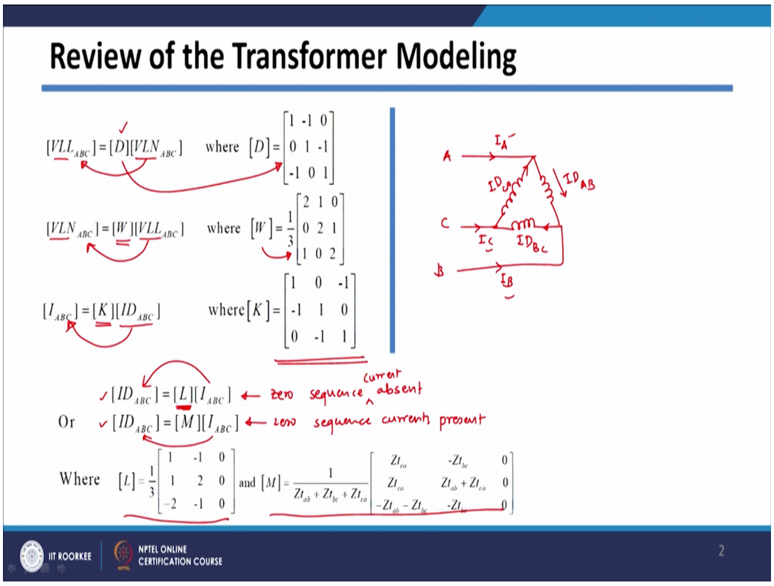
Electrical Distribution System Analysis
Dr. Ganesh Kumbhar
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 16
Modeling of Three-Phase Transformers.
Part III

Dear students, we were discussing about Modeling of Three-Phase Transformers. In the last class we have seen two connections those are grounded wye, grounded wye connection and delta grounded wye connection. In this class we will see few more connections, before going to those connections let us see review of some transformer modeling matrices.

(Refer Slide Time: 00:53)



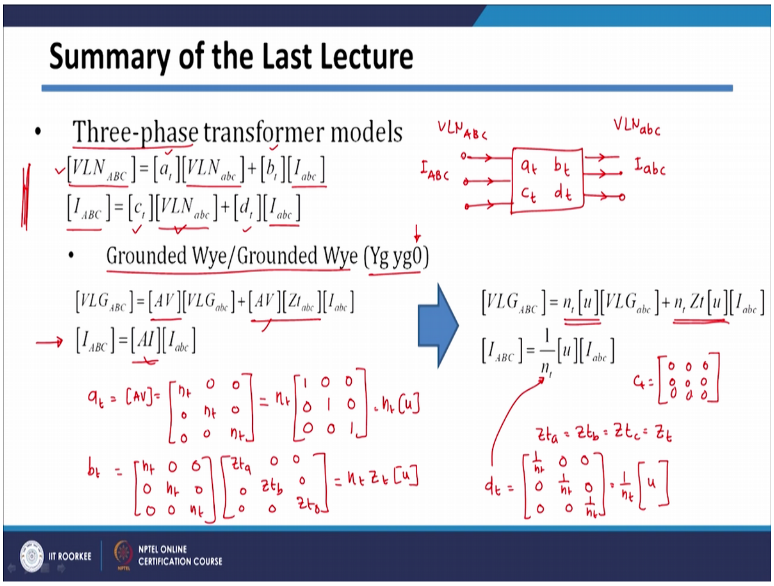
So, we have seen for this case, if you want to convert your line to neutral voltages to line to line voltages your conversion matrix is D. So, line to neutral voltages to the line to line voltages they can be converted using matrix D and this matrix D is given by this matrix here. And then to convert line to line voltages to line to neutral voltages you have this conversion matrix W and this W is given here, if you want to convert your delta phase currents to the line currents. So, these are the delta phase currents and if you want to convert into line currents the conversion matrix is K and the K is given by this matrix here.

In case if you want to converts line currents to the delta currents your conversion matrix is L and L is given here. And if we want to convert again this line currents with delta currents your conversion matrix is M here. However, the difference between these two cases are like this, if your zero sequence currents absent in your configuration then you need to use this L matrix here. And if zero sequence currents present so, if the zero sequence current present then you need to use this matrix here.

So, if you see the difference between ID ABC and I ABC this is like this. So, these are say three phase windings and these are your lines. So, this is say A phase B phase and C phase. So, this current will be I A, this is I B and this is I C, this current will be ID AB, this will be ID BC and this will be ID CA, so which will basically flowing like this.

So, these ID AB ID BC and ID CA these are nothing but ID ABC currents and this I A I B and I C they will be nothing but I ABC currents.

(Refer Slide Time: 03:45)



Then we have seen this grounded wye by grounded wye connection in the last class. Before that we have seen modeling of three phase transformers and we have seen that three phase transformers can be modelled using a b c d parameters, to are those are represented as a t b t c t d t. These are your secondary terminals of the transformer, these are your primary terminals of the transformer. Here the voltages are V capital ABC.

So, here line to neutral voltages I can represent. So, here the voltages will be VLN ABC capital letter for primary and secondary side they will be VLN small abc. Currents on primary side I ABC capital letters and on secondary side it will be I abc small letter. So, this is your I a I b I c here also this is I a I b and I c, and then to represent this in a b c d parameter terms.

So, primary side voltage is written in terms of secondary voltage and secondary currents then you get this a and b parameters of transformer. And when we write your primary side currents in terms of secondary side voltages and secondary side currents we get this c and d parameters. And we have derived this a b c d parameters for grounded wye by grounded wye connection in the last class and the vector group of this transformer is Yg yg0. This 0 represent phase replacement between primary and secondary voltages and we have got this expression here.

So, if we compare these equations with your standards equation of a b c d parameter this is your a parameter, this is your b parameter and this is your d parameter because there is no term related to here VLN abc into this expression here; that is why your c is 0. So, we can easily see your a t parameter will be nothing but AV and this AV matrix we have seen that it is n t n t n t diagonal entries and non-diagonal entries are 0.

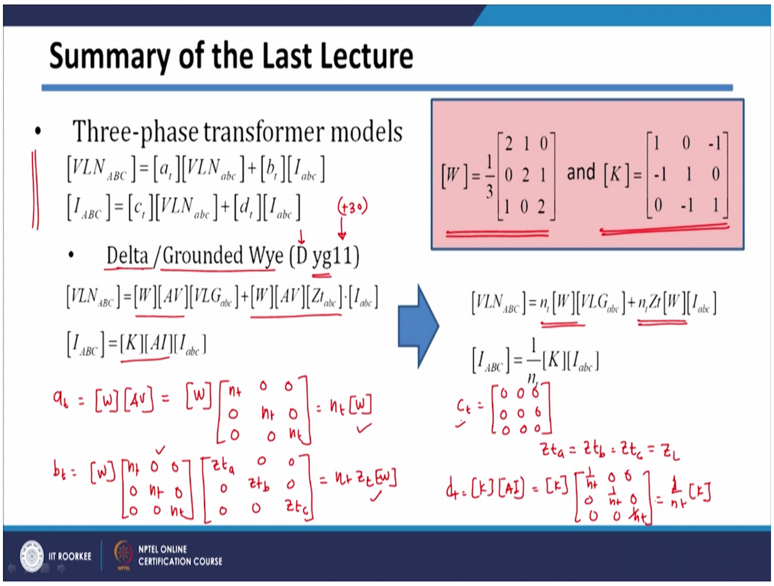
So, we can easily write it like this it will n t into your unit matrix because n t I can take it common so, it will be unit matrix like this. So, we can say it will be n t multiplied by u. Similarly, b t is having your AV matrix which is again n t 0 0 0 n t 0 0 0 n t which is AV matrix here. And then Zt abc matrix we have seen that this Zt a Zt b and Zt c diagonal entries and off-diagonal entries 0 here also.

Now, if I consider Zt a is equal to Zt b is equal to Zt c is equal to Zt. So, in that case I can take this Zt common out from this one so, it will we can write it like this n t multiplied by Zt into your unity matrix.

So, in this case you can see that a parameter can be converted into n t multiplied by u, b parameter can be converted n t multiplied by Zt multiplied by u; where this Zt a Zt b Zt c are the leakage impedances of the transformer. They are generally same in case of three phase transformer, they may be different in case of three different transformers if you are connecting, if you are connecting them in grounded wye grounded wye fashion, and then this AI matrix.

So, d t matrix we have seen it is AI and AI nothing but 1 by n t 0 0 0 1 by n t 0 0 0 1 by n t. So, in this case also we can take n t common out, so 1 by n t and your unit matrix. So, d t parameter can be depended by like this and your c parameter. So, c t parameter will be 0 0 0 3 by 3 in size. So, this was the derivation for grounded wye or grounded wye connection.

(Refer Slide Time: 08:54)



In case of delta grounded wye we had got this expressions and if you cons compare them with standard equations of transformer modeling; we can see that this is your a parameter, this is your b parameter and this is your d parameter. So your, a t parameter will be nothing but your W matrix multiplied by AV and we have seen that this W matrix is given by this matrix here.

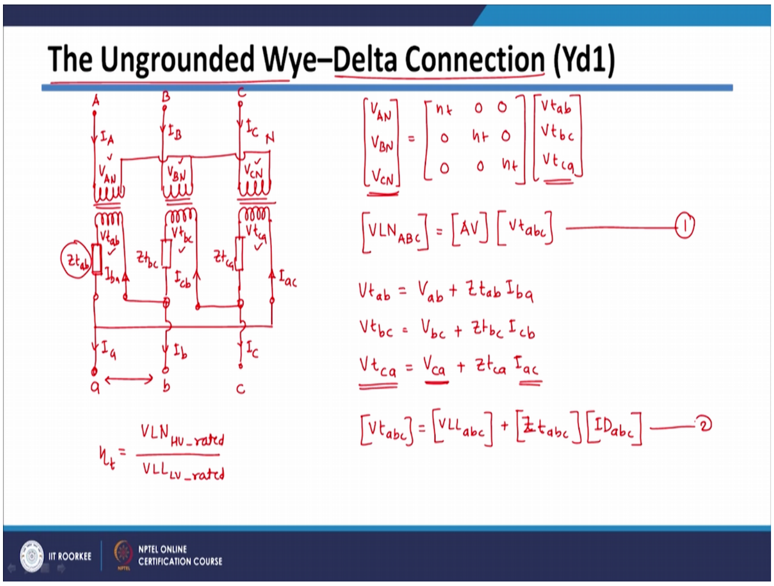
So, in this case your W I can keep as it is and AV matrix is nothing but n t diagonal entries. So, in that case n t can be taken out common and then it will be community matrix so, this will be just n t multiplied by W. And in case of b t it will be W multiplied by AV is again diagonal entries n t with off diagonal 0 entries. And your leakage impedance matrix is Zt a Zt b and Zt c diagonal entries with off diagonal entries are 0.

And so, in this case also if again Zt a equal to Zt b equal to Zt c we can have this Zt common out. So, in this matrix will become equal to n t I can take it common out from this one, Zt I can take common out from this one multiplied by W.

And then your c t matrix will be K multiplied by AI, where this K matrix is given by this term here. So, it will be K multiplied by AI, it is 1 upon n t 0 0 0 1 upon n t 0 0 0 1 upon n t. So, it will be equal to 1 by n t into your K matrix, sorry this is your d matrix and your c matrix will be equal to 0 0 0 0 0 0 0 0 0; 3 by 3 size.

So, for delta grounded wye connection that is D yg because y is grounded and it is 11, means its consisting of 30 degree phase shift between primary and secondary quantities. For those you can see that your, a parameter is given by n t multiplied by W here, your b parameter is given by n t multiplied by Zt multiplied by W here. And your d parameter is given by K divided by n t and c parameter is 0. So, these two connections we had seen in the last class.

(Refer Slide Time: 12:18)



Now, go for third connection which is ungrounded wye and delta connections. So, he in this case primary is ungrounded wye and secondary is delta connected. So, in this case let us see these are your three transformers and this is your secondary side; all the impedances we are referring to the secondary side. So, this is your impedance in one phase, impedance in other phase, impedance in third phase and this we are connecting in delta fashion. So, we can just connect it like this; will this and this winding I can connected to the first winding.

So, this will give us your delta connection and if you see the primary winding; so this is one terminal here, second terminal, third terminal and since we are connecting it in wye fashion we need to take this neutral connection.

However, in this case it is ungrounded wye means this neutral will be not be grounded. So, this is your A terminal, B terminal, C terminal HV side. So, this should be I A current I B, but it is capital letter and I C. This terminal ill will be small a, this terminal will be small b and this terminal will be small c. So, this current is I a I b and I c, but represented by small letters here. This impedance is Zt ab because it is between a and b phase, this is Zt bc and this is Zt ca.

This terminal voltage or voltage across this winding Vt ab, this is Vt bc and this is Vt ca. This voltage on primary side will be V AN V BN and V CN, the currents are like this. So, this current I can say I ba because it is flowing from b and a terminal, this will be I cb and this will be I ac.

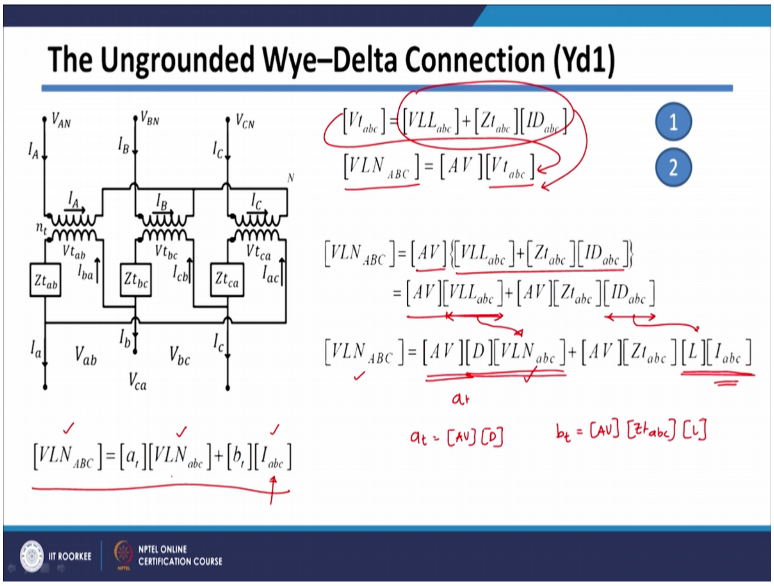
Now, if you write the expression for voltages which are basically these winding voltages. So, I can just say your voltage V AN V BN and V CN they will be just related by your turns ratio. And this turn ratio can be calculated in this case it will be n t turns ratio, will be equal to V LN because primary is wye connected HV related voltage divided by; since secondary is delta connected you need to take line to line voltages which are across the winding. So, LV and you take you need to take related voltage of LV side line to line voltage.

So, this will be equal to n t 0 0 0 n t 0 0 0 n t and this will be Vt ab Vt bc and Vt ca. Means these terminal voltages here this voltage, this voltage and this voltage will be related to V AN V BN V CN by just turns ratio. So, we can write this in short form. So, this voltages will be V line to neutral because this voltage your line to neutral voltages ABC capital letters; will be equal to this matrix we know that it is AV matrix multiplied by this is Vt abc matrix.

Let us say this is expression number 1, another equation I am getting from terminal voltages of the transformer. So, I can say terminal voltage V ab or we can write in opposite way. So, I can say so, I can say Vt ab will be equal to V ab plus Zt ab into I ba. So, basically this current here, this current multiplied by this impedance will give me impedance drop and plus this voltage Vt ab. So, V ab so, voltage here plus this impedance drop will give me Vt ab.

Similarly, I can say Vt bc will be equal to V bc plus Zt bc into I cb and third equation Vt ca will be equal to V ca plus Zt ca into I ac. Now, if you write them in matrix format so, these voltages are nothing but Vt abc. So, these are a Vt abc which will be equal to these are nothing but line to line voltages on secondary side. So, these VLL small abc plus this matrix of impedance says Zt abc matrix multiplied by these currents. If you observe they are ID abc because these are nothing but your delta currents.

(Refer Slide Time: 18:48)



So, let us say this is expression number 2. So, from 1 and 2 so, you have got this expression number ok. So, I am saying this is your 2 and this is expression number 1. So, this two expressions can be written like this like this and if you put this expression into this I will get this expression here.

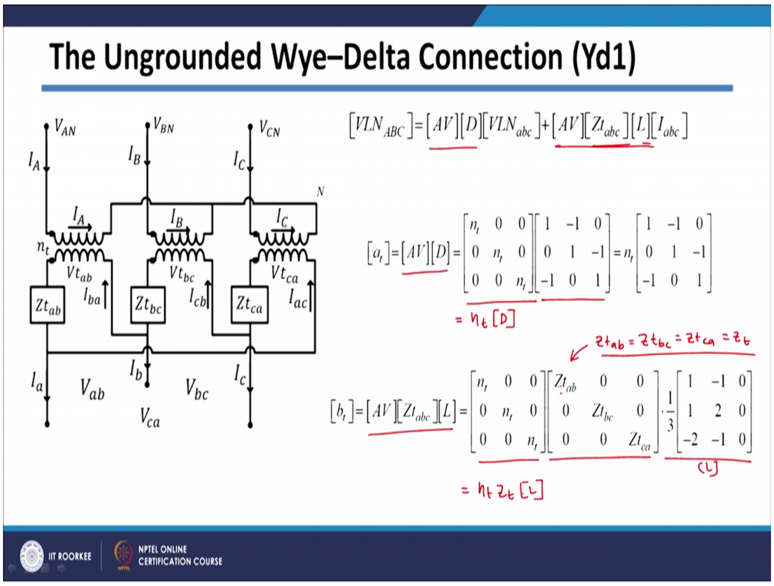
So, I can say VLN abc will be equal to AV multiplied by Vt abc is given by this term here, which I am putting it here. So, basically this is your this term. So, if you multiply this AV inside bracket. So, it will be AV multiplied by VLL abc and then AV multiplied by Zt abc multiplied by ID abc.

Now, if you see the standard form of equation so, standard form of a and b parameter is like this. So, here we need line to neutral voltages, here also we need line to neutral voltages and here we need line currents. So, if you observe this form here this is line to line voltages which need to be converted into line to neutral. And this part is delta phase current which need to be converted into line currents.

So, we have seen the conversion matrices. So, this line these line to line voltages can be written into line to neutral voltages by multiplying it with D. So, V line to line will be equal to D multiplied by V line to neutral abc and this ID abc can be written as L multiplied by I abc. So, ID abc can be replaced by L multiplied by I abc.

So, now we have got in standard format. So, here also we are getting line to neutral voltages on both sides and then here we are getting line currents. So, this will be your a t parameter. So your, a t parameter will be AV multiplied by D matrix and your b parameter will be AV multiplied by Zt abc multiplied by L.

(Refer Slide Time: 21:12)

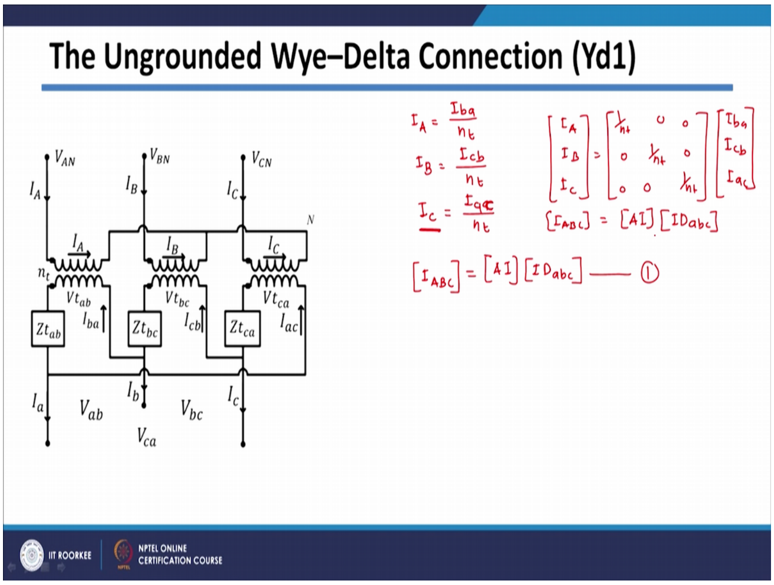


So, let us simplify it. So, we have got this a and this is your b. So, a is AV multiplied by D, AV we have seen it is diagonal entries n t and D matrix is given by this expression. So, in short form I can write, from this I can take n t common out. So, it will just remaining unit matrix, unit matrix multiplied by some matrix will be it will be same.

So, it will be just n t multiplied by D here and your b t parameter we have seen it is AV multiplied by Zt abc multiplied by L which is here. AV is again diagonal entries with n t, Zt bc is again diagonal entries of all this leakage impedances of three phases and this is your L matrix.

So, in this case also if your Zt ab is equal to Zt bc is equal to Zt ca is equal to Zt, I can take it common out. So, this I can write I can from this I can take n t common out. So, it will remain unit matrix from this if this condition getting satisfied I take Zt common out. So, this is also remain unit matrix and this is your matrix L. So, n t multiplied by Zt multiplied by your L matrix is nothing but your b parameter.

(Refer Slide Time: 22:43)

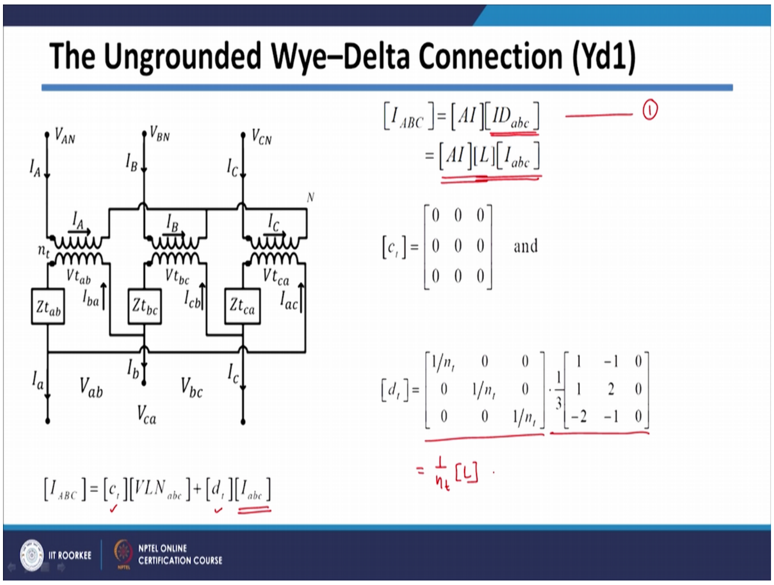


Now, to get c and d parameters we have seen that we need to take current creations into account. So, if you observe I can easily write your I A current will be nothing but your I ba current divided by n t. Your I B current will be nothing but I cb divided by n t. And your current I C is nothing but I ca divided by or I ac divided by n t.

And in short form I can just write it like this I ABC these are nothing but line currents. So, I can just say I ABC will be nothing but your AI matrix because this I can just like this. It will be I A I B and I C will be nothing but 1 by n t 1 by n t 1 by n t diagonal entries and 0 0 0 off-diagonal multiplied by I ba I cb I ac ok.

So, this is your I ABC in short form this matrix we know that we call it as AI and this is ID abc; because they has these are delta phase currents. So, this is nothing but your ID abc.

(Refer Slide Time: 24:31)

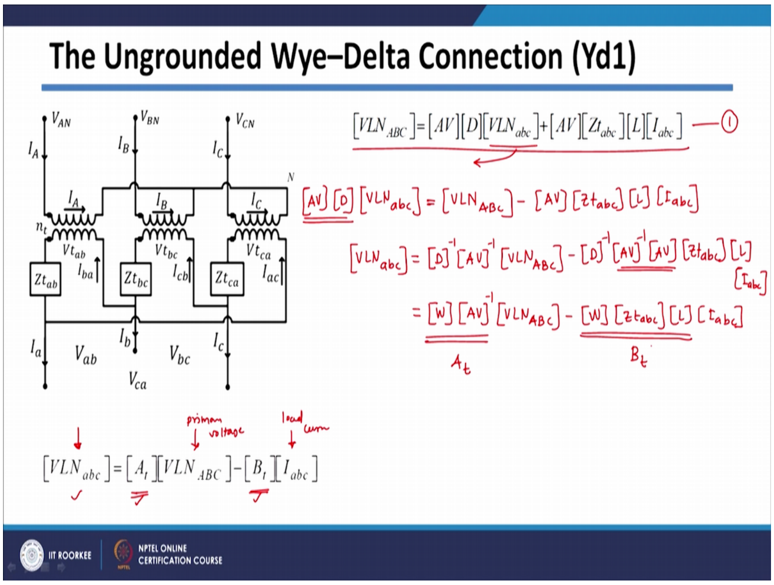


So, if you compare this with your standards. So, I will just take this on next slide. So, call it an expression number 1. So, expression number 1 I have taken on this slide here and if you compare this with standard equations in terms of c and d parameter.

So, here if you compare c parameter is 0; however, a d par to get the d parameter we need line currents here, but we are having delta currents here. So, these delta phase currents can be converted into line currents by using this L matrix and this delta phase currents can be written as L multiplied by I abc ok.

Now, we are getting in standard format. So, c will be 0 and your d matrix will be equal to AI multiplied by L. So, AI we have seen it is diagonal matrix 1 by n t and this is your L matrix. So, from this I can take n t common out. So, it will be 1 by n t remaining unit matrix multiplied by L matrix. So, unit multiplied by L will be just L matrix, so 1 upon n t multiplied by L. So, this is your d parameter here.

(Refer Slide Time: 25:40)



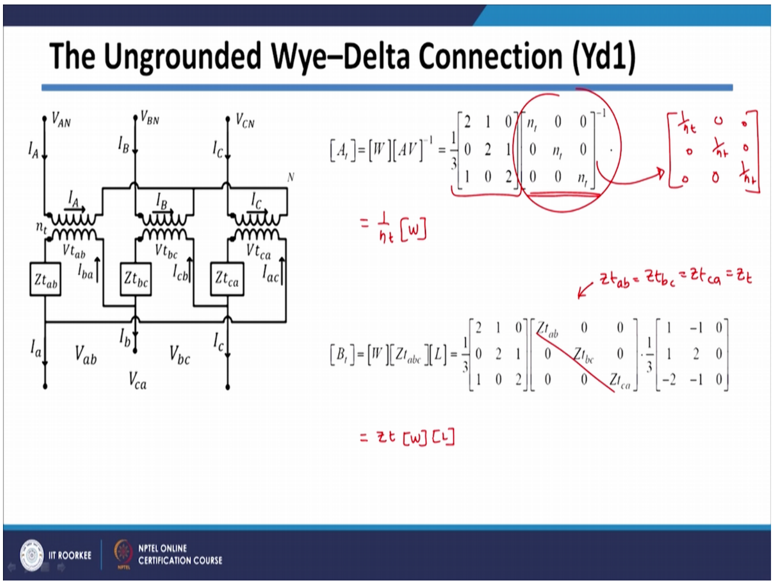
We have also calculated a capital A parameter and capital B parameter and these capital A and capital B parameters I have defined at like this; which are basically used to calculate line to neutral voltages on secondary side in terms of primary side voltages and the load currents. So, generally load currents are known primary voltages are known because those are applied voltages. Then load terminal voltages will be calculated using this capital A parameter and capital B parameter.

Now, to calculate it we can use this expression here which we have derived in earlier slides and what we need on left hand side is VLN abc. So, we can bring this on left hand side and all the remaining quantities on right hand side. So, in this case I can say VLN abc will be equal to VLN or here it is multiplication by AV multiplied by your D it will remain as it is; will be equal to VLN capital ABC minus AV multiplied by Zt abc multiplied by L into I abc.

If you see the secondary format we need only VLN abc on this side. So, we can just multiply this by D inverse multiplied by AV inverse. So, if you multiply D inverse multiplied by AV inverse to this term it will get cancelled out. So, it will be just VLN abc, this term will get multiplied with D inverse multiplied by your AV inverse into VLN ABC. This term also we will get multiply with D inverse and AV inverse, this AV is as it is and Zt abc into L into I abc. So, this AV inverse and AV will get cancelled out and we have seen that whenever there is D inverse term we get we need to replace by W. So, it will be W multiplied by AV inverse into VLN capital ABC minus your D inverse; means we need to replace by W into Z Zt abc multiplied by L into your I abc.

So, we can see that if you compare with standard equation this will be nothing but your capital A t term and this will be nothing but your capital B t term.

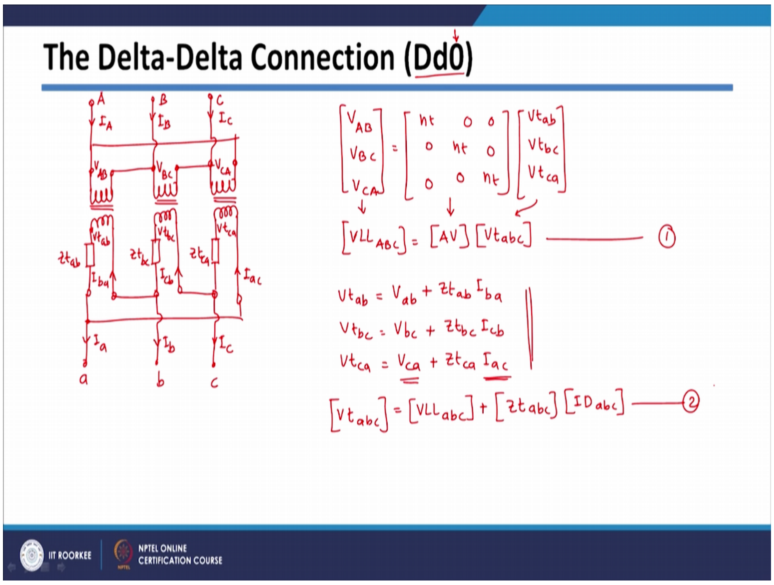
(Refer Slide Time: 29:09)



Now, capital ap A t term is W multiplied by AV inverse and capital B t term is W multiplied by Zt abc multiplied by L term. So, if you write them explicitly. So, this is your W matrix and this is your AV matrix and it is inverse. If you see the inverse of this part it will be just 1 by n t because they are only diagonal entries. So, it will be just 1 by n t 1 by n t 1 by n t of diagonal entries will be 0, if you take the inverse of these matrix. So, from this I can take 1 by n t common out which will get multiplied by W.

So, A t I can just write it like this and in case of B t also this is your W and Zt abc. So, if you consider Zt ab equal to Zt bc equal to Zt ca equal to say Zt. So, in that case these three entries will be equal which can be taken out common. So, it will be in that case Zt multiplied by your W matrix multiplied by your L matrix. So, in this case it will be Zt multiplied by W multiplied by L.

(Refer Slide Time: 30:35)



So, we have seen ungrounded wye by delta connection. Now, let us see your last connection that is delta delta connection and in this case I am considering vector group which is Dd 0 means, place displacement between primary and secondary quantities will be 0. So, they will be connected like this. So, you are having this transformer 1, transformer 2, and this is transformer 3. The impedances are referred on secondary side so I should represent them here and these are the terminals.

Now, we have seen that to connect delta we can connect them like this. So, this is delta connection on secondary side and this is your delta connection on primary side. And this terminal will be taken out as a capital A terminal, this will be capital B terminal, this will be capital C terminal.

Similarly, this will be small a terminal, this will be small b terminal and this will be small c terminal. Therefore, this voltage will be V AB, this voltage will be V BC, this voltage will be V CA. This voltage will be Vt ab, this will be Vt bc, this will be Vt ca. This impedance is Zt ab, this impedance Zt bc because it is connected between b and c phase, Zt ca and this current we are considering I ba.

This current is I cb and this current is I ac; on this side this will be I A I B and I C capital letters for primary side. And this will be I a I b and I c small letter for secondary side.

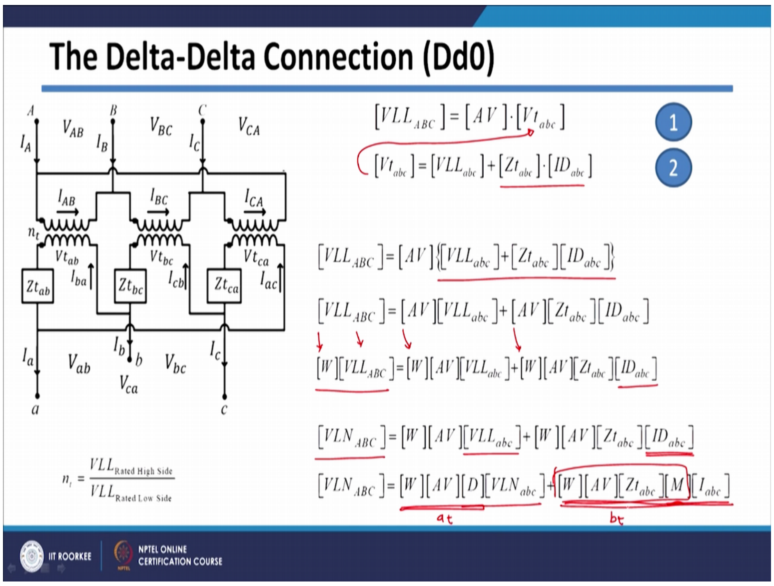
Now, in this case also we can write two expressions here, first is based on your turns ratio. So, V AB V BC and V CA they will be related to your Vt ab Vt bc and Vt ca voltages by just turns ratio.

Therefore, the diagonal entries will be n t and we know that this is nothing but line to line voltages. So, VLL ABC will be equal to this is your AV matrix and these are Vt abc voltages. So, in short form I can represent this by VLL this is by AV and this is by Vt abc; I can just say this is expression number 1.

And one more relation I can write in terms of terminal voltages and the voltages across the winding Vt ab. So, Vt ab will be equal to voltage V ab plus Zt ab multiplied by I ba current. Vt bc will be nothing but V bc plus Zt bc into I cb and Vt ca will be equal to V ca plus Zt ca I ac. And these three equations also we can write in short form like this.

So, these on left hand side I can represent it by Vt abc is equal to these three voltages will be nothing but line to line voltages on secondary side. So, VLL abc plus your impedance matrix Zt abc multiplied by this these currents, which are ID abc because they are delta phase currents and this I can write expression number 2.

(Refer Slide Time: 35:33)



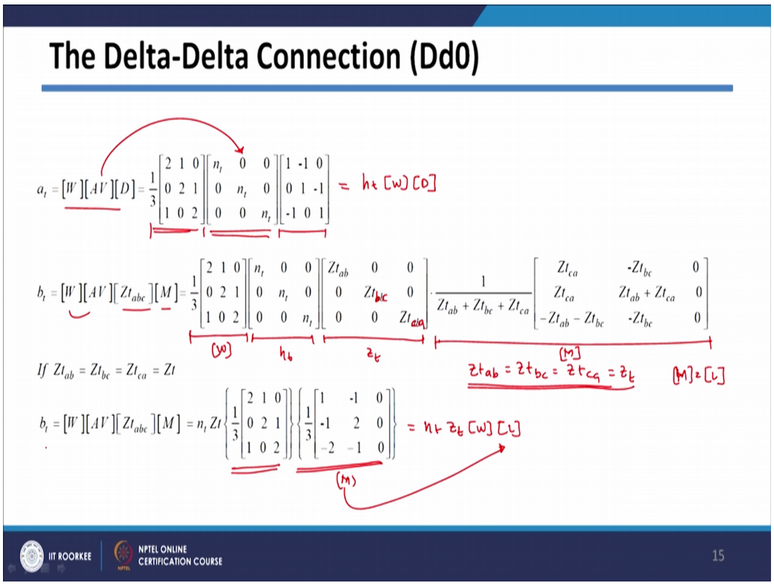
So, if you take this three equations on next slide so, we have got these two equations here. So, VLL ABC is equal to AV into Vt abc which one actually our first equation. So, this was our first expression and this was our second expression which I represent it here.

Now, I can put this Vt abc into this expression here. So, after putting this I will get this expression. So, Vt abc I have inserted into this expression here. So, in that case VLL ABC will be equal to AV multiplied by VLL plus AV into Zt abc. And then to convert this into line to neutral voltages we need to multiply whole expression by W. So, that is so, here if I multiplied by W I will get thi. These also we will get multiplied W and this also will get multiply with respect to W. And on this side I will get VLN ABC because W multiplied by V line to line voltages will be VLN ABC and these terms are as it is.

Now, let us take these terms on ok. So, here this VLL abc can be written as D multiplied by VLN abc. Similarly, these currents ID abc can be written as M multiplied by I abc. So, this ID abc currents can be written as M multiplied by I abc.

So, if you compare with standard equations. So, this must be your a t parameter and this must be your this must be your small b t parameter so, b t parameter.

(Refer Slide Time: 37:48)



So, if I take these parameters on next slide. So your, a t parameter we have seen it is W multiplied by multiplied AV by D. Out of this AV can be represented by diagonal entries of n t and this those can be taken it out. So, this can be written as n t multiplied by your W matrix multiplied by your D matrix because this term I can take common out.

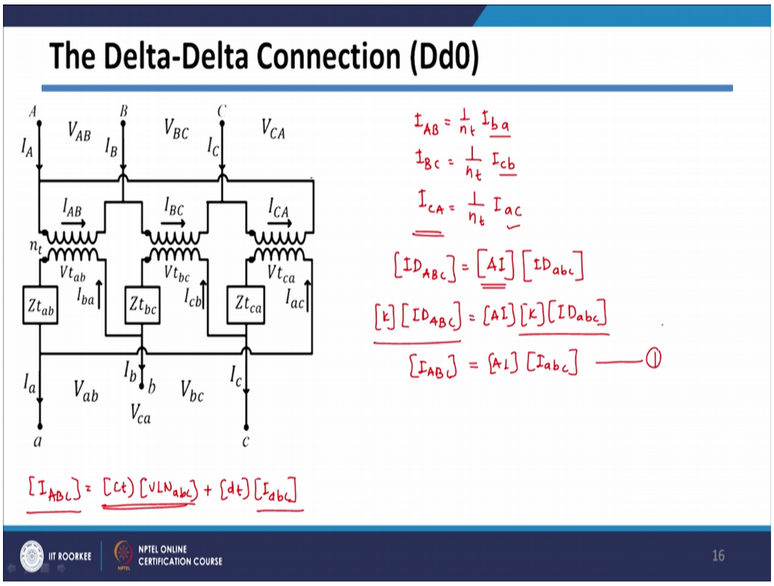
So, this is your W and this is your D. So, it will be n t multiplied by W multiplied by D and in case of b t we are having W multiplied by AV multiplied by Zt abc multiplied by

M. So, this is your W matrix, this is AV, this Zt abc and this is nothing but your M matrix.

So, if Zt ab is equal to Zt bc equal to Zt ca. So, this is actually Zt bc and Zt ca if they are equal to say Zt. In that case from this we can take Zt common out. From this we can take n t common out keeping W and this M matrix, but we have seen that if this impedances are equal your M matrix is actually equal to your L matrix.

So, in this case we can easily write. So, we can take this n t out Zt common out and this is nothing but your M matrix. Or this is nothing but your W matrix multiplied by this is nothing but your M matrix. However, since Zt abc and Zt bc Zt ca they are equal, I can represent this M matrix by L here.

(Refer Slide Time: 39:59)



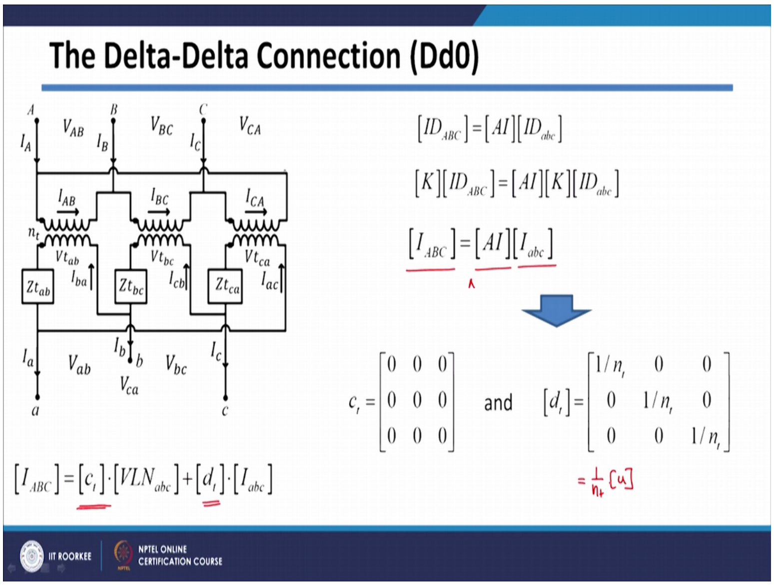
So, this is how we can get a parameter and b parameter, to get c and d parameter we have to take the current relations into account. So, in this case I can write I AB will be equal to 1 by n t into I ba. I BC will be equal to 1 by n t into I cb, I CA will be equal to 1 by n t into I ac and if you write them in short form.

So, these are nothing but your delta currents. So, ID capital ABC is equal to this will form your AI matrix because this will considered as a diagonal entries of AI matrix. And then these are nothing but ID abc which are delta phase currents in secondary winding.

Now, if you compare this with our standard equation. So, standard equation is like this I ABC is nothing but your c parameter c t into VLN small abc plus your dt into I abc. So, here this term is not there so, we do not have to worry. But here we need line currents and here also we need line currents. So, what we can do? We can multiply the both the matrices by K. So, it will be K multiplied by ID ABC which will be equal to since, it is just diagonal entries I can just extend the position of K.

So, instead of writing it first I can write this as second matrix. So, K multiplied by ID abc and this is nothing but your, I ABC. So, K multiplied by ID ABC is nothing but I ABC which will be equal to AI multiplied by, this is also I small abc. So, let us take this expression on next slide.

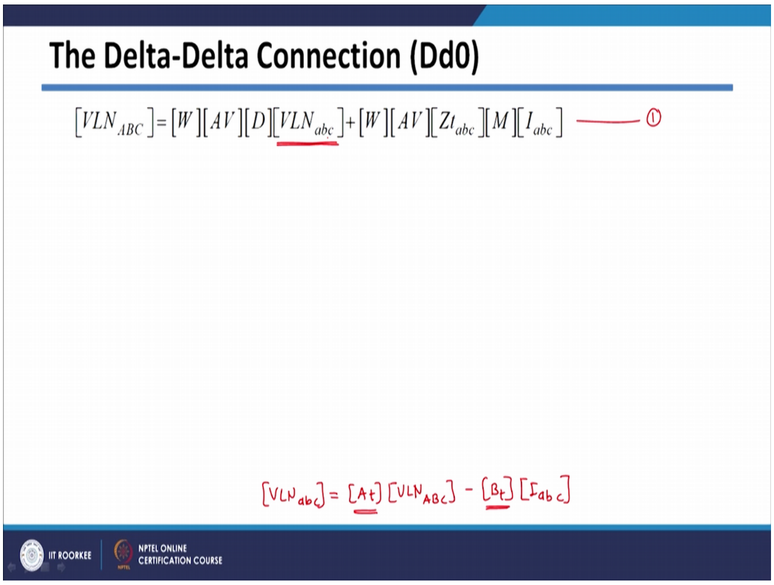
(Refer Slide Time: 42:26)



So, we have got this expression here I ABC which is equal to AI multiplied by I abc. So, if you compare standard equation c will be 0 because in this expression there is no term related to VLN abc. And your d t will be nothing but AI matrix and this will be equal to since diagonal entries we can take common out. So, it will be 1 by n t into u matrix which is identity matrix.

So, we have got a b c d parameters and as I told you we are also interested in calculating capital A and capital B parameters and those capital A and B parameters we need in this format.

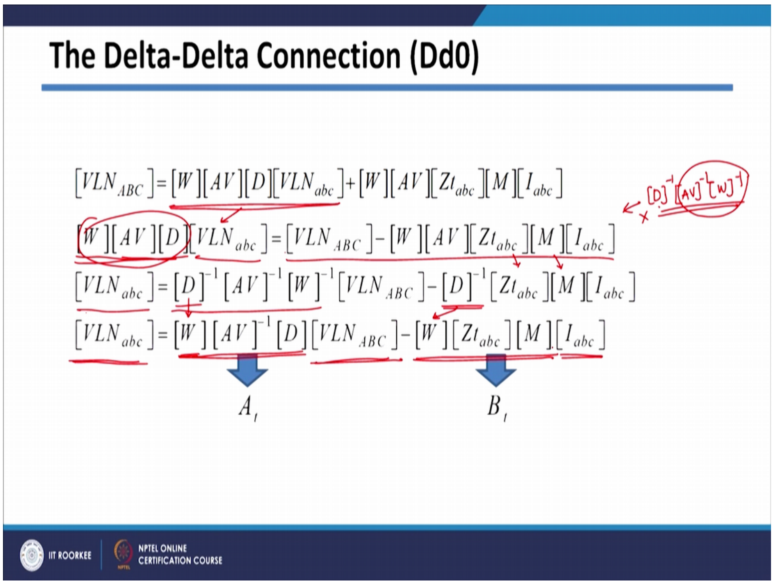
(Refer Slide Time: 43:15)



So, we need VLN small abc which is secondary side voltage which is written in terms of primary side voltage; which is VLN capital ABC minus your B t parameter into secondary side current I abc. So, we want actually expression to be in this format to get capital A t and capital B t parameters.

So, from your first equation which we have derived earlier slide, what we can do; we can take this term on left hand side and remaining terms on right hand side.

(Refer Slide Time: 44:04)

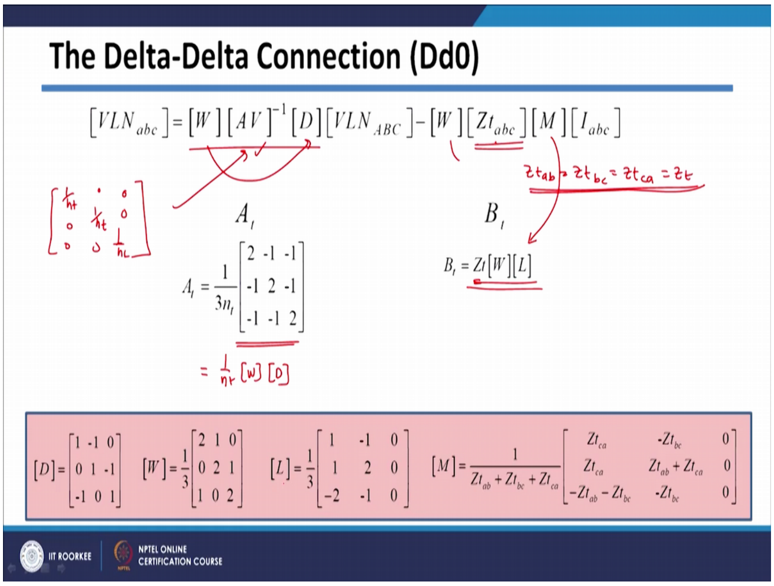


So, if you do this so, this term I am taking on left hand side; so here I, taken this and the remaining terms on right hand side which are basically this. Now, we want to eliminate this term here because on left hand side we want only VLN abc here. So, what we can do? We can multiply this whole expression by D inverse. So, whole expression will be multiplied with D inverse multiplied by AV inverse, multiplied by your W inverse.

So, if you multiply this whole expression by in this particular term here. So, here this will get cancelled out so, only VLN abc will remain on left hand side, this will get multiplied with this term. So, here there will be D inverse AV inverse multiplied by W inverse will remain. Here W and AV will get cancelled out bec with AV inverse multiplied by W inverse. So, only D inverse term will remain here so D inverse Zt abc as it is and M as it is.

Then we have seen that wherever D inverse coming will be replaced by matrix W. So, D inverse we are replacing with matrix W. So, here also this D inverse will be replaced by matrix W. So, we can see that we have got in the standard format. So, on this side secondary voltage in terms of primary voltage yes and load currents. So, this will be your capital A t parameter and this will be capital B t parameter.

(Refer Slide Time: 45:54)

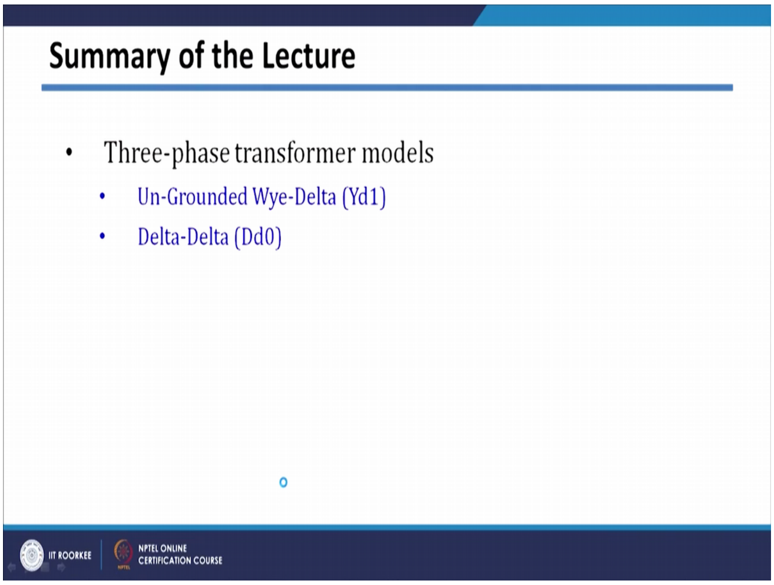


Let us see; what are these parameters. So, capital A t parameter which is W multiplied by AV inverse multiplied by D. And AV inverse we know that it is 1 by n t 1 by n t 1 by n t;

from that I can take n t common out multiplied by your W matrix multiplied by your D matrix.

So, if you do the multiplication of W and D matrix I will get this expression here. Or else I can directly write just write it will be 1 by n t which is coming from AV inverse multiplied by W multiplied by D here. And then from this side it is; if I am considering Zt ab equal to Zt bc equal to Zt ca equal to Zt. I can take Zt common out from this expression keeping W and M as it is, but we have seen that whenever this condition getting satisfied W will be replaced by L. So, B t parameter will be Zt multiplied by W multiplied by L where all these matrices are dependent it here.

(Refer Slide Time: 47:34)



So, in summary of this lecture two more connections we have seen; those are ungrounded wye by delta connections whose vector group is Yd 1. And then finally, we have seen delta delta connection whose vector group is Dd 0.

So, here we complete modeling of three phase transformers. In the next class we will see one example of typical distribution transformers and for that example we will calculate a b c d parameters. We will also see one more connections which is called as open wye open delta connections, which is also widely used to serve three phase as well as single phase loads.

Thank you.