Electrical Distribution System Analysis
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Lecture - 15
Modeling of Three-Phase Transformers
Part II

Dear students, we are seeing Modeling of Transformers in Distribution System. And in the last two lectures we have seen modeling of single phase transformers and basically to see the modeling of three phase transformer we have seen various types of convergence as well as representations. So, before going to actual modeling of different types of transformers those are actually wye wye connected or delta delta connected or delta wye connected. Let us revise whatever representation we have seen once again.

So, before going to actual modeling of this three phase distribution transformers which are having various different types of connection that is those are wye wye connection or delta wye connection or delta delta connections, before going to that let us see the representation of different voltages and currents. So, in case of grounded wye connection we are writing it as line to neutral voltages which will be represented by this one, sometimes if it is grounded neutral we can write V AG also.

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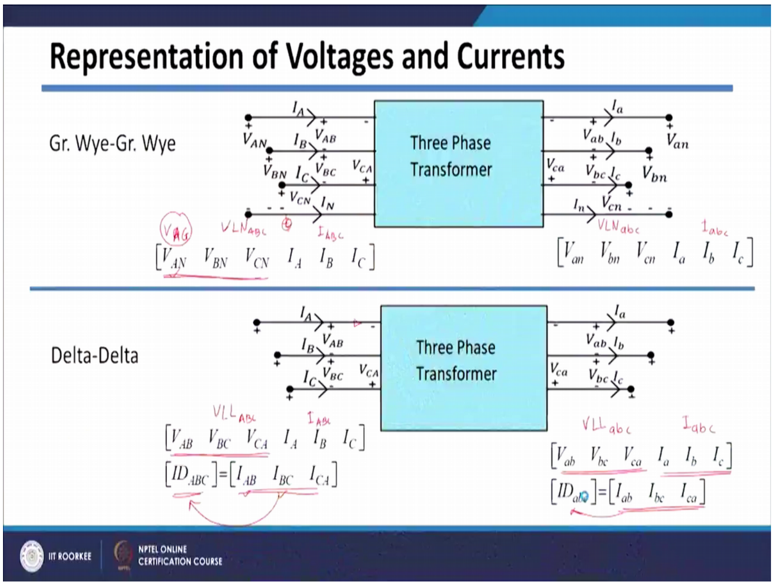

So, V AG represent way phase to ground voltage and if in case of ungrounded neutral. So, if this neutral is grounded neutral terminal available it is grounded then I will say V AG and if it is ungrounded means in this grounding is not there then I will say V AN voltage with respect to neutral terminal.

And as I told you we are going to call this three voltages collectively as V line to neutral voltage. So, I will say V LN ABC. So, these are basically line to neutral voltages of ABC phases and secondary side also it will be V LN small abc which represent secondary side line to neutral voltages. Then line currents will be represented like I ABC on primary side and I small abc on secondary side.

In case of delta delta connection we were seen that we are representing voltages using line to line voltages that is V AB V BC and V CA and collectively we can call it as a V line to line VLL ABC. So, when I am saying VLL ABC they will be line to line voltages of all the three phases, again this will be I ABC. However, delta current collectively I am calling ID ABC these are delta current which are after line current go inside this one there will be delta winding and in this line currents will get distributed. So, there will be different line currents and delta currents will be different. So, collectively this three current as I told you we can call it as a ID ABC.

Similarly, on secondary side these are the line to line voltages. So, this will be VLL small abc and these are line currents. So, it will be I small abc and this three delta currents these are basically ID abc this these related to representation and basically we are interested in getting abcd parameters to model these transformer.

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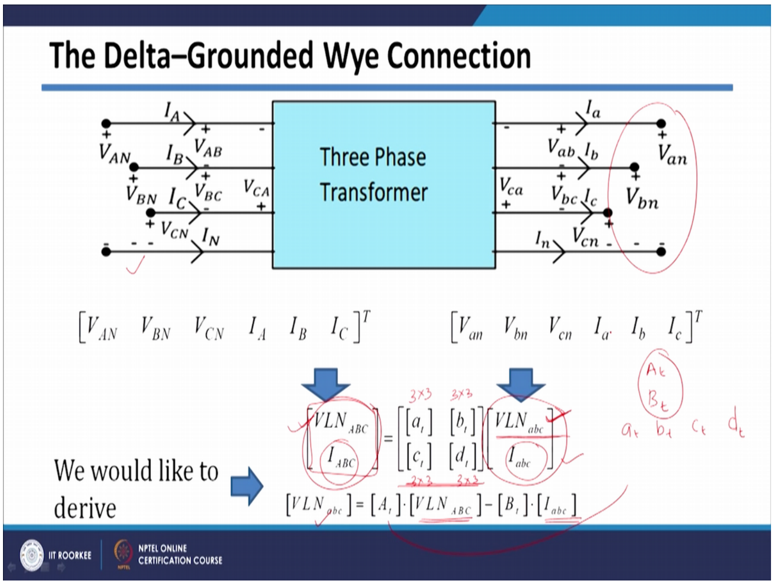

For various connections we want to get abcd parameters. So, we have calculated abcd parameter for single phase connection to winding transformer where abcd parameters are just one number. However, since if you observe these are actually three phase voltages which are three voltages, this also three voltages there are three currents on primary side and secondary side.

So, because of that you are a parameters will be having 3 by 3 matrix, b parameter will be 3 by 3 matrix, similarly c 3 by 3 and d also 3 by 3. So, abcd parameters will be basically 3 by 3 matrices we can get which we are interested in. So, always on this side we want to bring line to neutral voltages even though it is delta connection for that case also we need to bring abcd parameters in terms of line to neutral voltages. And secondary side also or we can say this secondary side voltages also we want line to neutral voltages. So, on this side of the parameters and on this parameters source side quantities also we want to be line to neutral and current to be line to line. So, always remember this.

One more thing we have seen many times in doing the calculation we want to know the load side voltages in terms of source side voltages and load current. So, to get this load side voltages in terms of source side voltages and load side current in that case we are writing this as capital A A t parameter and capital B t parameter. So, we are interested in small a t small b t small c t and d t and then capital A t and capital B t. Let us see how we can get it for different connections.

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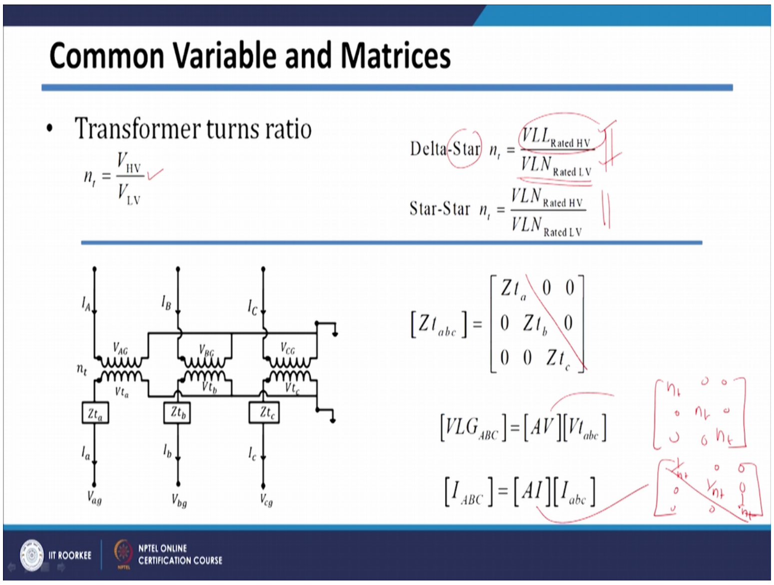

Before that I will just revise the terminology turns ratio we are going to see it is voltage on primary side or generally in case of distribution it will be HV side. So, voltage on primary side divided by rated voltage on secondary side. So, for different configuration depending upon phase voltage means voltage across the winding you have to see while calculate turns ratio.

So, in case of delta transformer voltage across transformer delta side winding will be line to line voltage. So, that is why V line to line rated voltage divided by in case of star connection it will be line to neutral voltage which will be appearing across the winding. So, that is why it will be VLN line to neutral. Then Zt abc was unit is diagonal matrix of impedances of three phases. There is possibilities of many times these impedances will be same. Then we have seen this AV transformation matrix which is basically your turns ratio.

In all the three phases generally distance ratio will be same and then AI will be inverse of your turns ratio that is 1 upon nt 1 upon nt 1 upon nt and all other. So, in this case also it will be AI also will be diagonal matrix, only thing is it will be 1 upon nt. And since the turns ration of all the three phases will be generally equal during operation also this three terms will be equal. Then we have seen various voltage transformations.

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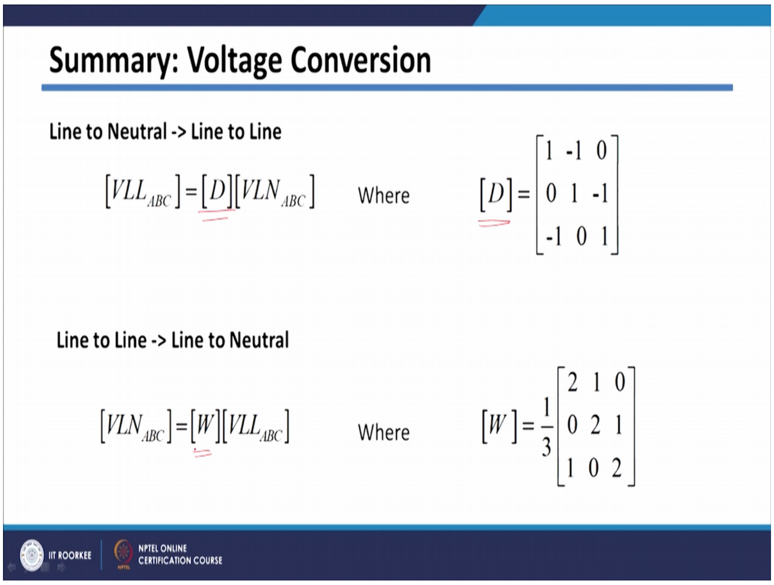

So, for transforming line to neutral voltage to line to line voltage we have got D matrix, then to convert line to line voltages to line to neutral voltages we have got this W matrix.

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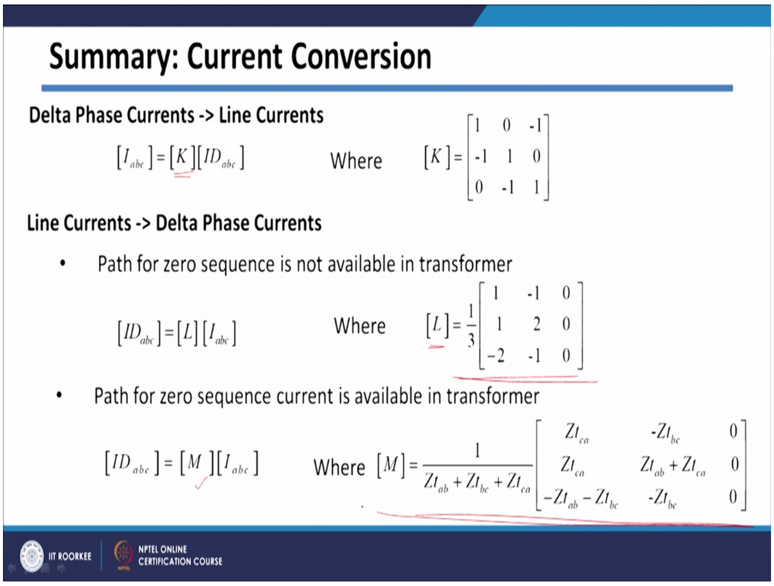

And in case of current convergent to convert delta phase currents to line currents we have got this K matrix and to convert line currents to delta phase currents we have got this L matrix. However, there are two cases because depending upon zero sequence current path is available or not we need to use L or M alternatively. So, whenever there is zero sequence current path is not available we have to go for L transformation and whenever zero sequence current path is available we have to go for M matrix ok.

Let us start with our first and simplest connection many times we will find this connections in power system application that is grounded wye by grounded wye connection, it is very simple. So, to show the connection so, that we can easily understand I am just drawing it here and then we will see the expression.

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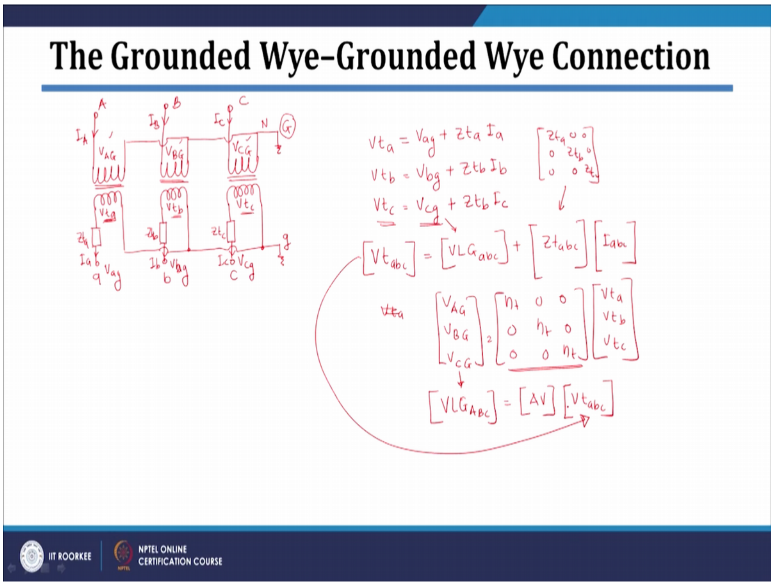

So, this connection will be something like this joining say these are the three transformers here ok, then as I told you we are linking this impedance on the secondary side. So, impedance of all the three transformers I am just showing at secondary side and since we are going to connect them in star both the winding.

So this is your a phase, this is your b phase and this is your c phase secondary side small letters and then we need to make this star point here. So this is connected to star point, this is star point and since we are using grounded wye connection we need to grounded ok, then representation of impedances will be like this. So, this is impedance in a phase impedance in b phase impedance in c phase Zt c so, I should say Zt Zt.

And then currents are basically current which are coming out of this transformer which is it is current is I a, this current is I b and this current is I c. So, small I a small I b and small I c. Now, primaries also grounded wye connection in that case you are having this three terminals which are basically capital A capital B and capital C. And when you to make star point by connecting this other three terminals together and which will be basically grounded and this terminal I am calling at G and this is small g, basically both terminals are same. However, to represent primary side and secondary side I am writing capital G and small g. However, it they are actually same terminals.

Then primaries are current will be I A, this current will be I B and this current will be I c. So, voltage across the winding it will be V AG because to represent grounded terminal instead of neutral, even though it is a neutral point to differentiate between ungrounded neutral and grounded neutral. So, if it is grounded neutral I am using instead of N I am using G. So it will be V AG, it will be V BG and V CG capital letters voltage across the terminals V ta V tb and V tc. And the voltage of this terminal will be V small ag with respect to ground, voltage of this terminal will be V bg with respect to ground and this will be V cg with respect to ground. So, this is how we can actually draw the different connections.

Now, if you see voltages at the terminal that is V ta V tb and V tc this voltages will be nothing but so, V ta will be equal to V ag plus your z ta into I a. And then V tb will be equal to V bg voltage plus Z tb into your I b and V tc will be equal to V cg plus Z tb into I c. And as I told you we can collectively write these three terms together. So, this will be nothing but Vt abc which represent all this three voltages which will be equal to these are basically as I told you line to ground voltages.

So, we can use term VLG because in this case also neutral is grounded VLG small abc. So, this three voltages will be represented by this plus your impedance matrix Zt abc which as I told you it will be diagonal matrix Z ta Z tb Z tc and non-diagonal matrix will be equal to 0. So, Z Zt abc will be having structure like this and then multiplied by your I abc all the three currents.

One more thing we know these voltages V AG V BG and V CG capital they will be just related by turns ratio with respect to your terminal voltages on secondary side. So, I can easily write so, V ta or its opposite way V AG V BG and V CG capital they will be just equal to turns ratio multiplied by your terminal voltages, those are basically V ta V tb and V tc.

So, in this we can again collectively write this terms as VLG capital ABC which are basically this three voltages will be equal to this matrix we have given name as AV. I explained this AV matrix we already defined it and this is nothing but your Vt abc.

And as I told you we for writing this or getting this model in terms of abcd parameter we need voltages in terms of line to neutral voltages and currents are basically line to line currents line currents ok. What we can do? We can take this Vt abc and put into this expression. So, I will just I am having these expression I will explain you.

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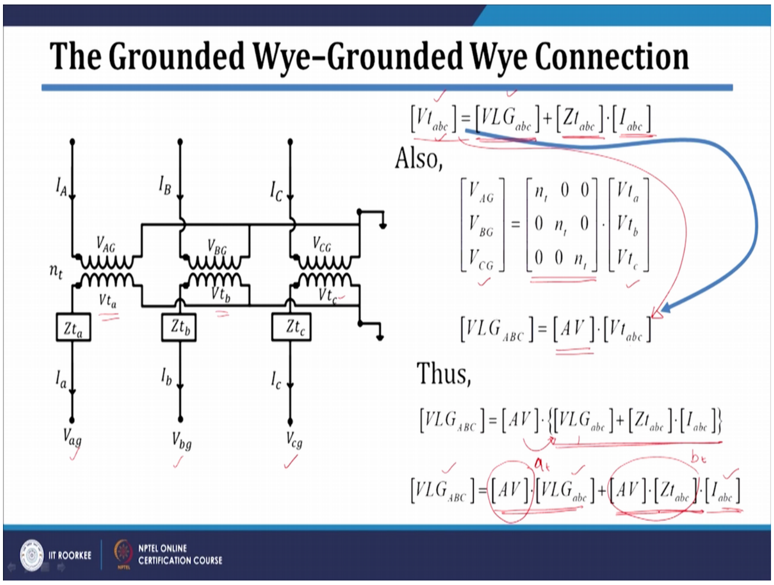

So, basically we have got this Vt abc which I explained is basically line to ground voltages of these terminals that is V ag V bg and V cg which is basically in this particular matrix. And all the three terminal voltages those are V ta V tb and V tc which are in this Vt abc matrix, this is impedance matrix and line current matrix we have derived in last slide.

Similarly, I explained you so, the terminal voltages and on primary side and terminal secondary side they will be just related by n t, means terminal voltages means I am talking about winding voltages. They will be just related by turns ratio and turns ratio matrix we defined at AV matrix and then what we can do this Vt abc I can just put into this expression. So, after putting I will get this expression so, here where Vt abc I am taking it from here. So, Vt abc will be replace by this here and then AV AV matrix I am taking inside the bracket. So, this term will get multiplied by AV and this term will get multiplied by AV multiplied your Zt abc.

So, if you observe this equation we have got this equation in required format that is voltages are actually line to neutral or line to ground voltages, ground we are as I told you we are using it alternatively to the neutral. So, these are basically line to ground voltages or line to neutral voltages and the this abc line abc this I abc is nothing but your line currents.

So, definitely this will become your a parameter a t and a t I am writing because transformer. So, a parameter of transformer I am writing a t, if it is for regulator I can say a r. So, a t and then it is this parameter is b t.

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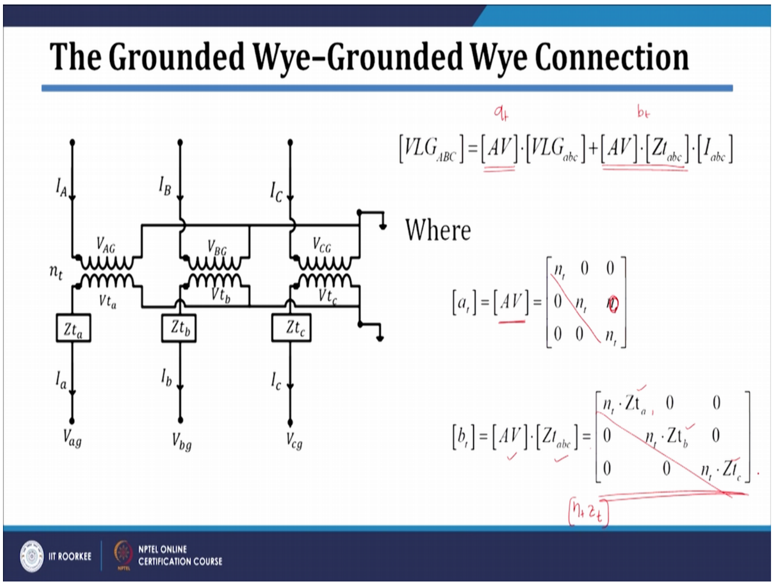

So, let us see what we are getting so, as I told you we are getting this as a a parameter a t and this is b parameter b t and AV. So, a parameter just AV and AV we have defined ok, here there mistake it should be 0 here.

So, AV matrix is just diagonal matrix of your turns ratio and b t we are getting AV multiplied by Zt abc. So, in this case also this is diagonal matrix of impedances and this is also diagonal matrix of turns ratio. So, in that case your b matrix will be just diagonal matrix of turns ratio multiplied by your impedances of this one. And many times as I told you impedances of all the three transformers are there same then in that case the entries will come out of the matrix and then there will be 1 1 1 here. And then I can take n t into Z t common out if the impedances are same and in that case it will be unity matrix.

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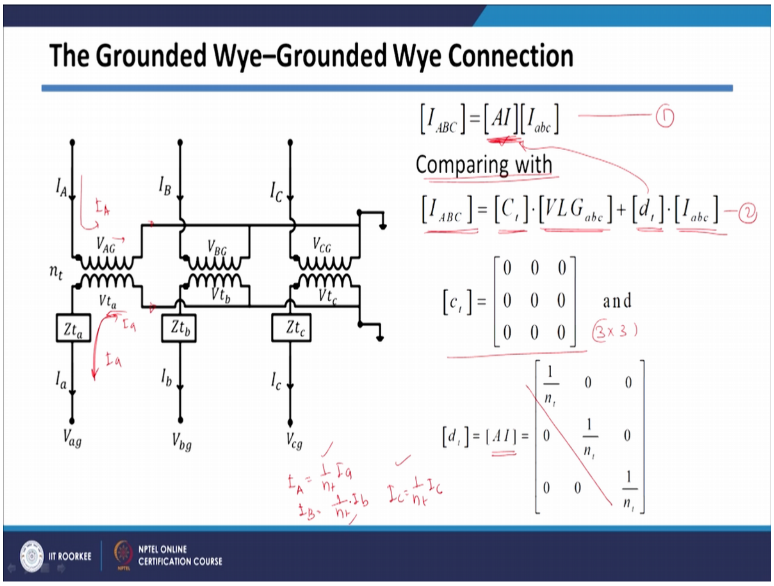

So, these are a t and b t parameter; let us see how we can get c parameter and d parameter; c parameter and d parameter we can get it from current equations. We have seen current transformation equation that is the current which is flowing in primary side and current which is flowing in secondary side; in this case current which is flowing in primary side is I A and current which is flowing in secondary side is a opposite direction it is small I a ok.

So, winding current also in this case this winding current is I A and this current winding is small I a and they will be rated by your turns ratio means your this I A will be equal to 1 upon n t into small I a. Similarly, I B will be just 1 upon n t into small I b and then I C will be equal to 1 upon n t into small I c. And in that case your is this as I told you this AI matrix is nothing but diagonal matrix of 1 upon n t terms which I am writing it here.

Now, if you observe it here if you compare this with our required abcd parameter matrix where we want actually your primary side current in terms of secondary side voltage and secondary side currents. If you compare these equation 1 and 2 you can see that there is no voltage related term means your C t matrix will be 0 and the current related term is AI.

So, in that case your d t matrix will be AI. So, dt will become equal to AI C t will be 0. So, C t will be 3 by 3 matrix of 0's and d t will be 1 as I told you AI, AI matrix we have seen 1 upon n t 1 upon n t 1 upon n t; this is basically coming from your current transformation.

So, we have got small t small d matrix also for this simple configuration which is grounded wye by grounded wye connection. Then capital B and capital A and capital B in that case as I told you we need to write your secondary side voltages in terms of primary side voltages and secondary side current.

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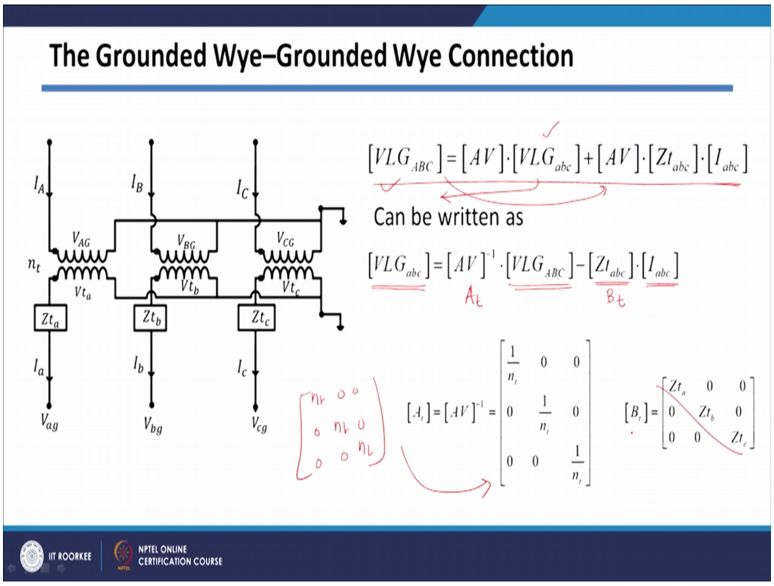

So, we are got this equation here and we want actually write this equation secondary side voltage in terms of primary side voltage and secondary. So, we can take this on left hand side. So, in that case if you take this on right hand side and then if you multiply everywhere by AV inverse, in this case it will become VLG abc secondary side voltage in terms of primary side voltage and secondary current. So, so we have got in required format.

So, this will be in nothing but your capital A t matrix and capital B t matrix. So, AV inverse AV matrix we have seen it is just n t n t n t diagonal elements and then when we are taking inverse of it will be just 1 upon n t 1 upon n t 1 upon nt in diagonal elements. And B t matrix is matrix of impedances so, three impedances in the diagonal terms so, we have got B t matrix.

Let us go for another configuration that is delta grounded wye connection. This is also widely used configuration where primary side is delta connected and secondary side is grounded wye connection, ok.

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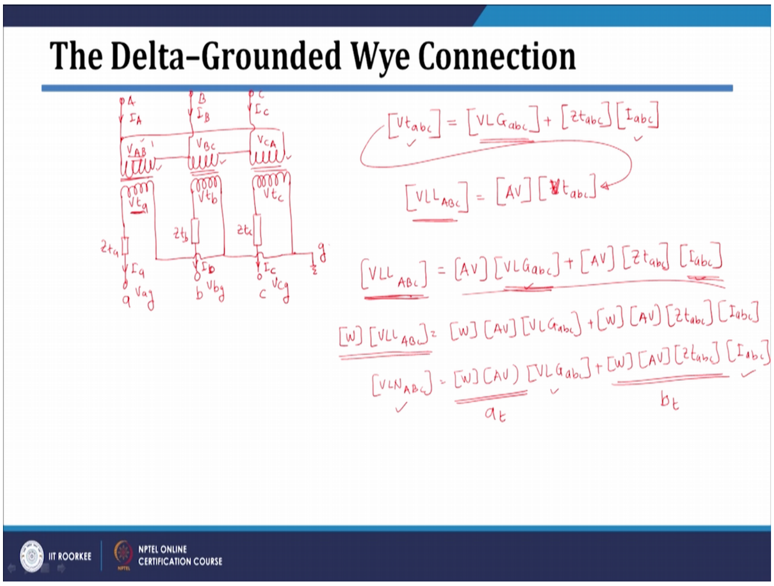

So, we will try to draw it here. So, I will first draw three transformers this is your secondary side and then as I told you we are referring your impedances on secondary side. So, all the impedances are connected secondary side and these terminal we have seen small a b and c. And since secondary side is grounded wye connections I can take this terminal together and make your neutral point. And we can ground it and I am calling it is point g in primary side it is connected in delta. So, in that case we can connect all these windings into delta fashion. So, they have now connected in delta fashion and then we can take each terminal out. This is your capital A terminal, B terminal and C terminal and the currents are this is I A I B and your I C.

Now, this voltage will be voltage across this winding will be V AB this will be V BC and this will be V CA. Similarly, these currents which are flowing through this winding are I this winding will be I AB, this will be I BC and I CA. So, these are delta basically delta current. This terminal voltages on secondary side it will be V ta because they are grounded wye connection. So, this is V tb and V tc this impedances are Zt a Z tb and Z tc, this current is small I a I small b and then this is small c.

So, in this case also first equation which will write the terms which are related to secondary terminal voltage that is in terms of V ta. So, now, in this case I am directly writing it. So, Vt abc which here basically terminal voltages we can easily write then they will be equal to VLG. Now since, this is actually ground connection abc plus your impedance Zt abc impedance matrix which is again diagonal matrix of three impedance entries and then line currents which are basically small I abc.

So, this includes all the three currents those are I a I b and I c on secondary side this include all the terminal voltages those are V ta V tb V tc as a column. This represent all the voltages on secondary side those are basically V ag V bg and V cg with respect to ground basically neutral connection. Since, neutral is grounded and I am using g term here.

And then one more equation I can write which relates your primary voltages, primary winding voltages with respect to secondary winding voltages. So, V ta and V ta and V ab they will be just related by your turns ratio. Means, I can easily write VLL because now, these voltages line to line voltages VLL ABC which basically V AB V BC in V CA on primary side.

They will be equal to the transformation ratio matrix which we were seen it is matrix AV multiplied by your Vt abc. So, Vt abc multiplied by your transformation ratio matrix will give me line to line voltages ok. Then what we can do exactly similar way what we did in case of earlier case whatever this Vt ab we can put it here. So, in that case your equations will be VLL ABC which will be equal to your matrix AV into VLG small abc plus AV into Zt abc into I abc.

Now, if we observe this equation here as I told you whenever we are writing whenever we want to get abcd parameters always there should be line to neutral voltages and line currents. So, if you observe this equation here this term this is actually line to neutral voltage. So, we do not worry this term also line to line currents. So, do not worry; however, on this side we are getting line to line voltages which need to be converted to line to neutral voltages.

So, we have seen that convergence matrix for converting this into line to neutral voltages, we need to multiply this equation by equation W matrix W which is basically convergence matrix which will convert your line to line voltages into line to neutral voltages. So, everywhere I am just multiplying this equation by W. So, if we multiplied by W, W multiplied by V line to line will be basically will become V line to neutral into ABC here and it will be actually W as it is W AV into VLG small abc plus your W into AV into Zt abc into I abc.

So, we have got in required format. So, this voltages are line to neutral voltages and currents are actually line currents. So, definitely this must be your parameter a t and this will be your parameter b t. So, let us see what are this parameters are we have seen this equations.

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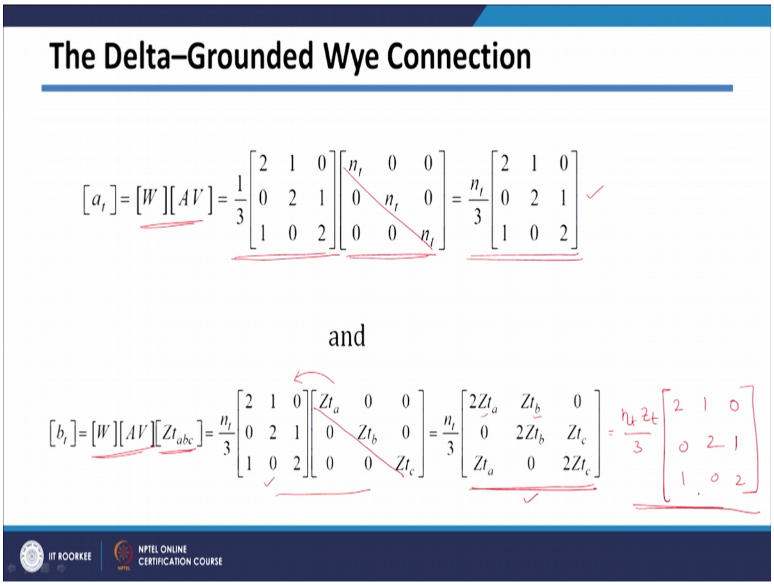

So, we have got a t which is equal to W multiplied by AV, W matrix we know which is this one and AV matrix is a transformation matrix which is n t which is diagonal matrix. So, we can easily write n t these are diagonal which will get multiplied to each of the entry basically it will come out of the matrix. So, it will be n t by 3 and to this particular matrix.

So, you are a t parameter will be given like this and we have seen that your b t parameter is W multiplied by AV multiplied by your Zt abc. So, in this case also this Zt abc diagonal matrix in that case this will get multiplied to each of the entry of this matrix. So, if we multiplied this two matrices basically since it is diagonal each entry will get multiplied with respect to diagonal entry of this one.

So, in that case the matrix which is after multiplying will get this matrix here and as I told you if impedances are same generally they are same in that case impedances will also come out. So, in that case your matrix will be n t into Zt if they are equal divided by 3 and this is 2 1 0 0 2 1 1 0 2. So, this will be your b t parameter. So, we have got a t and b t for delta and grounded wye connection.

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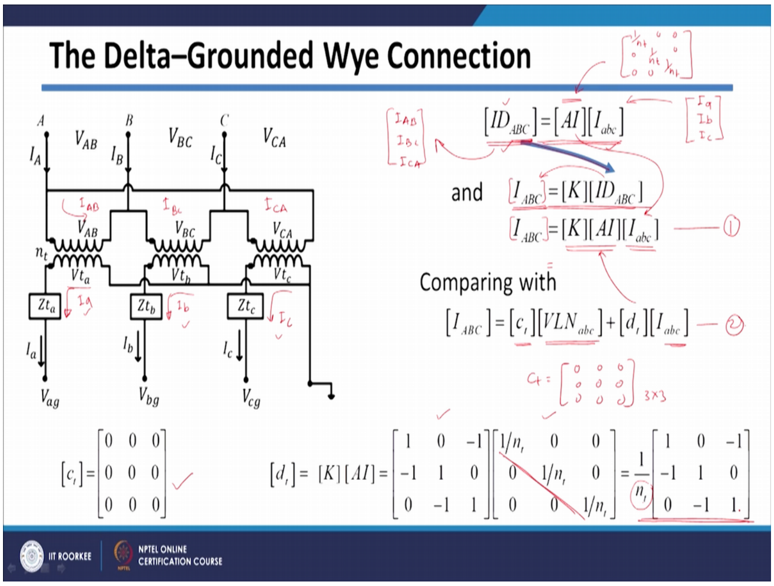

Let us see how we can get currents. So, in this case first we will take currents which are winding currents. So, as we know that winding currents are related by turns ratio. So, on this side winding current is I a. So, the current which is flowing through the secondary winding is I a and I b and I c.

However, on primary side the currents are I AB I BC and I CA which are basically delta currents. So, collectively I can write this delta currents which are basically I AB I BC and I CA and we have seen that whenever there delta current I am writing this D term here so, ID ABC represent these delta currents on primary side. So, capital letters are here and they are related to the secondary side currents which are basically I a I b and I c. So, this basically this term represent your currents I a I b and I c and they are just related by your turns ratio transformation which is basically AI matrix which is 1 upon n t 1 upon n t 1 upon n t in the diagonal terms and non-diagonal terms are 0.

So, we are getting this matrix now since there are delta can because as I told you for abcd parameters we need line currents. So, we need to convert this delta currents into line currents. So, basically this these are line currents so, we do not have worry this delta current need to be converted. So, this we can get we know this equation which basically converts delta currents to the line currents I ABC.

So, we can use this equation here. So, your I ABC and instead of this I ID ABC I can just put these equation here. So, it will become actually your K multiplied by AI multiply I abc. Now, we can compare with this your standard equation of abcd parameters. The standard equation is this one say 2 and this is which equation which you have getting 1 and if you compare this 1 and 2 you can see that there is no terms which related to VLN that is why c t matrix will be equal to 0 0 0 of 3 by 3 size. And your d t matrix basically is nothing but this K multiplied by your AI, AI is basically diagonal 1 upon n t terms.

So, we can easily see that your c matrix is basically matrix of 0 0 0 and your d t matrix is K multiplied by AI. So, K matrix we know which is given by this and AI matrix which is basically given by this and in this case also n t; actually these are diagonally entries which will get multiplied by corresponding entries into the matrix and since they are diagonal it will come common. So, 1 upon n t and this will remains same. So, this will nothing but your matrix d t here for delta to grounded wye connection. So, we have got all abcd parameters, let us see capital A and capital B parameters.

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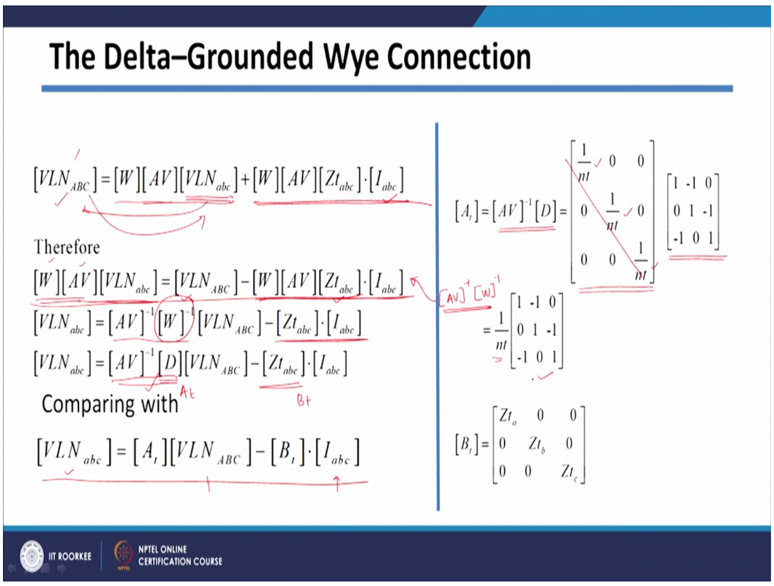

So, we want to make this load side voltages in terms of source side voltages and so, will just write that way by taking this on left hand side and this on right hand side. So, if you do that it will basically I will get this equation here. So, this term will be equal to this term minus basically this term. So, after doing that I will get this term here then what we need to actually remove this W and AV. To remove this W and AV what we need to do is we have to multiply whole equation by your AV inverse multiplied by W inverse.

So, if we multiply this whole equation by AV inverse multiplied by W inverse this term will get cancel out because AV we are multiplying by AV inverse and W inverse. So, on this side only VLN will remain this term will get multiplied by this AV inverse by W inverse which I have written it here. And in this term also if you multiply this term by AV inverse and W inverse only Zt abc will remain. So, here in this term only Zt abc will remain.

Now, as I told you whenever you are getting W inverse you need to use matrix T or whenever you are getting D inverse you need to use matrix W. So, here we are getting this W inverse here. So, this should be replaced by matrix D and then we can compare this with our standard equation of capital A and capital B parameter which basically gives secondary side voltages in terms of primary side voltages and load currents. So, we can easily compare.

So, this will become your parameter A t and this will become your parameter B t and by knowing AV inverse and D we can easily get your parameter capital A t which is basically AV inverse multiply by D. And AV inverse is nothing but because AV is just diagonal matrix of n t n t n t. So, it will be 1 upon n t 1 upon n t 1 upon n t which is basically AV inverse and this is your D matrix. So since, these are the diagonal entries it will be just 1 upon n t and this is your D matrix here.

So in summary of today's lecture: we are actually seeing various models of distribution transformers. To start with, we have revised all the transformation matrices which basically convert voltage and current transformations, those who have revised. And then we have seen two types of transformer connections which are basically grounded wye to grounded wye connection and delta to grounded wye connection. And in both this cases we derived a b c d parameters and capital A and capital B parameters, which basically used while modeling the transformer.

Thank you.