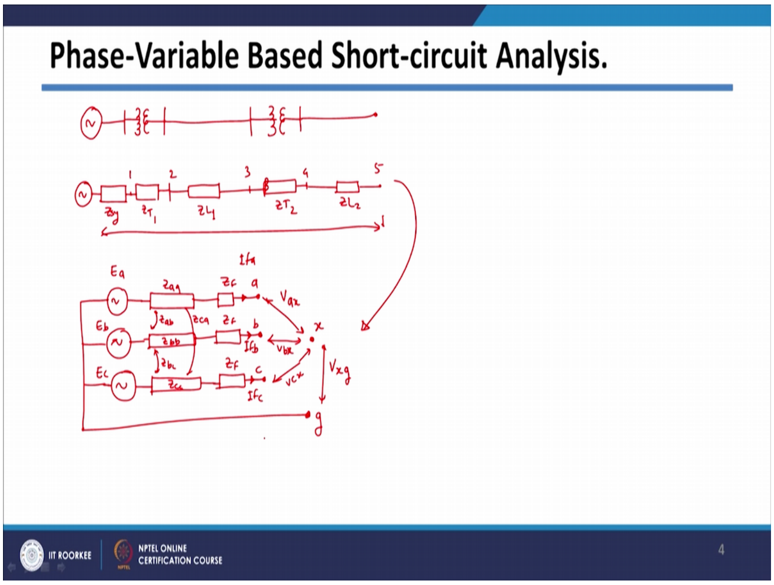

So, here let us say you are having one source; then there is one say transformer. And then there is say primary distribution line, then one another transformer which again step down and then there is secondary line which goes. And here there is some fault is occurred.

So, this network can be shown it like this. So, there will be source and then there will be impedance of this source, then there will be. So, it can be z system I can show in the because source will not be there in case of distribution system, it is may be 33 kv network which is incoming, so z system. Then this is z transformer one this will be then you are having impedance of your line one. And then this will be this will be your impedance of transformer 2, and then you will be having impedance of your line 2.

So, if this network will be something like this, various impedances depending upon where the fault occurs those many impedances will come into fault current path. So, there are possibility that fault may occur at 1, 2, 3, 4, 5. So, there are five locations where the fault can occur. And depending on where the fault is occurring, you can easily get the Thevenin’s equivalent circuit by adding those many impedances. Basically we will get Thevenin’s equivalence are three phase equivalent circuit which will be something like this.

So, you are having this say E a a phase and this is your E b; and this is your say E c three sources of three phases. Then up to for wherever the fault is occurring that much impedance will come into picture. So, if the fault is occurring at 0.5, all the impedances will come into picture; and those impedances will be shown it like this. And they will be mutually coupled with each other because these ranches three phase manches they will be mutually coupled. So, this is say z aa z bb z cc. And there will be mutual coupling that is z ab, here this mutual coupling is z bc, and there will be mutual coupling between these two it is z ca.

Now, there will be some fault impedance which I can show it here. So, this is z f, z f, z f which I consider as fault impedance. And then this is grounded and these ground have taken it here this i find I am calling at a g point. And here I am taking one x point. So, this is your a phase, b phase and c phase and this x point. This arrangement i have done that is a, b, c and x g, so that I can create different types of fault into this network and I can calculate fault currents easily.

So, the fault current which this is I F a fault current means v phase is I F b this will this current and this current is I F c. So, fault current I F a, I F b and I F c here. This voltage I am calling between a and x it is V ax; between b and x it is V b x; and between c and x, it is V c x; and between x and g, it is V xg.

So, for fault analysis purpose we have converted this system into this particular system and I have made this arrangement of this point x and point g, and this point a, b, c, so that for different types of fault I can make different connections.

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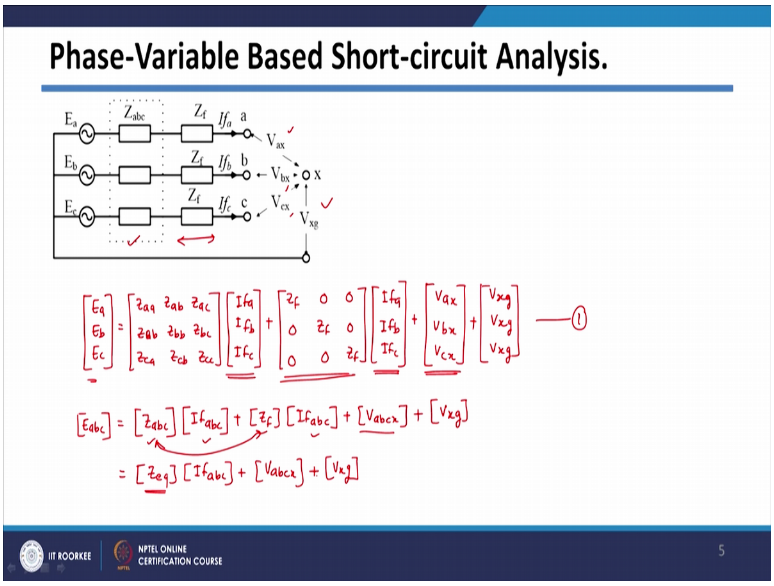

So, let us write the equation for this. So, if you write the kvl equations for this particular figure here. So, I can easily write on the right hand side, there will be sources. This is E a, E b, and E c, the voltage will be equal to the impedance drop across this impedance say that is Z abc Z abc.

As I told you consisting of impedances three phase impedance matrix Z ab Z bb Z bc Z ca Z cb and Z cc. All the three phase impedances they will get multiplied with fault currents I fa, I fb and I fc plus there will be a impudence drop across fault impedances, but fault impedances will not be mutually coupled with each other. So, they will be just diagonal entry fault impedances z f. And, this will again get multiplied with fault currents plus this voltage drop that is V ax, V bx and V cx, so it will be V ax, V bx and V cx plus this drop which is V xg which is getting added in all three phases which is 0 sequence of basically V xg V xg, V xg.

Now, I am writing this equation 1 in just short form. So, this short form of this part of the matrix I just write E abc which is basically this vector which is equal to this total matrix as I told you I am calling and Z abc matrix which is 3 by 3 this matrix. I can say if a, b, c all the three phase quantities plus this I can just say just z f matrix multiply again your I f abc fault current plus this matrix I can say V abc x because all the three quantities plus your V x g matrix.

In this case this is I f abc, I f abc, so I can just combine this Z abc and z fault. And I can just say it is Z equivalent. So, Z equivalent multiplied by I f abc. So, I am just taking these two impedances together and I am calling at a z equivalent plus V abcx which is this quantity plus V xg. So, I am just take the one next slide.

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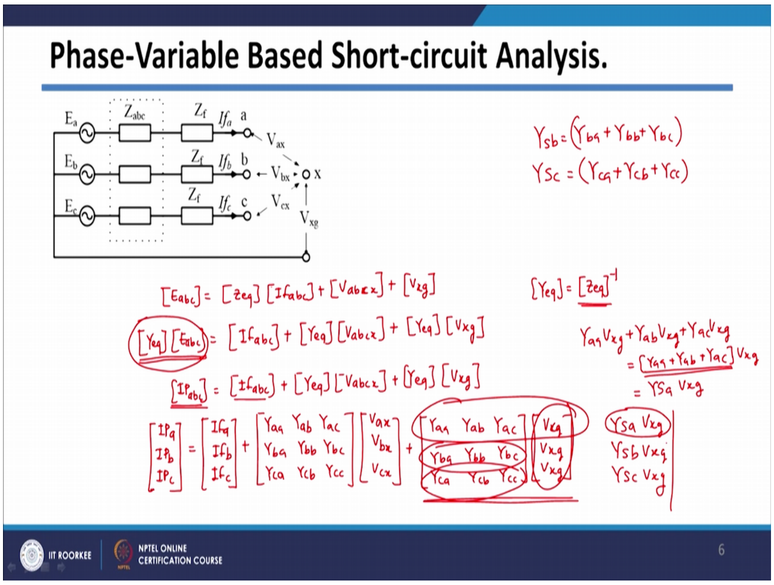

So, basically what we have got is this one E abc is equal to your z equivalent into if abc plus V abc abcx plus V x g matrix. Now, I am multiplying this whole equation with respect to Y equivalent which is basically equal to Z equivalent inverse. So, if I multiply with Z equivalent inverse with this whole equation, it will be Y equivalent multiplied by E abc. Here this will get cancelled out, so it will be I F abc plus this will get multiplied with Y equivalent, so Y equivalent into V abc x plus Y equivalent multiplied by your V xg.

Now, here of if you observe this quantity here, in this case Y equivalent will be known because it is just inverse of Z equivalent. And Z equivalent we have seen that it is just fault impedance plus impedance of the lines. So, impedance fourth impedance and impedance of the lines they are known as well as Thevenin's equivalent voltages E a, E b and E c they are known. So, basically we can get this of quantity. And this since it is admittance multiplied by voltages. So, it will be or having magnitude units of currents. So, I can just say this will be I P abc which will be equal to I F abc plus your Y equivalent into V abc x plus your Y equivalent into V x g.

Now, I can just write these equations in full form. So, in this case, this will be I P a, I P b and I P c all the three phase currents will be equal to this quantity is fault current in a phase, fault current in b phase and fault current in c phase plus this Y equivalent is with y admittance matrix Y ac Y a b Y bb Y bc Y ca Y cb Y cc, it will get multiplied with V ax

V bx and V cx plus again this Y equivalent is same that is Y aa Y ab Y ac Y ba Y bb Y bc Y ca Y cb Y cc multiplied by V xg V xg V xg.

Now, you can see that if you observe this part of this equation this Y aa, Y ab, Y ac they were getting multiplied with same term that is Y xg. So, I can just this term of this equation that is Y aa into V xg plus Y a b into V xg plus Y ac into V xg. So, here I can just take V xg common out and it could be Y a a plus Y a b plus Y ac multiplied by V xg. So, this I can called YSa. So, addition of this three, I am calling it Y S a into V xg.

So, this particular this particular term will get this multiplied by this particular term will get multiplied by so I can say Y S a into V xg because three are same. So, then corresponding to this row, it will be Y S b into V xg. And this is third row it is Y S c into V xg where YSb will be equal to Y ba plus Y b b plus Y bc. And your YSc will be equal to Y ca plus Y cb plus Y cc.

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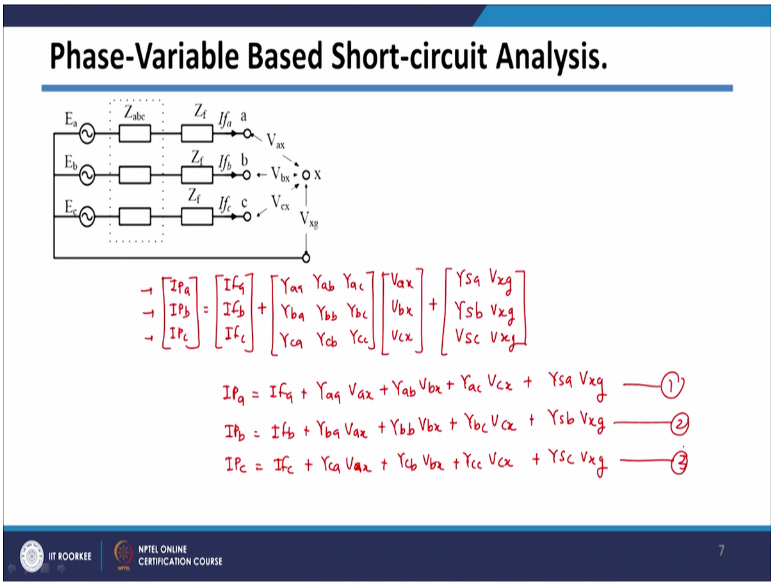

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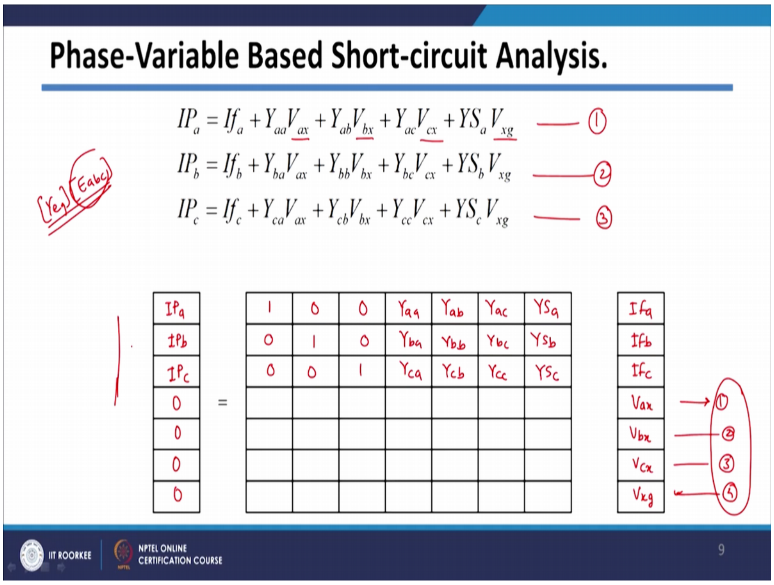

Now, if you write these equations, so we have got this relation here which is basically IP a IP b plus IP c is equal to I f a I f b and I f c plus your y metrics that is Y aa Y ab Y ac Y ba Y bb Y bc Y ca Y cb Y cc multiplied by your voltages V ax V bx and V cx plus as we have seen that this part we have converted into V Sa into YSa into V xg Y Sb into V xg Y ac into Vxg. So, it will be Y sa into V xg Y s b into V xg and Y sc into V xg.

So, basically here are three equations which we can consider. So, this is equation number 1, equation number 2 and 3. So, equation number 1, I can explicitly right it will be I f a plus Y a a into vax plus y a b into V bx plus Y ac into V cx plus Y sa into V xg. So, this is equation number 1. I P b will be equal to I f b plus Y ba into V ax plus Y bb into V bx plus Y bc into V cx plus Y sb into V xg say this equation number 2. And I P c will be equal to i fc plus Y ca into V ax plus Y cb into V bx plus Y cc into Vcx plus Y sc into V xg is equal to this equation number 3. So, we have got these three equations from that last slide which is is equation number 1, equation number 2 and equation number 3.

Now let us see; what are the unknowns which are available as I told you this part is available because this part we have got from Y equivalent multiplied by your Thevenin’s equivalent voltages. And since, Y equivalent is known which is calculated from calculated from impedance matrix of the line and fault impedance. So, Y equivalent is known as well as this one. So, this part is known.

So, from this 3 equation, I can just write I P a I P b and I P c. Here the unknowns are actually I f a in these three equations I f b I f c and these voltages V ax, V bx, V cx and V xg they depend upon various force. So, till we consider any kind of fault, let us consider them as a variable. So, V ax V bx and V cx and V xg they are also variable.

So, if you put these variables at least three equation into this matrix system; so these three equations. So, I P a is getting multiplied from first equation. So, I P a is 1 there is no term related to I f b and I f c. Then here it will be Y aa, Y ab and Y ac and Y Sa. So, this is first equation. Second equation I f a time is not there. So, I f b term is there and then here V ax is getting multiplied with Y ba Y bb Y bc and Y sb which is getting multiplied with respect to V xg.

And from the third equation we are having 001 here then Y ca Y cb Y cc and Y Sc. And say this is actually we are writing equation such that zero will come on left hand side. And this particular four equations 1, 2, 3 and 4, we can get it from particular fault because as I told you these variables V x, V bx, V cx, V xg they are related to we have created in that equivalent circuit 2 create different types of faults.

So, for different types of fault these variables will change. And these variables will basically give different types of faults. Let us say you are considering your LG fault into this circuit, so we can get these four equations for LG fault like this. So, this system of matrix, I am taking on next slide.

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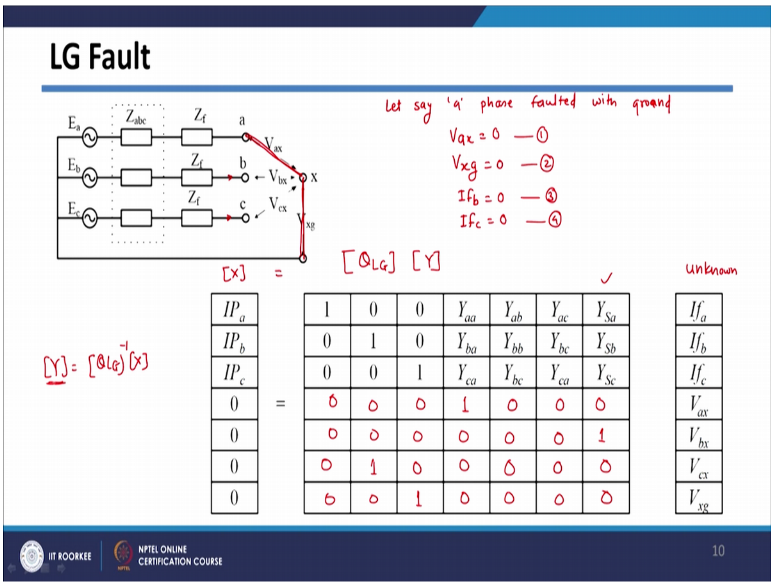

So, we have got this circuit matrix here. And if you are considering LG fault, so in case of LG fault let us say you are a four a phase is faulted; so in this case since with ground. So, in that case, your a phase and this will be grounded ok. So I, in that case I can say V ax will be equal to 0; there will be as well as V xg will be equal to 0.

Another thing I can say there will not be fault current through b phase and c phase. So, I f b will be equal to 0 and I f c will also equal to 0. So, now we have got these four required equations which need to be added in this earlier system equations or where we have got three equations. And these four more equations we have got which can be put it here. So, first equation if I am putting it here, so it is corresponding to V ax is equal to 0. So, V ax will come somewhere here. This will be 1; other elements will be 0. This makes V x is equal to 0.

Then another equation is V xg is equal to 0. So, from two V x is equal to 0 means 1 will come here or other elements are actually 0. And then third equation say I f b is equal to 0. So, basically I f b means it will come here 1 will come here other will be 0 and fourth equation says if c is equal to 0. So, 1 will come here and other elements will be 0.

So, here we have got the system of matrix where this is unknown. So, we can say this matrix is something like this. So, this I can say it is x is equal to this matrix Q corresponding to LG fault multiplied by this unknown matrix is say Y. And then you can get Y which will be equal to Q LG inverse into x. And as I told you Q LG is actually known. So, we can get these fault currents after solving this.

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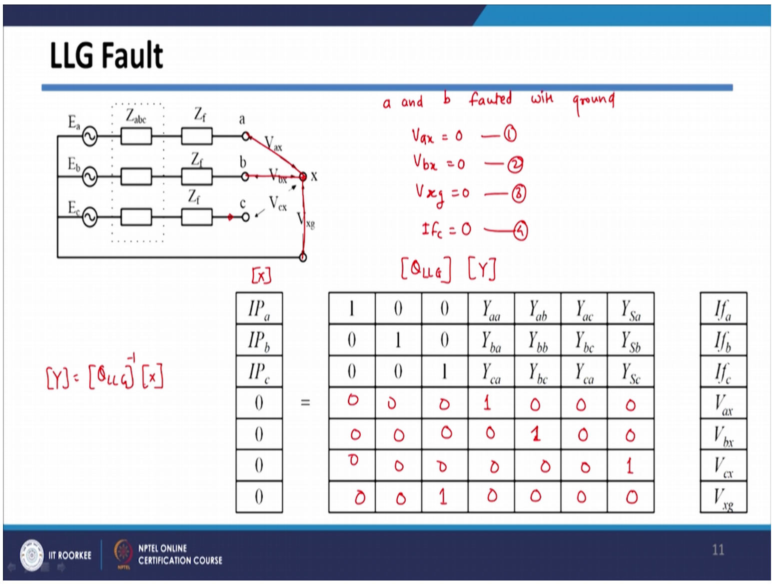

Let us say in case of LLG fault how we can get this remaining four equations. So, in this case as I told you two lines will be short circuited with respect to ground. So, these three voltages will be 0 those are so say a and b is faulted with ground. So, in that case your V ax will be equal to 0; v b x will be equal to 0; and v x g will be equal to 0. And one more equation we need that we can get that fault since there is no fault on space c in that case your fault current in I f c will be equal to 0.

So, we can put these four equation into this matrix system. So, first equation says this first, second, third and fourth. First equation says V ax is equal to 0. So, V ax comes somewhere here. Second equation say V bx is equal to 0. So, V b x will come somewhere here 1 will be here. V xg is equal to 0. So, this would be 1 and all other actually 0. And I f c is equal to 0. So, there will be one here and all other entries are 0.

So, in this case also we can do same thing. Let us see this is your x matrix; this matrix is Q LL G and this unknown matrix is say Y. So, we can get Y which is equal to Q LLG whatever we are getting its inverse multiplied by X.

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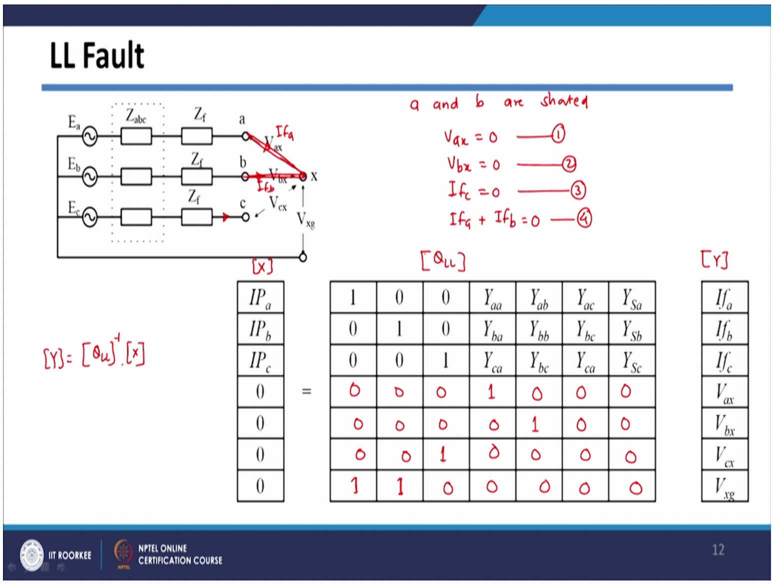

Let us consider one more fault LL fault. So, in this case say line a and b are shorted. So, in this case say a and b are shorted means this will be shorted with respect to this line. So, I can say V ax will be equal to 0; V bx will be equal to 0; I f c will be equal to 0, because there is not be current it is not faulted. And one more equation which we need that I can get whatever this current I a will be opposite of I b.

So, current I fa plus current I fb should be equal to 0th addition of both the currents will be 0 at these locations because your, I fb is coming from this side and I fa is coming from this side. This should be equal to 0. So, if we put say this equation number 1, 2, 3, and 4.

Equation number 1 say vx is equal to 0 so this will be 1; other are 0. So, we can say V bx is equal to 0. So, this will be 1; others will be 0. Third say I f c is equal to 0. So, here it will be one and all other zeros. And fourth equation say if a plus I f b will be equal to 0. So, there will be 1 here plus I f b and all other entries are actually 0. So, there will be one here and one here.

So, we have what actually a required system of matrix equations. And as I told you we are considering this is the x in this case we can call it a Q LL because it is L fault and this is your Y matrix. So, in this case again we can get Y which will be equal to Q LL and its inverse multiplied by x.

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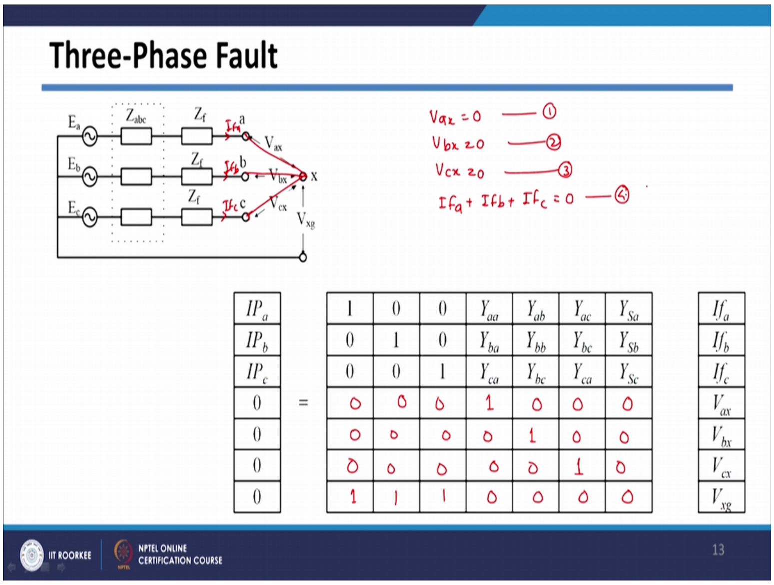

And let us say your last fault that is three phase fault maybe it is grounded or ungrounded. So, in case of grounded also we will see. So, in case of a three phase fault, in that case if a ground is not involved, this will be shorted, this will be shorted and this will be shorted. So, if three phase fault is occurring, all the three phases V ax will be equal to 0; V bx will be equal to 0; V cx will be equal to 0 and one more thing we can do is I f a I f b and I f c addition of all the three currents will be 0 because it is balance fault I f a plus I f b plus I f c will be equal to 0.

So, this gives me equation number 1, 2, 3 and 4. So, in this case from first equation, equation number 1, it will one here. From equation number 2, it will be one yet multiplied by V bx; third equation there will be one multiplied by V cx. And from fourth equation there will be 1 here, 1 here, 1 here and all of them entities are actually 0. So, this is how can get the for three phase fault and similar case.

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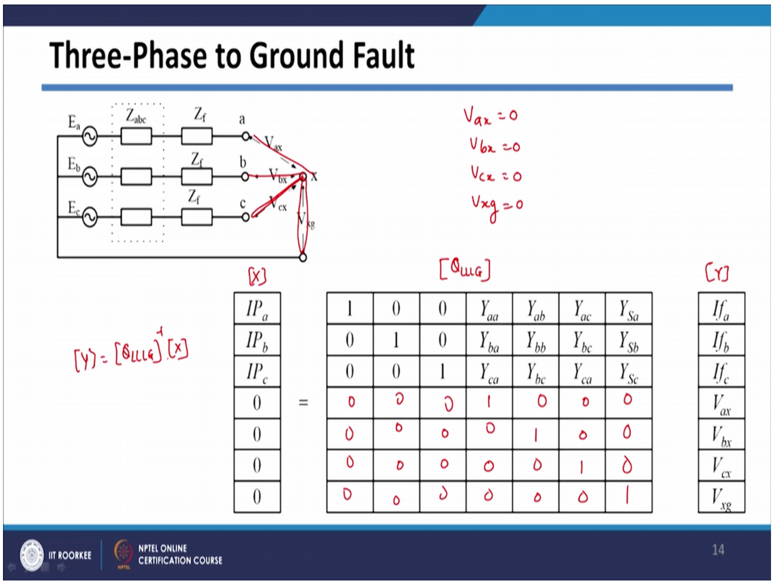

If there is three phase to ground fault then these all these three voltages they are getting short circuited. So, in this case, we can say V ax is equal to 0; V bx is equal to 0 V cx is equal to 0. And v x g is equal to 0. So, in this case, there will be 1 1 1 entry in this case all other entries will be 0. So, for particular fault, we can get this remaining four equation add to this system of equation which is written in matrix form and then called this has a x, this is Q LLG fault. And this is say Y, and then we can get y matrix which is unknown which will be equal to Q LLG fault its inverse multiplied by x.

So, in this particular lecture, we have seen that how we can use fair phase variable based analysis because we are seen there are some limitations of sequence component based analysis in case of distribution system that is basically because of unbalance in the system as well as transposition not existing in distribution system.

Moreover, there are some feeders which are single-phase, two-phase, three phase feeders as well as loads also might be single-phase, two-phase, three phase. So, this creates problem or a some it introduces some kind of error in distribution system analysis based on sequence component base theory that is why we have gone for phase variable based analysis. And phase variable analysis we have seen one simple method based on getting equivalent circuit and depending upon different types of fault, we can create system of matrix equations and solve it to get the fault currents.

However, if this method will be very cumbersome, if the network size is large because as I as I told you we are taken very simple system to analyze the fault currents, but distribution systems so we will be having thousands of nodes. And if you are concerned it is thousands of loads and if you want to get the sequent Thevenin's equivalent network till fault point it will be difficult task. Therefore, we need to sum or we need to analyze this fault systematically.

So, in next lecture, we will see direct approach based method which is very effective in calculating the short circuit currents and which is based on phase variable method. Again here also we are going to use instead of sequence component, we are going to use phase variable based analysis. So, next time we will see that.

Thank you very much.