

Electrical Distribution System Analysis
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Lecture – 29
Direct Approach Based Load Flow Analysis
Part I

Dear students, we are studying load flow analysis method for distribution systems. And from last 2 lectures we have seen method based on backward forward sweep and we have seen that it is very classical method and simplest of them. And we have seen this method for radial balance system first and then we have seen for radial unbalance system.

However, this method cannot be used for meshed system because in this case we need to use backward and forward sweep. We have seen that, backward sweep starts from your leaf node or a end nodes and we go towards the source node, and the your forward sweep start from source node and we end till end nodes.

So, therefore, we need to know the end nodes for this particular method. So, whenever there are a weakly mesh systems, it cannot be directly applied without any modification. So, because of that we will study another method today that is direct approach based method.

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Example: Backward/Forward Sweep Load Flow

Branch Number	Start Bus (i)	End Bus (j)	R (Ω)	X (Ω)	Load at End Bus (j)	
					kW	kVAR
1	1	2	0.279	0.015	0	0
2	2	3	0.444	0.439	1572	174
3	3	4	0.864	0.751	1936	312
4	4	5	0.864	0.751	189	63
5	3	6	1.374	0.774	1336	112

Before going to the direct approach based method, let us take one example how we can use it for distribution load flow. Now, you can see that your distribution load flow information will be something like this. So, here we are having this branch numbers.

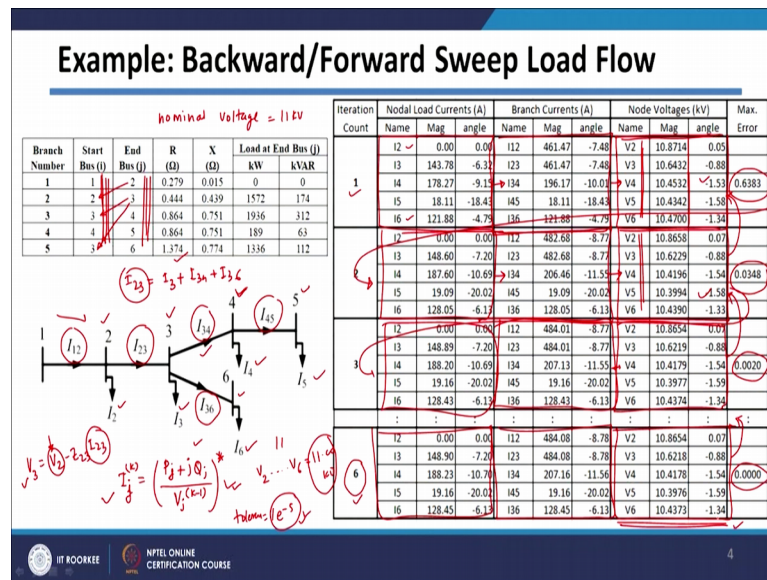
So, say there are 5 branches in the system, these are actually starting node of the this particular branch. So, for branch 1 starting node is 1 and ending node is 2, this is actually your resistance and reactance of line section and these are the load connected at the end bus. So, we call this as line data because it is related to line and this is this we call it as a load data, and this is load we have considered at j th node means it is just basically this load it is connected at number 2 means node connected at 2 we are not connecting any load because we have considered it 00.

And then at node number 3 we have connected this load at 4 and 5 and 6. So, basically this is at 4, this is at 5 and this is at 6. And if you observe this structure, we can easily find out a 1 is only appearing in this first column. So, we can find out this must be your source node, and you can see that this 5 and 6 is actually only appearing in the second a column or j th column of this matrix.

So, this must be your end node. So, we can recognize which are the start nodes and end nodes by observing this column, which is start end column start bus column and end bus column. So, if you plot. So, distribution system from this we can say that there is a line, which is connected between 1 and 2. So, there is line between 1 and 2 then there is line between 2 and 3. So, there must be one line is between 2 and 3.

And then there is line between 3 and 4. So, there will be line between 3 and 4 and there is a one line between 4 and 5. So, this is 4 between 4 and 5 and there is a one line between 3 and 6. So, there is a one more line, which is starting from 3 and it is ending with 6. So, by this seen we can easily find out the distribution system structure, and this represent your resistance or reactance of these sections. So, resistance and reactance of first section will be this one from 2 3 it will be basically this one.

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Now, if you want to apply your backward forward sweep algorithm, we know that so, this data I have taken from last slide, and this is your system which you have considered. So, these are basically a branch current I_{12} , I_{23} , I_{34} and I_{45} and I_{36} are basically your branch currents and these are the bus or node load currents and we have seen that this load current can be calculated like this. So, we have seen that your I_j any load current at any bus we have seen that it is $P_j + jQ_j$ divided by V_j^* and this voltage we have using from your last iterations. So, it was actually $k-1$.

So, when you are calculating current for k th iteration you have to use voltages of $k-1$ th iteration and in this case since we are considering constant power loads are not going to change. So, these loads are not of your voltage dependence. However, whenever there is voltage dependant load, you need to calculate load first from this voltage as we have described in case of load modeling and then you have to calculate the current. So, this how we can calculate the currents so, this currents for first iteration will be calculated as I told you by considering your voltage; voltage of the system we have consider a nominal voltage of the system is 11 kV.

So, nominal voltage of the system is 11 kV. So, initially we assume we have assumed all the voltages of all the buses from V_2 to V_6 they are equal to 11 kV and using that 11 kV, we have calculated your current which are flowing in each of the node. So, the load current is calculated since there is no load on this one. So, basically this will give when

you calculate load current using this equation, you will get all the load currents from I 2 to I 6. Then you have to need to calculate your branch currents by using the backward sweep.

So, for calculating backward sweep, we need to know the how the branches are connected. So, we can see that the connections of the branches will be like this. So, we can see that 2 is connected to this one. So, there is actually 1 2 branch is connected to 2 3, then 2 3 is connected to 3 4. However, there are 2 branches which are starting from the 3 ok. So, we can easily find out current I 2 3 will be equal to current I 3, which is load current at this bus plus current I 3 4 plus I 3 6.

So, if you add this 3 current I will get this 2 current into I 2 3; this is how we can calculate currents in all the branches by using the logic be based on the structure of these 2 columns, we can get all the branch currents. So, we if you calculate branch currents, we will get this branch currents and based on this branch current we can calculate the voltage drop. So, we know that voltage drop any voltage drop say V 3 in this case. So, V 3 will be equal to V 2 minus Z 2 3 into I 2 3 this currents are already calculated this voltages which. So, for when calculating V 3, V 2 is already calculated.

And that is how we know the voltages and then voltages all the voltages will be calculated in forward sweep. So, initially we calculate voltage of V 2, then based on V 2 we calculate V 3, and based on V 3 we calculate V 6 and V 4 and then based on V 4 we calculate V 5. So, using this we get the voltages then we check the error with respect to now earlier voltages are actually 11 kV we are used. So, with respect to 11 kV and this voltages we calculate maximum error between the 2 iterations and that maximum error comes out to be this one.

And in this case I have used 1×10^{-5} is your tolerance value. So, a tolerance value is a 1×10^{-5} . So, it is not less than your tolerance. So, you will get a go to the next iteration using this voltages again using this equation of the current, we can get the new current values which are basically this then get similar logic get branch currents, which are basically this. And then using a band current again using your forward sweep get the node voltages, which are basically this and then again calculate error between this voltages and earlier iteration voltages means basically this voltages and this voltages.

Maximum error comes out to be 0.00348 which is again more than your tolerance values. So, you have to go for next iteration. So, if you keep on calculating your node current based on this voltages, then corresponding branch current by using the logic based on start bus and end bus. And then you can calculate again the voltages and then again we calculate error between this 2, which is comes out to be this one. So, if you keep on iterating like this till error will be lesser than one exponential 5.

So, in this case if you keep on iterating at sixth iteration, we get this current here the branch currents using the backward sweep, and then there is of using forward sweep node voltages. And in this case if you compare the voltages of sixth iteration and your fifth iteration, you are getting error which is less than 1 exponential minus 5. So, in this case we can see that we have converged and the voltages are no more changing, more than 1 exponential minus 5.

So, we can consider that this is your solution. So, this is how your backward forward sweep further in the work and this is the example of backward forward sweep method.

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Introduction: Direct Approach Based Load Flow Analysis

Proposed by J. H. Teng [1]

$I_{45} = I_5$ — (4)
 $I_{34} = I_4 + I_{45} = I_4 + I_5$ — (3)
 $I_{36} = I_6$ — (5)
 $I_{23} = I_3 + I_{34} + I_{36} = I_3 + I_4 + I_5 + I_6$ — (2)
 $I_{12} = I_2 + I_{23} = I_2 + I_3 + I_4 + I_5 + I_6$ — (1)

Branch currents

$$\begin{bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ I_{45} \\ I_{36} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$
 BIBC
 (Bus injections to branch current)

$$[I_{branch}] = [BIBC] [I_{node}]$$

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[1] J. H. Teng, "A direct approach for distribution system load flow solution," IEEE Transactions on Power Delivery, vol. 18, no. 3, pp. 882-887, July 2003

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Now, let us go for a direct approach based load flow analysis. This proposed first by J. H. Teng in 2003 and that paper which he published is titled as a direct approach for a distribution system load flow solution. So, this is how it works.

Let us say consider similar example here you are consider 6 bus system. So, this is your 6 bus system you are having this node 1 node 2 node 3 node 4 node 5 and this is your node 6 and this we know that current I_{12} , current I_{23} current I_{34} , I_{45} and current I_{36} and the load connected here you see it is I_2 , load connection here is I_3 , load connected here I_4 , I_5 and at the 6 node say I_6 .

So, here we can write your equation I_{45} will be equal to current I_5 , because 3 these through this branch only I_5 current is flowing. Now, if you consider current I_{34} it will be equal to current I_4 , which is load at bus 4 plus current I_{45} and we have got I_{45} is equal to I_5 . So, it will be I_4 plus your current I_5 . Similarly I can write for I_{36} . So, I_{36} only current of I_6 will be flowing. So, it will be I_6 and then I_{23} will be equal to the load current at bus I_3 bus 3 which is I_3 plus your current which is going from 3 to 4, that is I_{34} and then current which is going from 3 to 6, which is I_{36} and both the currents we already got. So, it will be I_3 plus your I_{34} is actually I_4 plus I_5 .

So, it is I_4 plus I_5 and I_{36} is actually I_6 . So, plus I_6 ; and then I_{12} will be equal to I_2 plus I_{23} and we already got current I_{23} , which is basically this one. So, it will be I_2 plus I_3 plus I_4 plus I_5 plus I_6 . So, here we have got the 5 equations. So, I can just number it this is a 1 equation number 1, this is equation number 2 say this is equation number 3 this is equation number 4 and say this is equation number 5.

So, we have got this 5 equations let us put this equation into matrix format. So, if you put this equation starting from 1 to 5 into matrix format. So, on this sides there are branch currents I_{12} , I_{23} , I_{34} , I_{45} and I_{36} basically this current here which will be equal to say a some matrix here and then there are node currents which are I_2 , I_3 , I_4 , I_5 and I_6 .

So, if you see this equation number 1 you can see that all the currents are existing that is I_2 , I_3 , I_4 , I_5 , I_6 . So, it will get multiplied to all the currents. So, it would I_{12} is equal to I_2 plus I_3 plus I_4 plus I_5 plus I_6 if you see the equation 2 there is no I_2 . So, I_2 will be get multiplied with 0 and all other currents are present that is I_3 , I_4 , I_5 and I_6 if you see equation 3 4, there is only I_4 and I_5 . So, 00 it is I_4 and I_5 I_6 is not there.

And if you see I_{45} it is only I_5 . So, 0 0 0 1 0 and if you see I_{36} there is only I_6 . So, 0 0 0 0 and 1 now we have got this matrix we are going to call this matrix as BIBC matrix; Because if you observe this these are nothing, but your bus injections and these are nothing, but your branch currents. So, this matrix is basically converts your bus

injection to branch currents. So, this matrix is called as bus injections to branch current matrix. So, this is called as a BIBC or Bus Injection to Branch Current matrix.

So, I can write this as I_{branch} , which are basically these are branch currents which is equal to this your BIBC matrix, multiplied by these are nothing, but your nodal currents. So, I can just write I_{node} or I_{bus} . So, we have got now BIBC matrix.

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Introduction: Direct Approach Based Load Flow Analysis

$$V_2 = V_1 - Z_{12} I_{12} \quad \text{--- (1)}$$

$$V_3 = V_2 - Z_{23} I_{23} = V_1 - Z_{12} I_{12} - Z_{23} I_{23} \quad \text{--- (2)}$$

$$V_4 = V_3 - Z_{34} I_{34} = V_1 - Z_{12} I_{12} - Z_{23} I_{23} - Z_{34} I_{34} \quad \text{--- (3)}$$

$$V_5 = V_4 - Z_{45} I_{45} = V_1 - Z_{12} I_{12} - Z_{23} I_{23} - Z_{34} I_{34} - Z_{45} I_{45} \quad \text{--- (4)}$$

$$V_6 = V_3 - Z_{36} I_{36} = V_1 - Z_{12} I_{12} - Z_{23} I_{23} - Z_{36} I_{36} \quad \text{--- (5)}$$

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} -Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & -Z_{23} & 0 & 0 & 0 \\ -Z_{12} & Z_{23} & -Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & -Z_{45} & 0 \\ -Z_{12} & -Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ I_{45} \\ I_{36} \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = [BCBV] \begin{bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ I_{45} \\ I_{36} \end{bmatrix}$$

$$[\Delta V] = [BCBV] [I_{branch}]$$

(Branch Current to Bus Voltages)

Now, let us consider one more matrix which we can write which is called as a BCBV let us see how we can write it. So, we know that your voltage V_2 is actually equal to voltage V_1 minus Z_{12} into I_{12} this is basically this drop which is Z_{12} multiplied by I_{12} .

And then you are having this V_3 will be equal to your V_2 minus Z_{23} into I_{23} ; however, you have got this V_2 which is basically V_1 minus Z_{12} I_{12} multiply by I_{12} . So, I can just write it will be equal to V_1 minus Z_{12} into I_{12} minus Z_{23} into I_{23} . So, basically instead of this V_2 here I just replace by V_1 minus Z_{12} multiplied by I_{12} ; similarly I can write for V_4 .

So, V_4 will be equal to V_3 minus your Z_{34} into I_{34} which will be equal to V_3 we already got it I can just take it as it is that is V_1 minus Z_{12} into I_{12} , minus Z_{23} into I_{23} minus, Z_{34} into I_{34} exactly similarly way if I can write for V_5 .

So, V_5 will be equal to V_4 minus Z_{45} into I_{45} . So, this will be equal to your V_4 we already got it which is this one, V_1 minus Z_{12} into I_{12} minus Z_{23} into I_{23} minus Z_{34} into I_{34} , minus your this Z_{45} into I_{45} and then we can write remaining that is V_6 voltage at bus number 6 will be basically 6 is connected to bus number 3. So, it will be V_3 minus I_{36} or Z_{36} multiplied by your I_{36} .

So, V_3 is you already got it is actually this one here. So, V_1 minus Z_{12} into I_{12} minus Z_{23} into I_{23} minus I_{36} or refers Z_{36} into I_{36} . So, here again I have got this 5 equations say this is number equation number 1, this is say equation number 2, this is say equation number 3 4 and equation number 5; now we can put this 5 equations into the matrix format. So, if you can put it will be like this.

So, on the left hand side we are getting V_2, V_3, V_4, V_5 and V_6 which will be equal to and then in every equation we are getting terms related to V_1 . So, it will be V_1 in all equations minus here we are getting from matrices and then it is getting multiplied by your branch current because your, you are see that there are branch currents. So, $I_{12}, I_{23}, I_{34}, I_{45}$ and I_{36} .

Now, if you observe first equation it is V_2 is equal to V_1 minus Z_{12} into I_{12} even we already taken it. So, it will be Z_{12} multiplied by I_{12} . So, it is just getting multiplied by I_{12} other terms are not there. So, from first equation we can write it like this, from equation number 2 we are getting 2 terms one is getting multiplied to I_{12} and another is I_{23} . So, it will be Z_{12} and Z_{23} all other currents term are not there [noise.]

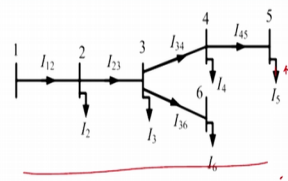
Then in the third equation we are getting 3 terms Z_{12}, Z_{23} and Z_{34} ; So, Z_{12}, Z_{23} and Z_{34} . In equation number 4 we are getting 4 terms those are basically $Z_{12}, Z_{23}, Z_{34}, Z_{45}$ and 0 there is no term related to I_{36} into this equation number 4 and then if you see the equation number 5, there is Z_{12}, Z_{23} and Z_{36} . So, here we are having $Z_{12}, Z_{23}, 0, 0$ and when the term which is related to is Z_{36} .

And this we can write I am just calling this matrix as BCBV matrix, we will see the significance of this. So, here we are getting this if you write it is by taking this one right hand side and if you take of multiplied by minus 1. So, it will be V_1, V_1, V_1, V_1, V_1 we can write it like this minus your voltages V_2, V_3, V_4, V_5 and V_6 will be equal to this is your BCBV matrix multiplied by your $I_{12}, I_{23}, I_{34}, I_{45}$ and I_{36} .

So, we can see here this is nothing, but the subtraction of voltages from bus voltage number voltage at bus number 1 minus different bus voltages; and this particular term I am calling it as a delta V matrix, which is equal to your BCBV matrix multiplied by if you observe this these are nothing, but your branch currents. So, I can write I branch and this BCBV matrix is called as branch current to bus voltages; basically this matrix if you observe it is converting your branch current to your bus voltage terms; So, this that is why it is called as a branch current to bus voltage matrix.

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Introduction: Direct Approach Based Load Flow Analysis



$$[I_{branch}] = [BIBC] [I_{node}]$$

$$[\Delta V] = [BCBV] [I_{branch}]$$

$$[\Delta V] = [BCBV] [BIBC] [I_{node}]$$

$$[\Delta V] = [DLF] [I_{node}]$$

$$I_j^{(k)} = \frac{P_j + iQ_j}{V_j^{(k-1)}} \quad \text{for } j=2, 3, \dots, N$$

$$[I_{node}^{(k)}] = \begin{bmatrix} I_2^{(k)} \\ I_3^{(k)} \\ \vdots \\ I_N^{(k)} \end{bmatrix}$$

$$[\Delta V^{(k)}] = [DLF] [I_{node}^{(k)}]$$

$$[V]^{(k)} = [V_1] - [\Delta V^{(k)}]$$

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So, basically we have got two equations one is called I branch is equal to you may write BIBC matrix, which can convert branch injection to bus injection to branch current multiplied by your I node. And one more equation we got that is delta V voltage difference will be equal to your BCBV matrix multiplied by your branch currents. So, basically your BCBV matrix convert your branch current to bus voltages.

And then if you can put this I branch equation here. So, I can say your delta V will be equal to BCBV multiplied by your BIBC matrix multiplied by your I node, and this particular multiplication I am calling matrix DLF Distribution Load Flow matrix multiplied by your I node. Now we can see that these matrices that is BCBV and BIBC they are just depend on your topology of your distribution system.

So, you can see that this BIBC matrix is basically having the form of 1 and 0's, and then your BCBV matrix is basically having similar structure. However, it is matrix of a

impedances and if you multiply them, you are getting this DLF matrix. So, during the iterations basically this DLF matrix will remain constant as long as the configuration of the system is not changing. So, if the configuration of the system is remaining constant DLF matrix will not change during the iterations.

So, what will change is only your I node currents and your delta V s during the iterations. And delta V we have seen that it is basically difference of voltages. So, basically it is difference of V_1 minus V_2 , V_1 minus V_3 , V_1 minus V_4 like this, V_1 minus V_n if there are n number of nodes in your system. So, in this case if you are using this algorithm or using this equation we need to calculate first I node matrix.

So, I node matrix as I was shown you it is actually nodal current matrix where the node current is calculated at say k th iteration depends upon your current equation, which is basically P_j plus Q_j divided by your V_j which is calculated at $k-1$ th iteration and just take star of it, and then for all the j s starting from 2 3 up to n we go.

And here then your I node matrix you have seen that I node matrix will be equal to your I_2 at k th iteration, I_3 at k th iteration up to I_n at k th iteration. And then we can get directly the delta V matrix of k th iteration which is basically this matrix of at k th iteration will be equal to your matrix DLF, which is not going to change it during the iteration multiplied by your matrix I node, which is nodal current matrix, which you can built using this one and then we can. So, it is at the k th iteration.

So, once we get delta V we can update the voltages. So, voltages at various buses at k th iteration will be equal to your voltage V_1 minus your delta V_k which you have got it here. And then we get this updated voltage at k th iteration and we can check your thousand value if it is not satisfying you can again calculate currents and keep on this iterating till we get converge these; we will see the detailed step that is to call it as them in the next class.

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Summary of the Lecture

- Direct Approach Based Load Flow Analysis
 - Radial balanced distribution system
 - Introduction

Handwritten notes in red:

$$\begin{aligned} & [BIBC] \\ & [BCBV] \\ & [DLF] = [BCBV] [BIBC] \\ & \underline{[\Delta V]} = [DLF] [I_{node}] \leftarrow \end{aligned}$$

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So, in this particular lecture we have seen direct base approach based load flow analysis. We have seen the introduction and introduction, we have seen that we if you form this matrix, which is called as BIBC matrix, which basically converts your bus injection to branch currents and then another matrix, which is BCBV matrix which basically converts your branch current to bus voltages.

And then DLF matrix which is basically your BIBC matrix sorry BCBV matrix multiplied by your BIBC matrix, DLF matrix. And from this we have seen that your delta V is actually equal to your DLF matrix multiplied by your nodal currents. So, this algorithm is more appropriate to use it for distribution system next time we will see how we can build this BIBC and BCBV matrices from your system data.

Thank you.