Electrical Distribution System Analysis
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Lecture – 17
Modeling of Three-Phase Transformers
Part IV

Students, we are studying Modeling of Three-Phase Distribution Transformers; it may be full three-phase transformer or three single-phase transformers are connected in different fashions. And what we did, we try to derive the abcd parameters of different connections of the three-phase distribution transformer. In that basically last two lectures, we have seen four different connections, and we have derived abcd parameters for those four different connections.

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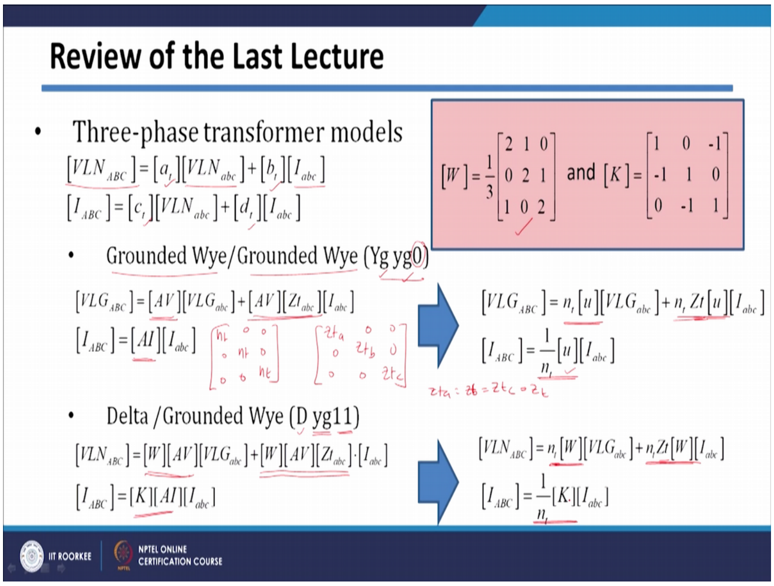

To derive the abcd parameter, we have seen that we need to get voltages in terms of line to neutral voltages, and currents in terms of line currents. So, and they those define your abcd parameters. And for various combination from last two lectures, we have derived various abcd parameters for different configurations.

First configurations, which we had considered was grounded wye by grounded wye connections. It is both side it is wye connection l v side as well as h v side, and both side the ground is there. And it is 0 connection, there is no phase shift between primary and secondary contribute, so vector group is Y g y g 0. And for that we have derived abcd parameters, and basically we have got this is your a parameter a multiplied by Z t, it is B parameter, and this is your D parameter, c parameter is actually 0 0 0 here.

And for that if your Z abc is basically matrix of Z t a Z t b and Z t c, which are basically impedances in all three phases. And if you consider them equal, you can get simplified form of these equations, where n t you can take it common out, because A V is also matrix of your turns ratio that is n t n t n t, which are basically diagonal elements. So, n t can be taken it out. So, in that case your A V will become n t multiplied by unity matrix, and A V multiplied by Z t abc, will become l t multiplied by Z t.

Here I am considering your Z t a is equal to Z t b is equal to Z t c. So, in that case, your Z t will come out, which is basically equal to Z t, and then unity matrix will remain here. Exactly same thing we can apply for A I, A I is actually matrix of 1 by n t in diagonal elements. So, in that case also 1 by n t will come out, and it will be just unity matrix. Exactly similar way, we have got abcd parameter for delta grounded wye connections. So, in this case, it is D yg is grounded wye and 11, so vector group is D yg 11.

And for that configuration, you have your a parameter, which is W multiplied by A V; where this W matrix is given by this one, which basically converts into your line to line voltages into line to neutral voltages. And then your b parameter will be W multiplied by A V multiplied by Z t abc. And your d parameter will be K multiplied by A I.

Again apply similar simplification by considering same tap position in all the three transformers, and in that case, n t will come out. Similarly, considering same leakage impedances in all the three transformer your Z t also will come out. And in that case, your simplified matrices, A matrix will be n t multiplied by W, B matrix will be n t multiplied by Z t multiply by your W matrix, and your d parameter will be K divided by n t.

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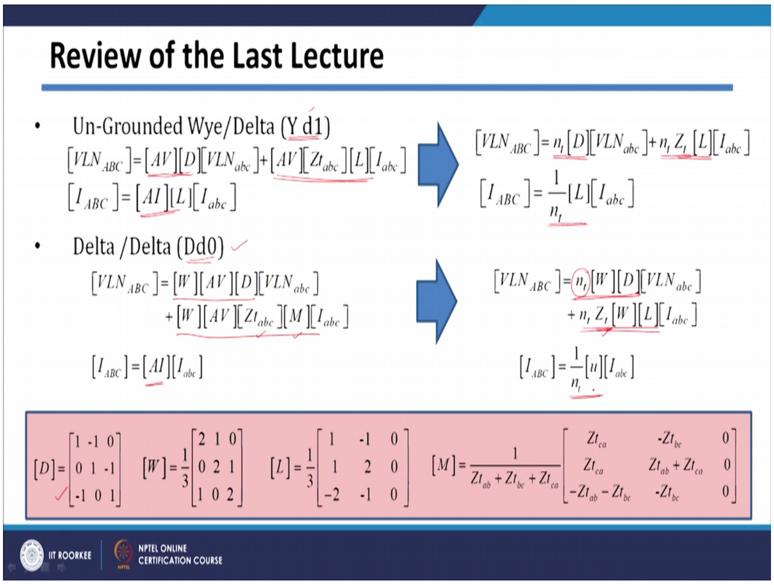

In the last lecture, we have derived for two different transformer, those are actually ungrounded wye by delta connection, so this particular vector group is Y d and 1. And for that connection you have got A parameter, which is A multiplied by D, where this D matrix is done by this one, which basically convert your line to neutral voltages to line to line voltages. And then your b parameter is, A V multiplied by Z t abc multiplied by L. And your D matrix is, A I multiply by L.

In that case also, applying simplification that is all the transformer on the same tap, if they are single-phase transformers. And self-impedances, if they are same, they can taken it out. So, your a parameter will be simplified to just n t multiplied by D. And your b parameter will be just simplify to n t multiplied by Z t multiplied by your L matrix, and then your d parameter will be L divided by n t.

And then lastly, we have seen delta delta configuration, and vector group is D d 0. And in that case, your a parameter will be W multiplied by A V multiplied by D, and your b parameter W multiply by A V multiplied by Z t abc multiplied by M. And your d parameter will be just A I, which is just diagonal entries 1 by n t 1 by n t and 1 by n t.

So, in this case also, considering all the transformers, if their single-phase on same tap position; And your Z self sorry leakage impedance of all the three transformer, if it is same, then it can be taken common out. And then this is your simplified a parameter, this is your simplified b parameter, and simplified d parameter, c parameter is always 0.

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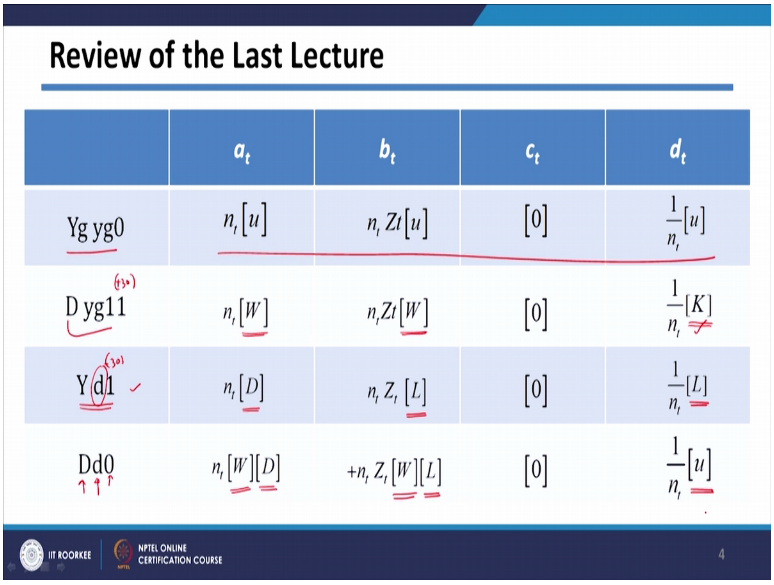

So, in last two lectures, you have seen four connections of the transformers. One is grounded wye by grounded wye connection, which is given by this expression Y g y g 0 vector group. Then we have seen delta grounded y connection, which is represented by D yg 11, which represent plus 30 degree phase shift. And then we have seen ungrounded wye delta connection, which is having minus 30 degree phase shift, so by so it is Y d 1. And then we have seen finally, delta delta connection, which is having phase shift of 0. And the abcd parameters, which we got for these transformer connections are summarized here.

So, for an grounded wye by grounded wye connection. These are your four parameters, ABCD. In case of delta grounded wye connection, we need one transformation from voltages, which are basically delta voltages to the line voltages. In that case, we need that W matrix, so that a conversion is represented by this W matrix here. Similarly, we need current conversion also. So, delta currents in primary will be connect converted into line currents, and those conversions will be represented by matrix K.

In wye delta connection, un grounded wye delta connection, which is 3. We again, we need on a left hand on low voltage side one conversion that is line to neutral voltages to line to line voltages, and that is done by your D matrix here. Similarly, we need current conversion, and that is represented by L matrix, where we need line currents, we need to be converted into delta phase currents.

Similarly, here we need L matrix. And in case of delta delta transformer, we need two conversions is first line to neutral voltages will be converted into line to line voltages. And then line to line voltages will be converted into again, line to neutral voltages. So, here we will get both the matrices that those are W and D, because conversion happening twice.

Current conversion is also required that is why, your you are having this L matrix here. And here, in case of a D parameter, you will be having 1 divided by n t multiplied by u, because when we are transforming current two times, you are you are you are actually cancelling your L matrix, with respect to your K matrix. So, in that case, it will be 1 divided by n t multiplied by your u.

Let us see one example. In this particular example, I am considering our general transformer, which is 3-phase 200 kVA, voltage rating is on primary side is say 11 kV, secondary side say 4 433 volts, frequency 50 hertz, connection on HV side say delta, connection on LV side star with neutral buttered, which can be grounded [vocalize

Depending upon the connections, vector group may change. So, in this case, we are considering Dyn I also remember that all the matrices, which we are derived that is D W A V, all the matrices maybe different for different vector group. So, we need to particular vector group, we need to derive this matrices. So, in this in this particular vector group, we are already derived, we can take them directly. Then percentage impedance is actually 4, and its angle is 87 degree

So, by knowing this data of general any general transformer, we can derive the abcd parameters for them, and you can use this a abcd parameter for any kinds of analysis. So, let us see how we can get abcd for parameters for, the general transformer, which is we may find in various our Indian distribution systems. So, we can easily find out turns ratio.

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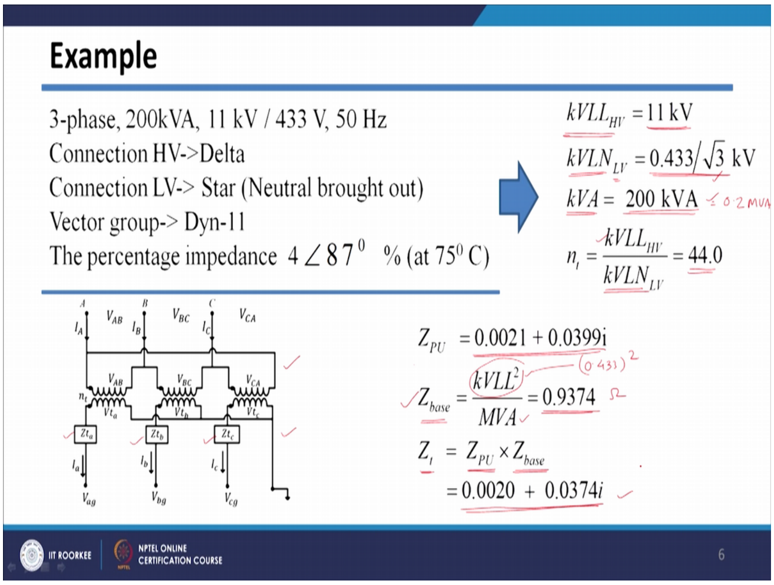

So, for turns ratio, we need to know, because on a primary side, it is delta. So, across the winding, there will be line to line voltage, and secondary side is star connected, so across the one winding, the voltage will be line to neutral voltage.

So, in this case, kV line to line voltage, we know that it is 11 kV. And kV line to neutral voltage on LV side, it is of 433 is actually line to line voltage. So, we need to convert into line to neutral voltage, so we can, we have to divide it by root 3. And the kVA, rating given is say 200 kVA. So, we can get the turns ratio from this, which comes around 44.0 something, so we can take that 44 here.

Then I have drawn this connections. So, primary delta current connecter, secondary star connecter, which is basically grounded, and then we need to calculate this Z t a, Z t b, and Z t c. We can assume them equal, because it is full three-phase transformer, I am considering, I am not considering three different transformer, which we are connected in delta star (Refer Time: 12:26).

So, it is full three-phase transformer. So, in that case, we can assume that the leakage impedances are same, and those can be calculated using the percentage impedance, which is given. So, percentage impedance is basically 4 angle 87 degree. And if you convert into complex number form, it will be this number that is 0.0021 plus 0.0399 i, it is complex number.

Z base impedance, we know that it is kV line to line square divided by MV rating of the transformer. So, from the kVA rating MVA rating will be actually 0.2 MVA. So, by doing that we can get your base impedance, which is 0.9374 ohm. And then actual impedance of the transformer will be nothing but your per unit impedance multiplied by your Z base, which will give actual impedance.

And we have assuming that in the all the three phases, this impedance is there. And this impedance, which have we have calculated on the secondary side. So, this is Z base, which you have calculated on secondary side, because I am taking here, kV line to line voltage, which is basically secondary side that is 0.433 its square So, it is on a secondary side. So, we have got actually impedances, which are applied on secondary side.

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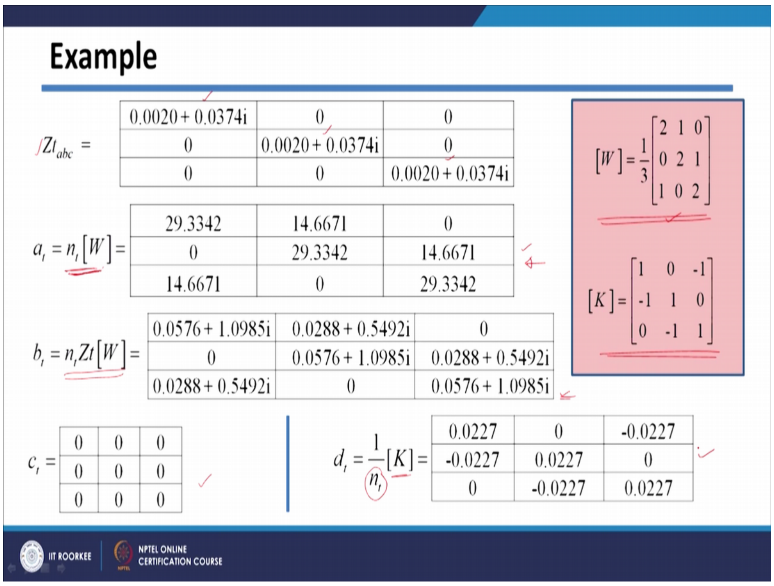

And then we need to get Z t abc, so Z t abc is nothing but these three terms, which are basically diagonal entries of Z t abc matrix, so this same impedances I am putting at those. And as we have derived, your a parameter for the vector group, which you have considered that is D yg 11, which is basically Dyn 11 here.

For that we have got at a parameter, which is n t multiplied by W. And we know W matrix, which is given by this, which you have derived, which basically converts your line to new line voltages, line to neutral voltages. And n t multiplied by W, n t we have got, which is 44 and W, which you have got this matrix here. So, 44 multiplied by W matrix, will give me 80 matrix. So, we have got a parameter matrix.

Then b parameter matrix, we have n t multiplied by Z t multiplied by W. So, basically this term, we need to multiplied by your Z t matrix. So, basically this is multiplication of this matrix multiply by your this matrix. So, we can do this, you will get matrix, which is which will basically this, so this is nothing but your b parameter here. And as I told you c parameter will be 0, because we are not considering any shunt branch here.

However, if you consider that magnetizing shunt branch, there will be some component in c matrix also. So, we in since, we have neglected your magnetizing branch, c matrix is always we are getting 0 for all configurations. And d parameter, we have seen this is K matrix divided by n t. And K matrix you have derived for this vector group, which is given by this. And if this is divided by 44 will give me this matrix, which is basically d t matrix.

So, you have got all the parameters that is abcd parameter, and those parameters we can use to get various other parameters. So, till now, we have seen the configurations, which are three-phase configurations. Now, sometimes we need to use what is called as open wye-open delta connection. It is basically valid if there are three single distribution transformer, or you can say if there is load, which is having some load with single-phase load, and some load is the three-phase load. In that case, to supplied at three-phase load, we need to have a three single-phase transformers.

However, that three-phase load can also be supplied by considering only two transformers, and that particular connection is called as a open wye-open delta connection. So, let us see how we can use only two transformer to supply both the types of load that is single phase load as well as three-phase load.

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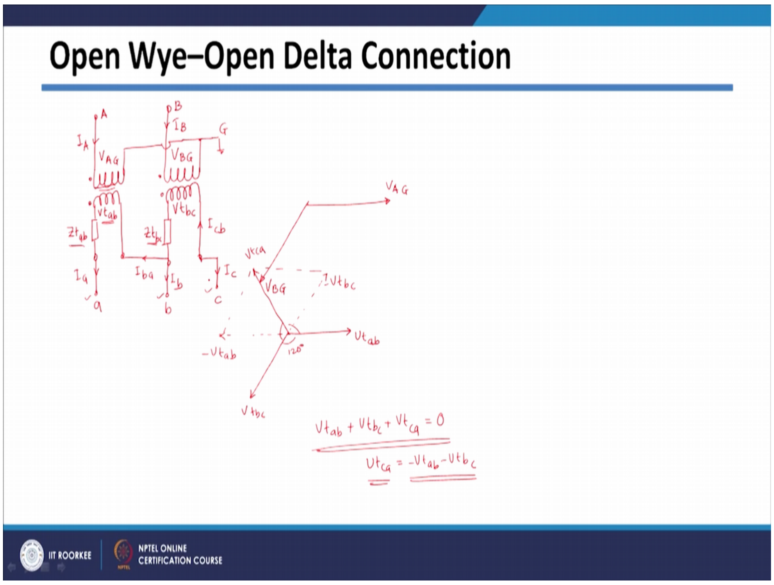

So, in this case we as I told you we are going to use only two transformers. So, let us say, these are the two transformers here. And then I am taking the impedances on secondary side, so these are the impedances.

Now, connection on primary side is wye connected. So, we need to take this two terminal out, and to make the common point for wye connection, and so two windings, two terminals will be shorted, and then it will be grounded. So, this will be basically your ground connection here, this will be your a terminal, this will be your b terminal.

And on secondary side, we need to connect them as a delta, so we can connect them here. And third transformer is not there, so it will be open delta here. And this will be taken as your c terminal, the current which is flowing through this is I am calling I c. This will be taken as your small a terminal, current is I a. And this terminal is taken as your b terminal, and this current is I am calling, it is small I b. This current will be current I b a. And this current will be current I c b. And I c b, and I c, they will be opposite in direction.

In this case, case this will be your current I capital A. And this will be I capital B. This impedance Z t a b, because we read between terminals a and b. And it is Z t b c, because it is terminal between b and c. This voltage across winding, it is V t a b, this is V t b c. In this case, if you observe, this voltage across, this it will be V capital A G, and this will be capital V B G. Now, if you can see, how it is converting your two single single-phase voltages into required three-phase voltages.

So, in this case, if you see, suppose your vector V A G, it is basically in this direction. Then your V B G voltage will be 120 degree shifted with respect to this, and this will be your V A G and V B G. Then we know that voltage V t a b, will be just in phase with V a b, if you consider dots here. So, V t a b will be in phase with V A G. So, your V t a b will come somewhere here.

In phase with V A G, and V t b c will be in phase with V B G, so this will be your V t b c. Your V t a b plus V t b c plus V t c a, they will be equal to 0. So, in that case, V t c a will be equal to minus V t a b minus V t b c. So, your minus V t will be will come somewhere here, so this will be minus V t a b. And minus V t a b will come somewhere here, so this will be your minus V t b c.

And if you add these two vectors, we will get third vector, because V t c a is a addition of these two minus quantities. So, V t c a will come somewhere here, and if you calculate the angle between them, they will be 120 degree shifted. This I am considering during no load condition, this condition will be valid during no load condition. So, during no load condition, at the terminal we are getting completely three voltages, which are basically 120 degree phase shifted.

So, required three-phase voltages, we are getting from only two transformer that is why, sometimes instead of using three transformer, we can say one transformer cause by putting only two transformer, which can supplied both the types of load, those are single-phase loads, as well as three-phase loads. So, that is why your open wye, and open delta connection sometimes become important.

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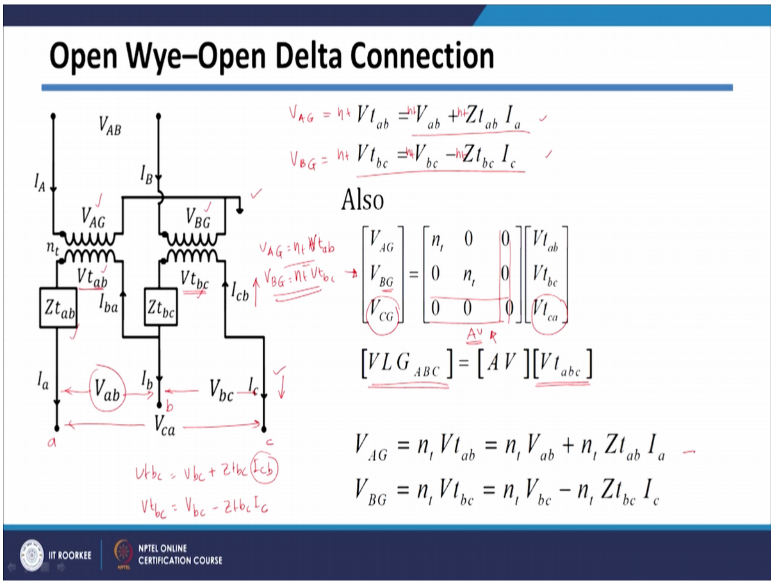

Let us see, how can we derive, the abcd parameters for this particular matrix here, this particular transformer here. So, in this case, again I am taking that transformer on primary side, which is basically wye connected open wye, because third transformer will not there. In this case, secondary is delta connected open delta, because third transformer will not there, which is basically makes you delta closed.

Now, if you write the equation for this voltage, this is basically V t a b, will be nothing but voltage V a b, which is basically between a and b terminal, V b c is between v and b terminal, and V c a is between c terminal. So, this is basically small a terminal, small b, and small c. So, V t a b will be nothing but this voltage plus voltage drop, which is happening across this. So, basically it will be given by this equation, where the current is I a.

In this case, this voltage drop V t b c, will be nothing but, so V t b c will be nothing but V b c plus Z t b c multiplied by I c b. However, I c b is nothing but minus of I c, because direction of I c b is like this minus direction of I c is like this. So, in that case, it will be V t b c will be equal to V b c minus Z t b c into I c, so basically you will get this equation here. Now, we can put this two equations into matrix forms, before putting matrix form, let us derive the this turns ratio condition also. As I told you this V A G voltage, and V t a b, and V B G, and V t b c, they are just related by turns ratio.

So, V A G will be nothing but V A G will be equal to your n t multiplied by V V t a b, and V B G will be equal to n t multiplied by V t b c. And if you put these two equations into matrix form, basically I will get this equation. There is no V C G voltage, because third transformer is not there, so we can put 0 0 0 as a column, as well as row. And V t c is also not there, so that is why, just we have augmented this matrix by putting 0 0 0.

So, in that case, your AV matrix will be basically this just modified AV matrix. And this will be nothing but your V L G A B C, because they are line to ground voltages, and on this side, this is V t a b c.

And now, these two equations I can what I can do, I can just multiply this equation, everywhere by n t, both side n t. And this we know that n t multiplied by this will be V A G and this is V B G. So, basically this two equations after multiplying n t, I am getting two equations here, which then I can put into matrix form.

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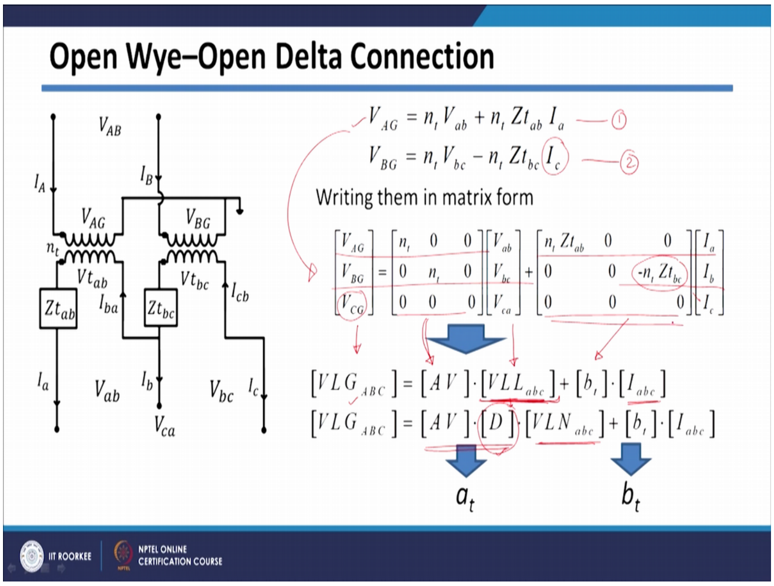

So, these two equations, 1 and 2, which are taken from last slide. And if you put them into matrix form, so 1st equation if you put into matrix form, it is basically this equation and 2nd equation, which is basically this equation. And in this case, there is no term, which related to C G, so that is why, this row I make in 0. But, however there is entry related I c, so this term will get multiplied by I c. So, if you put these two equation into matrix form, I will get this equation here.

And in condensed form I can write these equation. So, these are nothing but line to ground voltages of all the three phases. This I am calling matrix A V, these are basically line to line voltages. On secondary side that is why, V L L small a b c, and this is nothing but your b matrix, and this is I a b c. Only here, this is ok, because V L G A B C, we are getting a required form, and that is line to neutral.

However, these are voltages line to line voltages, which need to be converted into line to neutral voltages. So, this can be converted by writing, this term as I writing D multiplied by V L N, because here, we need to convert a line that we would not need to get line to line voltages from the line to neutral voltages. In that case, your conversion matrix is D here.

So, thus your a t matrix, will be A V multiplied by D and in that case, A V is little bit modified as compared to earlier AV matrix. D matrix is same, which we have seen earlier. And your b matrix will be basically this matrix for open delta open wye-open delta configuration.

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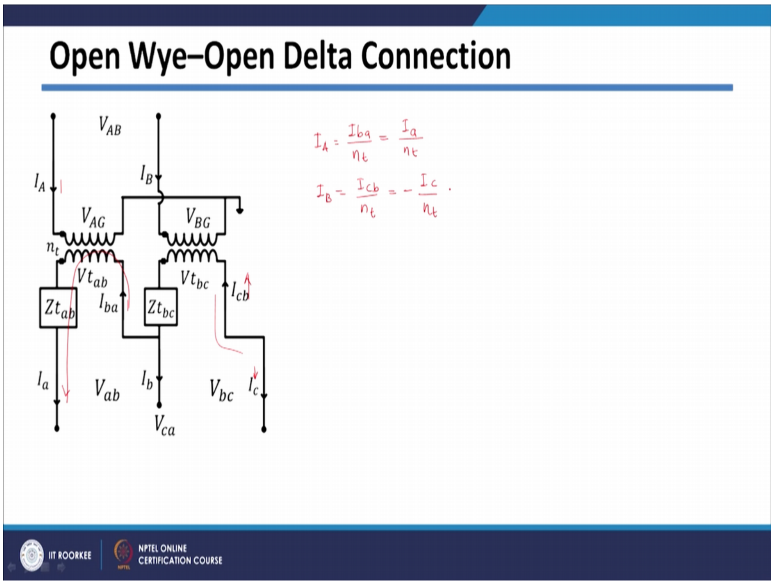

Now, let us try to derive for c and d. So, as you have seen to get c and d parameter, we need to get the relation between currents; so, in this case, if you this current I A, and current I b a, they just related by your turns ratio.

So, your I A will be equal to I b a divided by your n t. And your I B will be equal to I c b divided by n t. However, this I b a is same current, which is coming and going here. So, I b a, and I A, they are having same direction, as well as same magnitude. So, I b a can replaced by I a, so it will be I a divided by n t. And I c b is having opposite direction as compared to your I c, I c in the direction I c b in this direction. So, I can say, (Refer Time: 28:42) it will be minus I c divided by n t.

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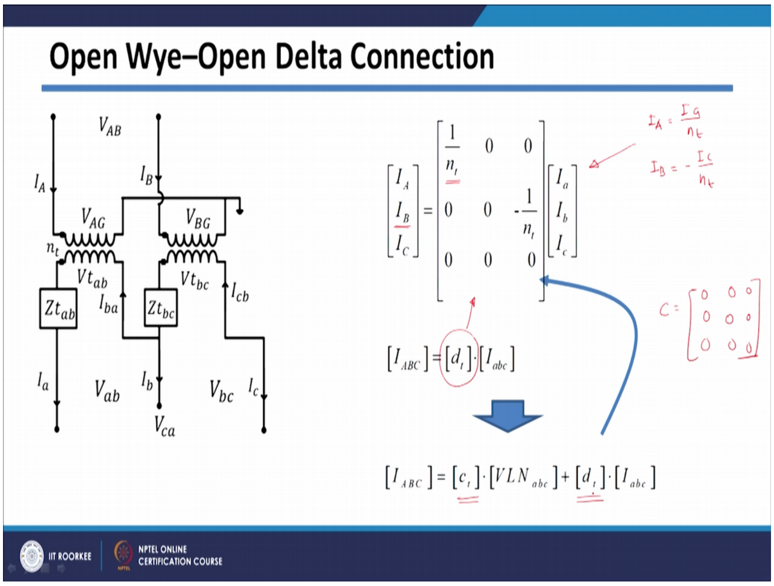

So, I can write this into matrix form, so which we have got actually I A, I A will be equal to I a divided by n t, and we have got I B will be equal to minus I c divided by n t. So, these two equation, which we have got on last slide, I put into matrix form.

And in this case, this will become your d matrix. And c matrix, will be 0 0 0, because again in this case, we are not having any shunt branch. So, in that case, your c matrix will be 0, and d matrix will be this one. So, we have got c matrix, and d matrix also.

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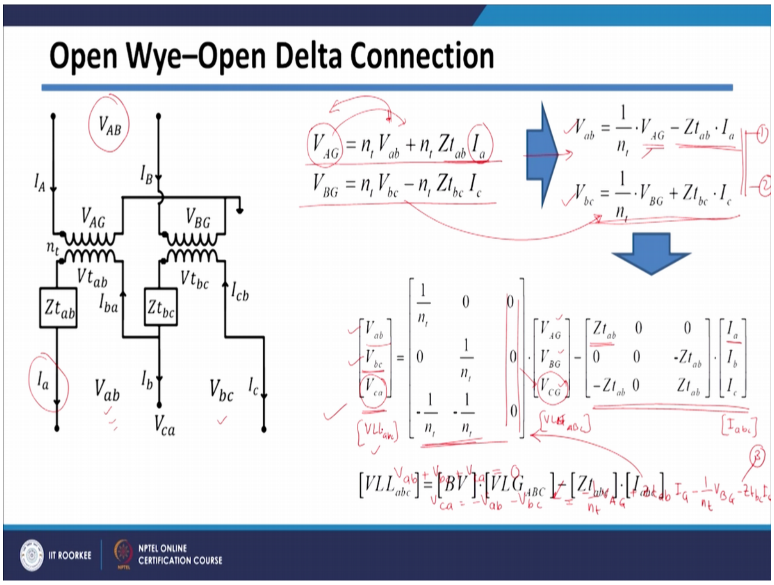

Let us see how we can get capital A and capital B matrix. So, as I told you for getting capital A t, and capital B t matrix. We need to write equations of V secondary voltages in terms of primary voltages, and secondary currents. So, basically this equation, we need to convert into secondary voltages, basically this voltages I need to write in terms of primary voltage, and I a. So, we can just rewrite them by taking V a b on this side, and V A G on this side. So, basically if you rewrite them it will be V a b will be equal to V A G divided by n t, and minus this term that is Z t a b dot I.

Again if you write this equation, I will get this equation here. So, it will be we can write them into matrix format. So, in this case also, on this side all the three voltages, I have written. However, we are not having equation for this V c a equation. So, and on this side, we are having V A G, V B G, and V C G, and there is no term into this equation related to V C G. So, that is why, all the three entries, which are getting multiplied to V C G, they are 0. And then V a b is equal to V A G divided by n t, which term here and then Z t a minus Z t a b into I a.

And from 2nd equation, we are getting this term here. And 3rd equation, we are getting from adding all the three voltages, on the secondary side. So, basically you are getting this by adding the all the three equation. This is basically we know that V a b plus V b c plus V c a should be equal to 0. And in that case, your V c a will be equal to minus V a b minus V b c. So, V a b is this one, V b c is this one, I can put into this equation, and from that I will get this 3rd equation, which I am putting it here.

So, it will be basically if I add this two quantities, it will be minus 1 divided by n t into V A G plus Z t a b into I a minus 1 divided by n t into V B G minus Z t b c into I c. So, basically this 3rd equation, which I am writing it here, is basically coming from this term. So, this 2, 1, 2, and this 3rd equation, if I put into matrix form, I will get this one.

Now, if you observe this equation here, we are getting line to line voltages. So, I can write this is V L L a b c, this is ok. So, the here we are getting V L N or V L G capital A B C, so this is ok. And here, we are getting line to current, so small a b c ok. So, here we need to convert this line to line into line to neutral voltages.

So, basically here, so this will be actually V L L a b c. This I am calling BV matrix, because it is little bit different than our earlier AR matrix AV matrix. And here, this is V L G capital A B C, because line to ground voltages, and these are line currents. So, here we need to convert this line to line voltages into line to neutral voltages.

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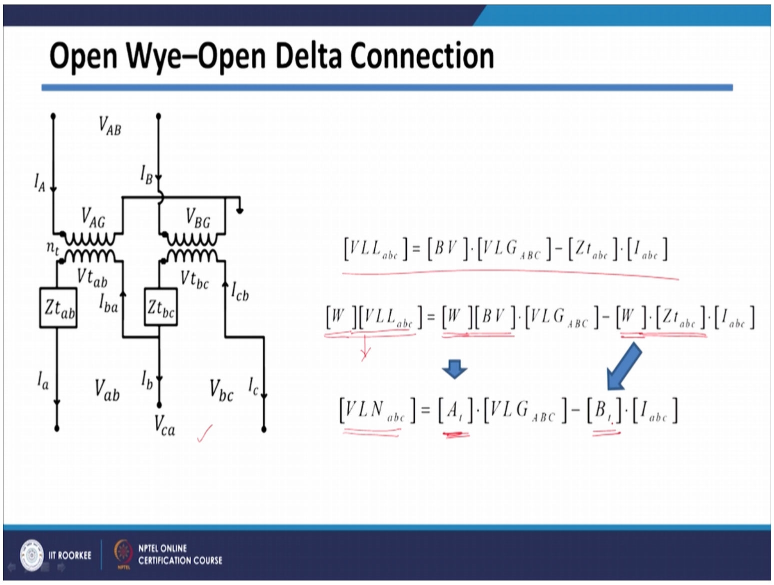

So, we know that the multiplication matrix, which will be basically W. So, every term into this equation need to be multiplied by W, to get line to neutral voltages on left hand side. So, this W multiplied by V L L will become line to neutral voltages. On secondary side, basically this side, this W multiplied by BV, will become your capital A t matrix, and W multiplied by your Z t a b c will become capital B t matrix. And BV matrix basically we have seen on the earlier slide. So, this is your actually BV matrix, which need to be used here.

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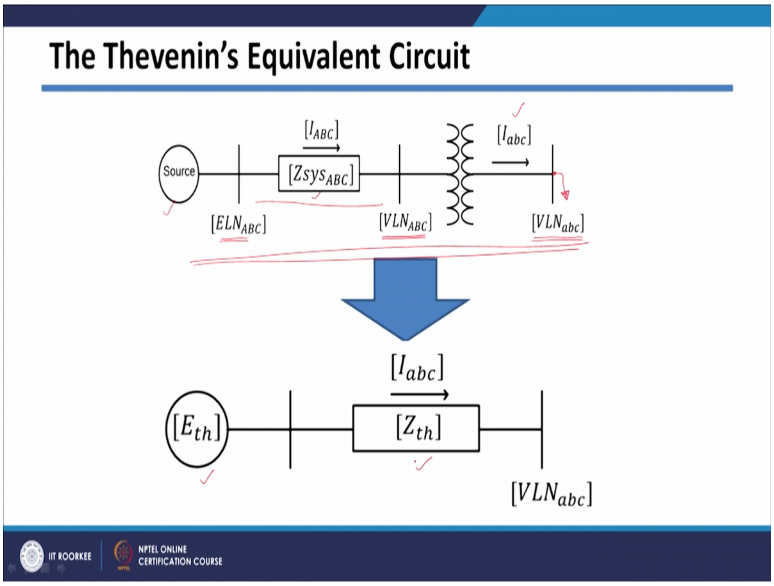

Now, see many times in basically short circuit calculation. We need to know, Thevenin's equivalent circuit at that particular short circuit point. Say in this case, what I shown is there is source here, then this bus voltage is V L N A B C, I am E L N A B C I am calling. And then say there is a transmission line, which is having impedance matrix also a Z system A B C. So, 3 by 3 matrix of your transmission line

Primary transformer voltage V L N A B C and secondary side, it is V L N small a b c. And secondary currents side current is I a b c. Let us say fault has occurred at this point, and we need to get the Thevenin's equivalence circuit till this point. We want to convert this whole circuit into just Thevenin's equivalent source, and Thevenin's equivalent impedance.

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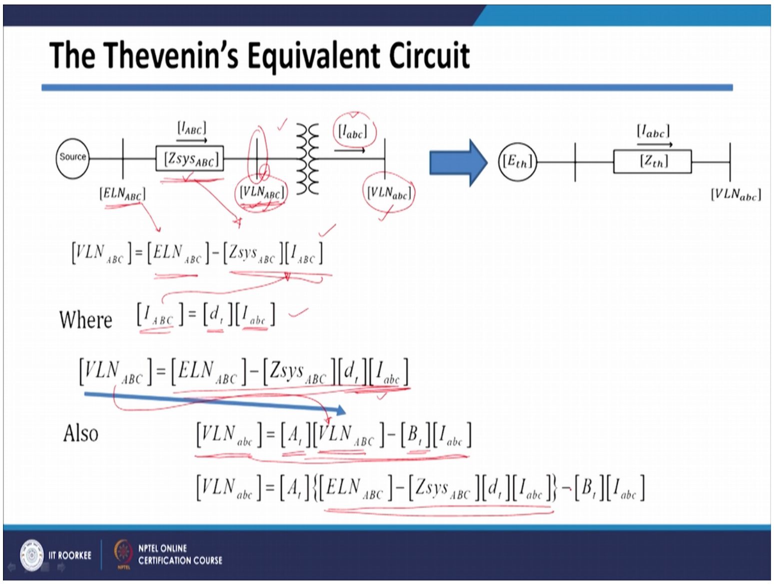

So, let us see how we can get this impedance here. So, if you see, the voltages V L N a b c, this voltages will be nothing but this voltages here, minus your impedance drop, which is happening across this transmission line. So, that can be written by this equation. So, this voltages will be equal to this voltages here, minus voltage drop, which is happening across this line, so this is nothing but your voltage drop across this. So, V L N a b c, we got voltages at this point.

We also know that the currents on a primary side of the transformer, and currents on a secondary side transformer, they are related by just d t matrix, because c t matrix, they it is always 0. So, I a b c currents on a primary side will be nothing but your d t matrix multiplied by current on secondary side. So, you always you already seen many times this one.

So, what we can do actually, this I can put into this expression. So, here, it will get multiplied by d t multiplied by I a b c instead of I capital A B C, it will be d t multiplied by I small a b c. Then the voltage relations, we know that secondary side voltages of the transformer that is V L N a b c can be written into primary voltages, and your secondary current.

And we were seen that for that particular case, we need to use capital A t, and capital B t matrices. So, this voltage V L N a b c, it will be equal to a t matrix multiplied by this voltage, minus d t multiplied by your line currents. So, basically you can get this equation here. And in this equation, whatever V L N, which we have obtained earlier, I can put it here. So, instead of V L N a b capital A B C, I can put this term here, which is basically this term here.

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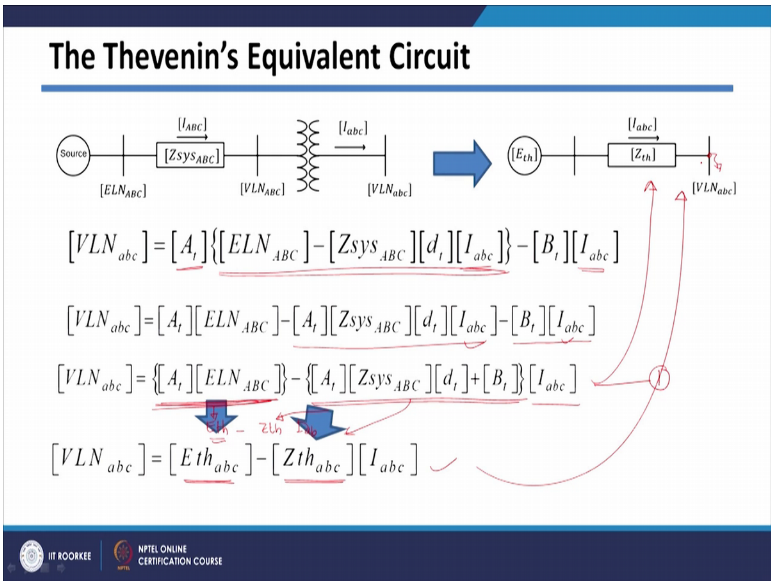

And then you just simplify it. So, that the terms relative current I a b c, I can take it together. So, I can just expand it by multiplying this A t matrix inside, this bracket here. And from that this is term, which is related to I a b c, and this is I a b c (Refer Time: 38:26) I can combine it together. So, basically I am getting this term here. So, I a b c is taken common out of from this two term. So, in bracket we get this term that is and remaining term is this one.

So, by seeing this expression here, say equation 1, we can easily get Thevenin's equivalent circuit. So, basically if you observe this, this equation will be nothing but something like this, where it will be E t h, so this is nothing but your E t h minus this will nothing but your Z t h into I a b c. So, if basically this will be nothing but your Z t h, which is this bracketed term. And E t h, which is Thevenin's equivalent voltage source, which is given by A t multiplied by E L N A B C.

So, as I told you this will become your Thevenin's equivalent voltage source, and this will become your Thevenin's equivalent impedance. So, in the summary, we have finished your transformer modeling part. And in this particular lecture, we have seen that summary of three-phase transformers how they are modeled. So, we have summarized all the four connections or four configurations, which you have considered, we have summarized them in one of the table. Then we have seen one example, where normally use transformer from Indian distribution system, I have taken it here.

And we have derived abcd parameter for those transformer. And then we have seen finally, we have seen what it is called as open wye and open delta connections. And I have told you the applications of this particular connection, where we say one single-phase transformer. And we have derived again abcd parameter for these configurations.

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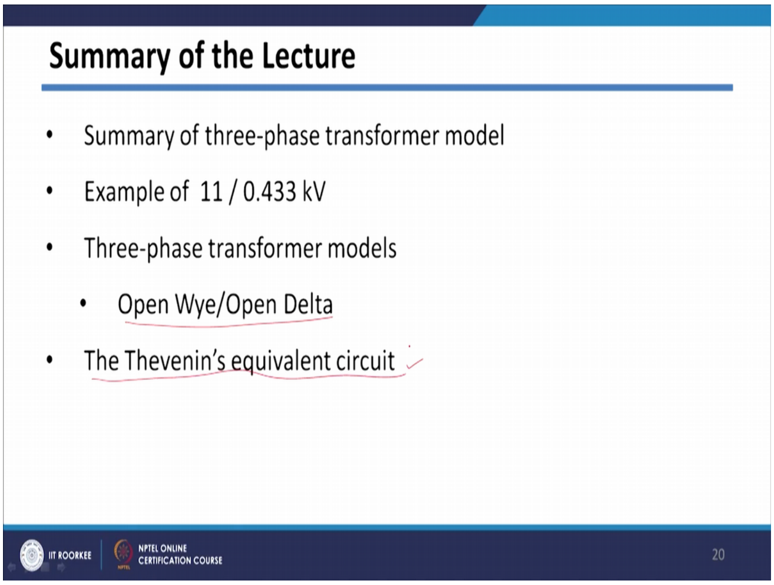

And finally, we have seen that how we can get Thevenin's equivalent circuit, if they are if there are transformers in into your circuit. And if you want to get your Thevenin's equivalent circuit till your point of interest, or where you want to calculate a short circuit current, you can get Thevenin's equivalent circuit.

So, in this particular case, I have considered only one transformers. So, even if there are many transformer into the your network, as well as many transmission line. We can easily find Thevenin's equivalent circuit by using the methodology which explained, which is explained in this particular lecture. So, here we finish our transformer modeling part, and next time we will see how we can model the voltage regulators.

Thank you.