Electrical Distribution System Analysis
Dr. Ganesh Kumbhar
Department of Electrical Engineering
Indian Institute of Technology Roorkee

Lecture – 33
Gauss Implicit Z - Matrix Method

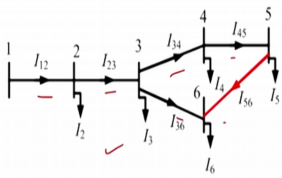
In the last lecture we have seen direct approach based method, which is applied to weakly meshed system and in today's lecture we will see Gauss Implicit Z-Matrix Method for distribution load flow analysis. So, before going to the Gauss implicit Z bus method we will just see what we have seen in the last class.

So, last class as I told you we have applied direct approach based load flow analysis for weakly meshed system; and we have seen that whenever there is weakly meshed what we need to do is we have to get BIBC meshed matrix form as well as a BCBV matrix form for these systems. So, what we do actually initially we remove the loop forming branches and then get BIBC matrix and then we modified BIBC matrix such that we can include those or loop forming branches like I have shown in this particular figure.

(Refer Slide Time: 01:21)

Example: Direct Approach Weakly Meshed System

Branch Number	Start Bus (i)	End Bus (j)	R (Ω)	X (Ω)	Load at End Bus (j) kW	Load at End Bus (j) kVAR
1	1	2	0.279	0.015	0	0
2	2	3	0.444	0.439	1572	174
3	3	4	0.864	0.751	1936	312
4	4	5	0.864	0.751	189	63
5	3	6	1.374	0.774	1336	112
6	5	6	0.896	0.155		



BIBC

1	1	1	1	1	0
0	1	1	1	1	0
0	0	1	1	0	1
0	0	0	1	0	1
0	0	0	0	1	-1
0	0	0	0	0	1

BCBV

0.279 + j0.015	0	0	0	0	0
0.279 + j0.015	0.444 + j0.439	0	0	0	0
0.279 + j0.015	0.444 + j0.439	0.864 + j0.751	0	0	0
0.279 + j0.015	0.444 + j0.439	0.864 + j0.751	0.864 + j0.751	0	0
0.279 + j0.015	0.444 + j0.439	0	0	1.374 + j0.774	0
0	0	0.864 + j0.751	0.864 + j0.751	-1.374 - j0.774	0.896 + j0.155

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE

Source: Edited by B. Das, *Power Distribution Automation*, IET Power and Energy Series, 75, London, 2016. 3

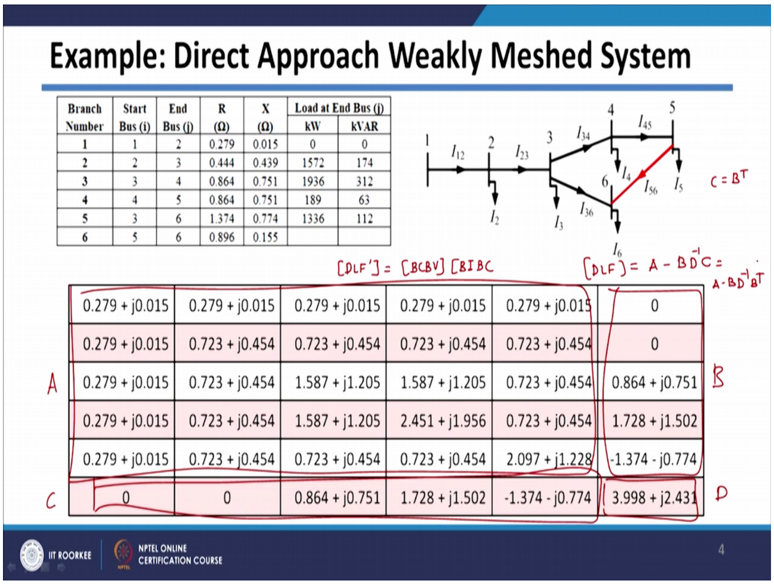
We have seen that whenever actually say suppose in this case this branch between 5 and 6 it is forming the loop. So, initially to get the BIBC matrix we remove that branch and get the radial BIBC matrix we have seen how to get that. So, we have seen that this column is corresponding to bus 2 this column corresponding 3 4 5 and 6 and we have seen that to get this 7th column of this BIBC matrix. So, this is your BIBC matrix and to get this seventh column of BIBC matrix, since this branch is connected between 5 and 6, so you have to subtract column 6 from column 5. So, if you subtract this column 6 from column 5 you will get column 7. So, here subtraction will be 0 because 1 minus 1 0, 1 minus 1 0 here it will be 1 here it will be 1 and here it will be minus 1 and since last column is corresponding to branch current which is forming the loop.

So, here we are getting corresponding to I 5 6 and we have seen that we have to put 1 here, and all other column we need to put 0. So, this is how we get BIBC matrix for weakly meshed system, similarly we have seen BCBV matrix also. So, as I told you initially we have to get the BIBC matrix for radial system, so here again this row is corresponding to 2, this is 3, 4, 5 and 6.

In this case also to get this seventh row we need to subtract this row number 6 from row number 5. So, this is corresponding to this loop forming branches, so here Z 1 2 minus Z 1 2 is 0, Z 2 3 minus Z 2 3 is 0, here I will get Z 3 4, here I will get Z 4 5 and here I will get Z minus Z 3 6 because, we have subtracting row 6 from row 5, and here we will get Z corresponding to loop forming branch that is Z 5 6 and the other entries in the other rows they are 0.

So, this is modified BCBV matrix including your loop forming branch and once you get this you multiply BIBC and BCBV so in one system I have taken. So, for this particular system by taking the R and X parameters of different lengths of the line and for that if you get BIBC matrix for this system. So, this is a BIBC matrix and BCBV matrix by considering particular impedances of the line you will be having this BCBV matrix.

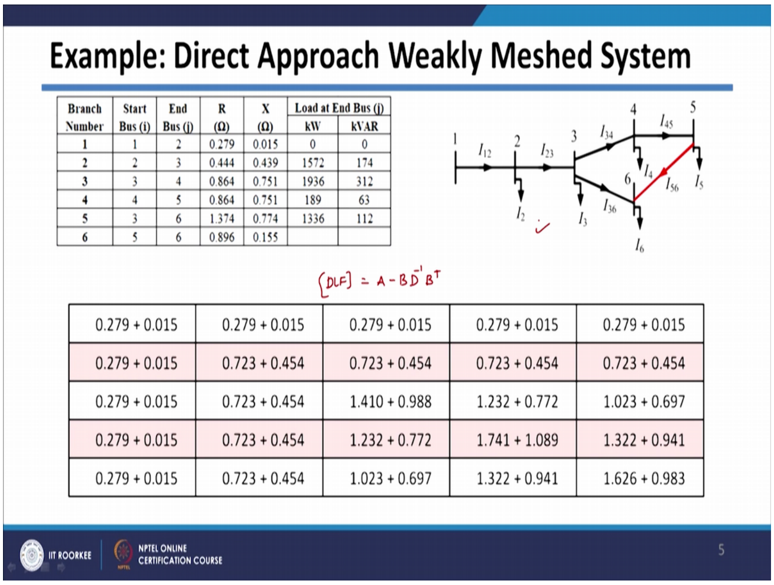
(Refer Slide Time: 04:25)



And then if you multiply so I can say DLF dash, because this is not actual distribution load flow matrix, which is BCBV multiplied by BIBC which you have got earlier in the this particular slide we can multiply them BCBV multiplied by BIBC I will get this DLF dash and then what we did?

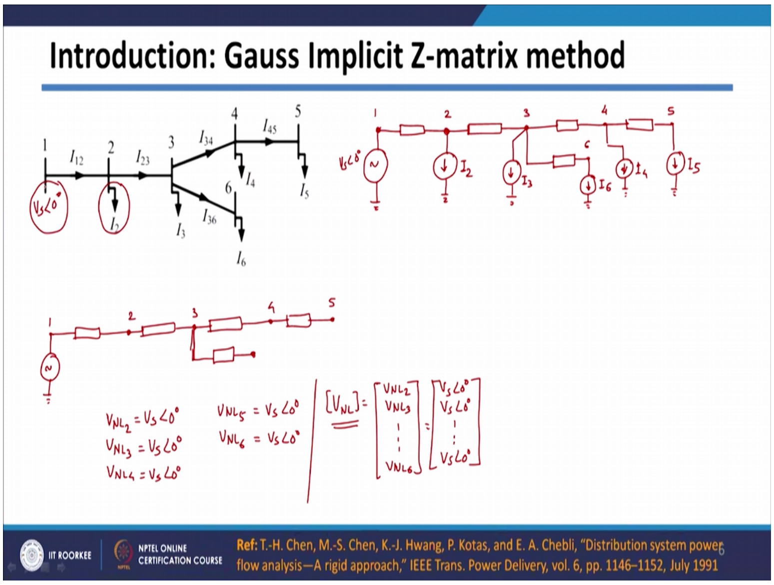
We divide this matrix in 4 parts and then we have seen that this part of the matrix which is basically a radial part we called matrix A this part, we called matrix B, this part we called C matrix C and this part we called matrix D. So, in this case modified DLF matrix so actual DLF matrix we have seen that it is A minus B into D inverse C, but C is actually equal to B transpose. So, this will be equal to A minus B into D inverse B transpose.

(Refer Slide Time: 05:41)



So, if you do that that this particular operation, that is A minus B into D inverse B transpose you will get DLF matrix for this particular solution and then we can go normal procedure which you have seen or normal algorithm step by step algorithm can be used for this also we already discussed it. Now today we are going to see another method that is Gauss implicit Z matrix method, this method is based on a superposition theorem. So before going to the super position theorem suppose, you are having this system here again that same system of 6 buses I am taking it here, and then various currents are shown in this figure.

(Refer Slide Time: 06:25)



So, if you see if you model this system as a circuit, it will look like something like this. So, this is node number 1 and then impedance Z 1 2 node number 2 this is this is node number 3 then node number 4. So, this is 1 2 3 4 and 5 and then there is one more branch from 3 which is going to impedance say 6, so this is your 6 here, and then source is connected here.

So, this is V S angle 0 degree with source is connected, so here this is V S angle 0 degree source is available and then we are having this loads I am modeling as a current sources into this network. So, the this will be modeled as a current sources so this current will be say I 2, then there is 1 current here current source here which is I 3, this current source here is I 6, this current source here is I 4 and this current source here is I 5.

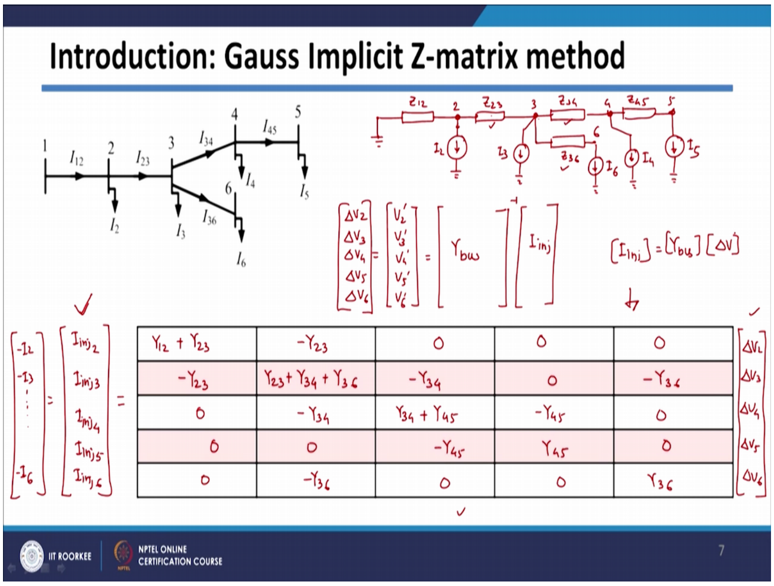
So, all the sources all the loads are modeled as current sources and here we are going to apply superposition theorem. So, initially we will consider all the voltage sources in the network and then we will consider all the current sources into the network. So, if you consider say in this case voltage source into the network the circuit will be something like this. So, initially so all the voltages by considering only the voltage sources, so in that case if you consider the only the voltage source your network will be something like this.

So, this is your 1 2 3 4 and 5 and then you are having this voltage source here, since we are considering only voltage source, and then next time we will consider the current sources and then we will calculate voltage at each of the node by both these methods and a add it together. So, in this case since we are considering only voltage sources your node to a network we will look something like this because, current sources are considered as a open circuit while we are doing the super applying the superposition theorem.

So, all the current sources will be replaced by infinite impedances. So, here they are not be connection with respect to ground. So, if we calculate voltages with respect to ground of each of this node VNL 2 node load voltage 2 will be in this case V S angle 0 degree VNL 3 will be also equal to V S angle 0 degree VNL 4 will also equal to V S angle 0 degree, because there is no path available to ground. So, no current will flow so each of this terminal will get same voltage. So, VNL phi will be V S angle 0 degree and similarly VNL 6 will be equal to V S angle 0 degree. So, all the voltages all the node voltages will be V S angle 0 and I am calling VNL because, it is no load voltage we are not considering any loads here all the current sources which are considered as a load they are removed. So, only no load circuit is there which satisfy V no load.

So, this equivalently I can say VNL matrix which forms VNL 2 VNL 3 up to VNL 6 which is basically we are getting. So, V S angle 0 degree V S angle 0 degree and all the entries are V S angle 0 degree. So, as we can say that this VNL will be available with us because, this voltage is known during the load flow solution. Now as I told you second step is to consider all the current sources and remove the voltage sources. So, in that case your network will be something like this.

(Refer Slide Time: 11:02)



So, here I am giving this voltage source will by which is having negligible impedance, so it will be directly grounded then these are the impedances and then there is a one more impedance which is starting from here Z 3 6 that is Z 1 2, Z 2 3, Z 3 4, Z 4 5 and Z 3 6 and we now we are considering the all the current sources into earlier figure. So, there is a it is current source here which is load current of bus 2 this is load current at bus 3.

So, this is I 2 this is I 3 this is I 6 because this is node 6 is 4 is 5, so this is your I 6 this current source is I 4 this current source is I 5. So, now I want to solve this network to get the voltages at these nodes ok, so this voltages I can say V 2 dash V 3 dash V 4 dash V 5 dash and V 6 dash that can be calculated by using say Y bus inverse multiplied by your current injected. So, if you know this current injected and Y bus inverse I can get the voltage sources.

So, basically we need to then get the Y bus of this, so in this case this is a say your current injected so current injected will be Y bus. So, this is current injected at bus 2 current injected at bus 3 I injected at bus 4 at 5 6 and this here will get multiplied with respect to this V 2 dash V 3 dash V 4 dash, but I am calling this as actually delta V 2 because, this is voltage difference when we are considering a load only.

So, here I am calling it as a delta V 2 delta V 3 delta V 4 delta V 5 and delta V 6, so this multiplied by your Y bus will give me currents injected. So, this is nothing but here delta V 2 delta V 3 delta V 4 delta V 5 delta V 6 these are nothing but node voltages when we are considering only the current sources. So, delta V 2 is voltage at bus 2 it is not difference it is voltage with at bus 2, when we are considering only current sources of the network.

So, in this case this will get multiplied with respect to this voltage here, so this is your voltage these are these current injected. So, here this must be your Y bus matrix so we can easily get Y bus matrix we already studied how to form Y bus matrix. So, for this particular network I can explicitly write so if you consider this node number 2 here.

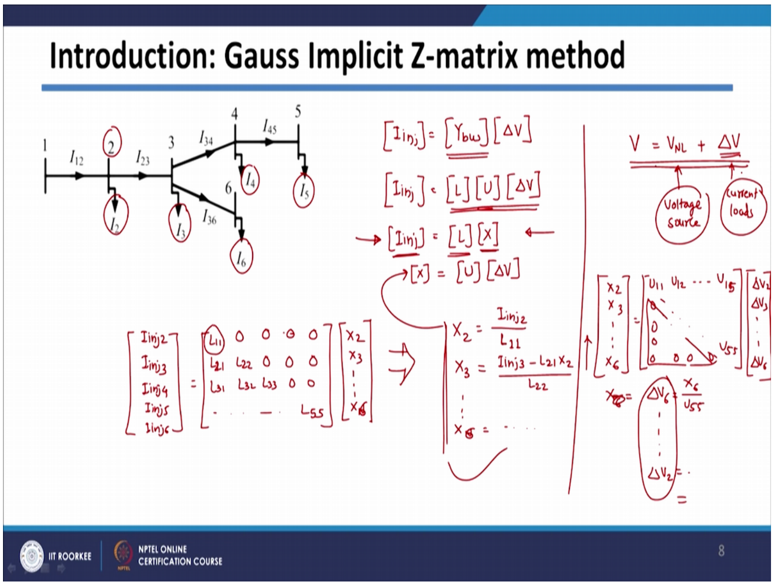
So, there are two impedances connected admittances connected those are Y 1 2 plus Y 2 3 and the impedance between 2 and 3 it is Y 2 3, then when we consider other entries will be 0 here. Then when you consider node 3 so there will be 3 admittances connected Y 2 3 Y 3 4 and Y 3 6, so for node 2 it is Y 2 3 plus Y 3 4 plus Y 3 6 then there will be between Y 2 3 is between 2 and 3. So, here there will be minus Y 2 3 here it will be minus Y 3 4 and it will be minus Y 3 6 we call it is between 3 and 6.

Then if you consider node number 4 there are 2 impedances connected 2 admittances or 2 admittance is connected those are Y 3 4 plus Y 4 5. So, here between 3 and 4 this is Y 3 4 and between 4 and 5 this is Y 4 5 all other entries are 0 and then when consider node 5 only one admittance is connected that is Y 4 5 and there will be between 4 and 5.

So, minus Y 4 4 will come here and between 3 and 6 there is a only 1 admittance. So, Y 3 6 will come here and there will be minus Y 3 6 here all other entries are 0 and we can form Y bus matrix like this. So, here the current injected if you observe these current sources they are in opposite direction of current injected. So, these are nothing but minus I 2 minus I 3 minus I 6 because, current injected is in toward the node and the direction of the load current which are we are taking up to this.

So, whatever load current we are calculating we need to take minus sign here. Now we have got this matrix which is basically if you see this matrix it is the structure of this matrix is less something like this, I injected matrix which is basically this part is equal to your Y bus matrix which is this we know how to form the Y bus and then this matrix I am calling delta V, basically structure of this matrix is something like this.

(Refer Slide Time: 17:30)



So, I injected is equal to your Y bus matrix multiplied by delta V, these are the voltages at this node which we are interested in; and then if you see these Y bus can be splitted into 2 part using led composition there are many methods. So, I injected will be equal to U decomposing this into lower and upper triangular matrix multiplied by U delta V and then this can be again written as.

Now, here U multiplied by delta V I am writing it as a matrix x. So, matrix X we know that it is U multiplied by your delta V. So, since here if you observe this it is low we know that this L is lower triangular matrix, and I injected current they are known because, as you told you as I told you these are actually just opposite of your negative sign of your load currents. So, this matrix is known so I injected at various buses is known t then.

So, by solving this we can get X and it is easy because here this now only lower triangular matrix. So, in this case if it is I injected at bus 2, I injected at bus 3, I injected at bus 4, I injected at bus 5, I injected at bus 6 will be equal to your lower triangular matrix, I am considering this matrix here.

So, lower triangular matrix will be L 1 1 and then all the entries will be 0 here and then here L 1 L 2 1 into L 2. So, here and then L 3 1, L 3 2, L 3 3 0 0 0 and up to L 5 5 and this is getting multiplied with your X 2 X 3 up to X N which are unknown I am starting from 2 because, we are starting from bus number 2 that is why I am not taking X 1 here x, so it will start from X 2.

So, in this case I can easily write your X 2 will be equal to your I injected at bus number 2 divided by L 1 1 matrix this element of this matrix and this is known because we have derived this LU matrices from Y bus only. So, L 11 is already known current injected is known. So, 2 X 2 can be calculated similarly your X 3 can be calculated like this. So, it will be L I injected at bus number 3 minus L 2 1 into X 2 divided by your L 2 2 and like this you can go on calculate up to Xn or in this case it is X 6.

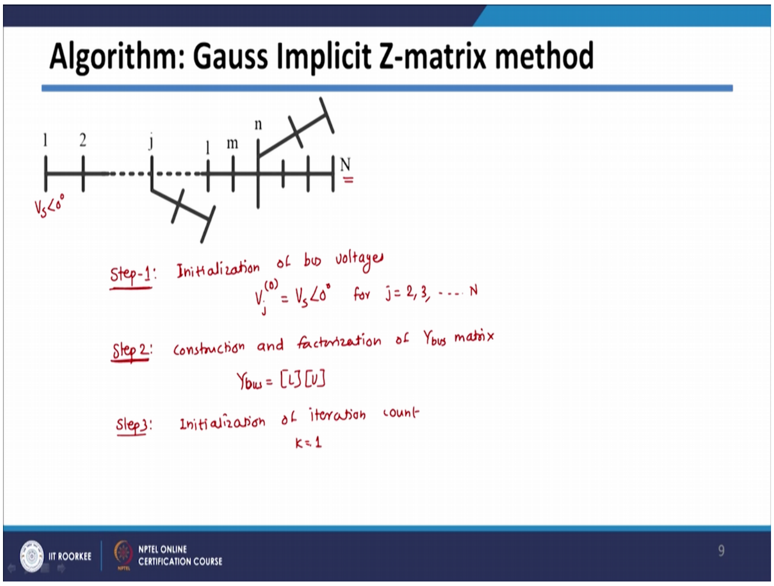
So, we can get up to X 6 all the values and once you get all the values of X we can put this into this matrix. So, in this case this next matrix will be X 2 X 3 and up to X 6 that will be equal to and here this upper triangular matrix means U 1 one U 1 2 and up to U 1 6 U1 5 and up to it U 5 5 and this elements will be 0 here all this.

So, it is just upper triangular matrix so here also we can use same procedure to get this right hand side of this one that is delta V 2 delta V 3 up to delta V 6 and so we can use same scheme here you can bus in vertical direction that is first start from X 6. So, X 6 will be just you can say sorry and delta V 6 will be equal to X 6 divided by U 5 5 and you can go on calculating till delta V 2.

So, in thus this is how we can actually get the all the voltages now you have to apply superposition. So, here you have got after doing this you have got your delta V matrix, which is just by considering the current sources and already you have got this VNL matrix, which is just by considering the voltage sources.

So, this is due to current sources which are basically loads and this is due to applied voltage source and if you add them together, I will get voltage which is applying the superposition. So, here we are applying super potion, so we are going to add this 2 voltages which is due to voltage source and due to load currents. So, V will be equal to V no load plus delta V, now let us say what will be the steps of these algorithm this algorithm.

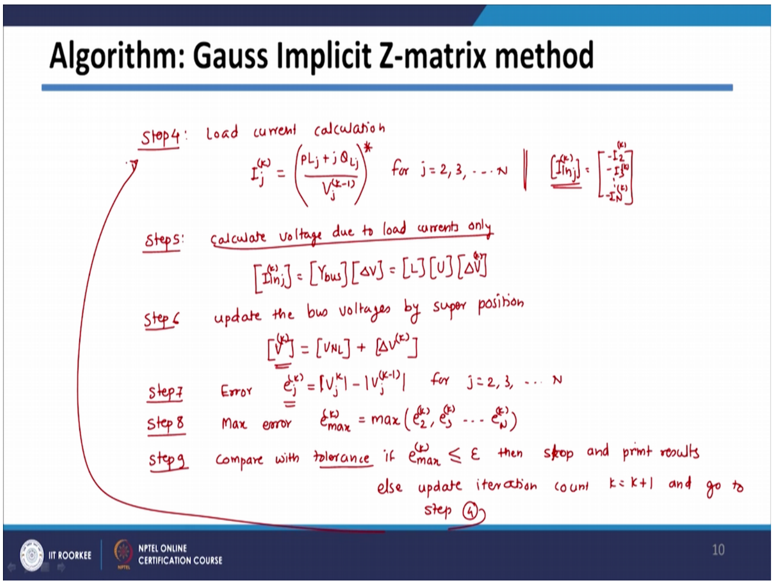
(Refer Slide Time: 23:14)



So, here say this is your generalized system which is having N number of buses here in this case the step 1 will be. So, here voltage at this node is known so V S angle 0 degree. So, step 1 like your all other algorithm, so step 1 will be initializing initialization of bus voltages. So, we can initialize that we can say Vj at 0th iteration will be equal to V S angle 0 degree and this is for j going from 2 3 up to N means all buses will be initialized with voltage V S angle 0 degree in a first step.

In second step construct your Y bus matrix. So, you can say construction and factorization both we are doing factorization of Y bus matrix. So, once you get Y bus matrix we can factorize means we can say your Y bus matrix will be L multiplied by U that is LU decomposition, we have de factorization we have doing then step 3 will be initialization of iteration count. So, initialization of iteration count iteration count k will be equal to 1; first and then step 4 will be calculation of load current at each bus.

(Refer Slide Time: 25:11)



So, load current will be calculated we know that load current calculation. So, we know that current Ij at any kth iteration will be calculated by knowing the load at jth bus. So, PLj plus jQLj is load at jth bus divided by your voltage at jth bus, but which is obtained at k 1 th iteration and you have to take star of it and this will be done for all j goes from 2 3 up to N. So, all the load currents will be calculated and then we know that your I injected matrix will be nothing, but your matrix of minus I 2 minus I 3 to minus N.

So, from this I injected matrix also in this particular step. So, now I injected matrix is known and so in step 5 we use that L U decomposition and get the delta V. So, calculate voltage due to load currents only modeling as current sources. So, we have seen that I injected matrix is actually equal to your Y bus matrix, but we have seen that it is actually decompose into 2 parts, so that is actually equal to your L matrix multiplied by U matrix into your delta V.

So, here we can get the delta V matrix and then step 6 will be update the bus voltages by superposition. So, in this case superposition still will be applied, so we know that V j or sorry here voltages of the buses at kth iteration. So, this is the injected current so delta V we have calculating at kth iteration by considering injected current which is taken at kth iteration. So, all these are related to kth iteration.

So, voltages of the bus is at kth iteration will be equal to V NL no load voltages as I told you they are not changing at all they are remaining constant. So, V no load plus whatever delta V which you are getting it here at kth iteration. So, so this delta V which is at k th iteration, then step 7 will be comparing or calculating an error. So, error in the voltages so error in jth voltage, in kth stage kth iteration will be equal to voltage of jth bus at kth iteration which you are getting it here.

So, this is 1 entry minus voltage of jth bus at k k minus 1 th iteration and that will give us error in jth and this step will be done for j is equal to 2 3 up to N. So, we can get the error in all the bus voltages and step 7 the step 8 will be to calculate max error. So, in this case max error will be e max at kth iteration will be nothing but maximum of all the errors which you have calculated in earlier step. So, here we are calculated e 2 e 3 e 2 e 3 up to e N.

So, it will be e 2 comma e 3 up to en which is we have calculated at kth iteration. So, that is why I am iterating k here, so we have got this maximum error and we know that in step 9 we will compare this maximum error with tolerance compare with tolerance. So, if e max at kth iteration if it is less than or equal to epsilon which is basically a tolerance value, then stop if this condition getting satisfied then stop and print the result.

So, we have got the results else update the iteration count, count that is k is equal to k plus 1 and go to step 4. So, here we have to go back and again calculate new currents injected currents then get the delta V, if I consider a injected currents only then update the voltages again compare the voltages keep this loop till the convergence happens. So, your Gauss implicit Z matrix method works like this.

So, in today’s lecture we have seen Gauss implicit Z bus matrix method and initially we have seen the introduction of this how we can get it, so we have seen that we need to use it superposition theorem here. So, initially the no load voltages will be calculated removing all loads and then in second step all the loads will be modeled as a current sources and you will get your delta V. So, no load voltage plus delta V will give actual voltage and we have to keep iterating till the convergence will happen.

So, here we completed the load flow studies and if you remember load flow studies we have compared the algorithms of transmission systems, load flow algorithm and transmission systems and load flow algorithm distribution system. So, you have seen that in case of transmission system we have used Gauss Seidal method Newton Raphson method or fast decouple load method and as explain there those methods can be used for distribution system with little modification.

However because of high r by X ratio convergence of these methods is very slow and other advantage of distribution system is they are radial 1 and then we can because of this radial structure we can develop some simple and efficient algorithm for distribution systems. So, that is why instead of going for traditional methods like Gauss Seidal and Newton Raphson we have gone for other methods which are efficient for distribution system and we have seen that those methods are basically backward forward sweep algorithm we have seen it for balanced system first and then we have used it for unbalanced system also considering all the 3 phases together.

Then we have gone for direct approach for load flow analysis and we have seen that from the topology of your circuit or graph of your circuit we can get BIBC matrix and BCBV matrix which is basically consisting of impedances and multiplication of this BCBV multiplied by BIBC you have got distribution load flow matrix and using this matrix we can get your update of voltages and it is you can say next step of backward forward algorithm.

So, we have seen direct approach based load flow also first we have seen it for radial systems it is balanced as well as unbalanced and then we have gone for using it for a weakly meshed system. So, we have seen one example also for radial as well as weakly meshed system and then finally we have seen Gauss implicit Z bus matrix method today, which is again widely used for distribution load flows. So, here we will complete the load flow portion and next class we will start with short circuit analysis.

Thank you.