Electrical Distribution System Analysis
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Lecture - 14
Modeling of Three-Phase Transformers
Part I

Dear students, last time we have started Modeling of Three-Phase Transformer. Basically, we have started with single phase transformer. And then we have seen how we can use that theory for modeling of three-phase transformer. So, just we will revise what we have seen in the last class.

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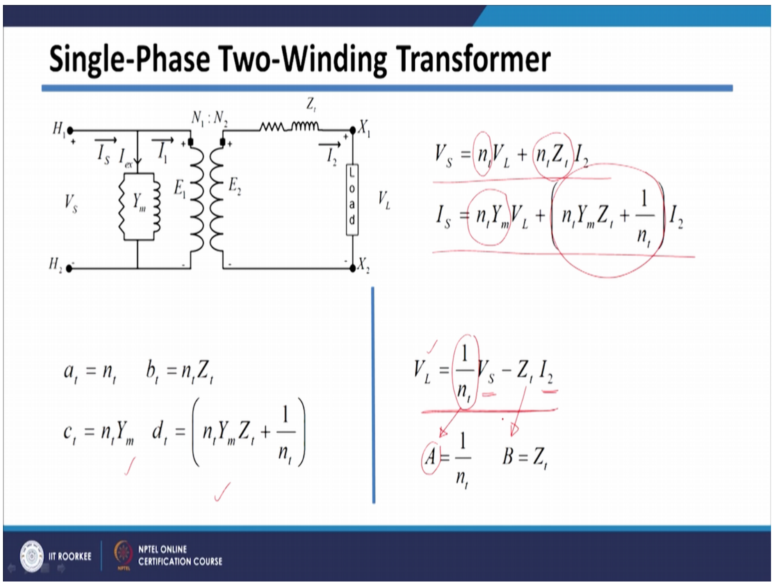

In the last class, we have seen two equations which we have written by seeing your equivalent circuit. And from these two equations we get a, b, c, d parameter. So, what we have got a, b, c, d parameter. This is a parameter; this is your b parameter; this is your c parameter; and this is your d parameter. So, I have written them here

Similarly, many times we need to calculate load voltage in terms of source voltage and load current. So, in that case we have defined them in terms of capital B parameter. So, this capital B parameter which basically we have calculated in last class so that your load voltage will be modeled in terms of source voltage and your load current; so this gives me your capital A parameter and this gives me your capital B parameter.

Then we have seen for modeling of three-phase transformer; we have seen how we can represent this quantities those are actually basically voltages and current in the three-phase transformers.

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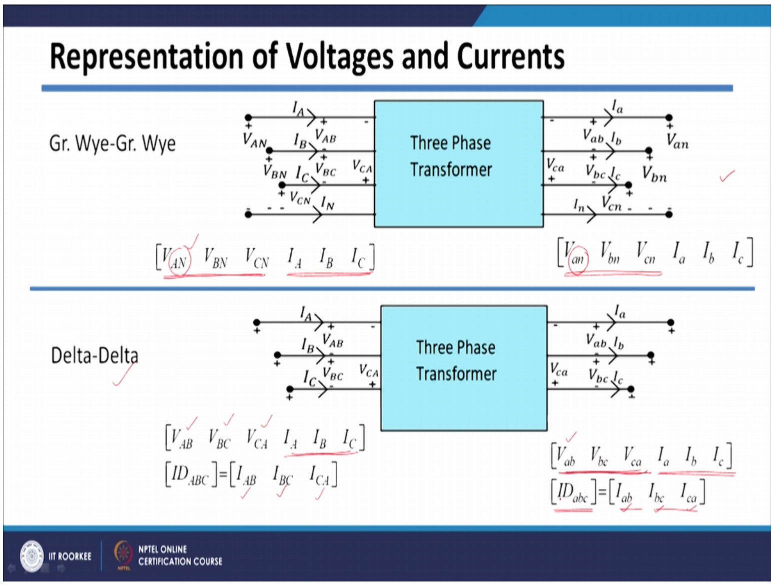

So, we have seen that the voltage which are generally represented by with respect to your ground terminal that is V AN, V BN, V CN and then current will basically your line current that is those are I A, I B and I C. And as I told you in the last class, we are using capital letters to represent primary side quantities.

Basically, in distribution system they will be high voltage side quantity. So, primary quantities will be represented by capital letter. And then similarly your line to neutral voltages on secondary side they will be represented by V an, V bn, V cn again this small letters are used for representing the secondary side.

And in case of delta winding, we have seen that we are we can represent the line to line voltages. Those are basically, V AB, V BC and V CA those are line to line voltages and your line currents which are I A, I B and I C. And on the secondary side, again it is delta connected I am considering because, it is I have written there and then these voltages will be V ab, V bc, and V ca line to line voltages and then line currents will be I a, I b and I c.

Now, as I have explained in last class when this line current entrains into transformer windings will be delta connected. So, these currents will get divided into your delta phase windings and then we have seen that these delta currents are represented by I AB, I BC and I CA, those are three-phase currents in the delta winding. I ab small ab, small bc and small ca, there are basically your secondary side delta currents. And collectively, I am calling them as ID ABC. So, whenever I am writing ID ABC these are nothing but your delta winding currents.

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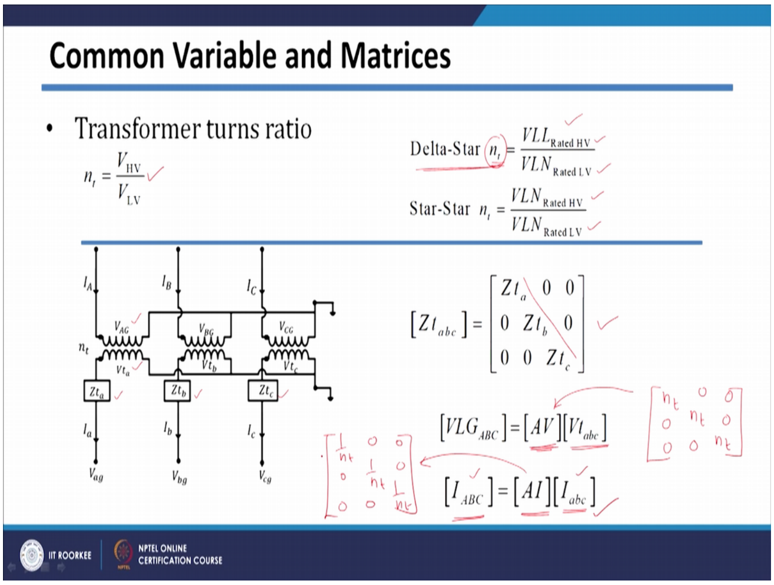

Then we have seen other terminologies. So, in case of transformer ratio n t we have seen that we are representing it by rated voltage on HV side divided by rated volt voltage on LV side. So, to represent say turns ratio of transformer which is delta to star n t will be represented at v line to line voltage, because phase voltage in that case it will v line to line voltage rated on HV side divided by v line to neutral voltage on LV side which will basically your turns ratio in case of delta-star transformer. And in case of star star-star transformer it will be just line to neutral voltage on HV side divided by line to neutral voltage on LV side.

Then we have seen how to represent your leakage impedance of the transformer and we have seen that this impedance is represented on secondary side of the transformer. So, we have represented both the trans impedances on secondary side and since there are three-phase matrix these are represented by diagonal element of Z t abc matrix.

Then we have seen your transformation matrix which basically transforms your terminal voltages on secondary side of the transformer to the terminal voltages on the primary side to represent this we are using this AV matrix. And we have vs we have seen that this AV matrix is nothing but, your diagonal element matrix which is having n t terms at the diagonal and all the non diagonal entries will be 0. So, your AV matrix will be having structure something like this; then your AI matrix which basically converts your line currents or winding currents on secondary side to the winding currents on a primary side.

So, these are the winding currents on primary side and these are the winding currents on secondary side and this basically AI matrix we have seen that it is one upon n t element in the diagonal which n t is again turns ratio and all other entries are 0. So, only diagonal elements will present which are one by n t. So, this explained in the last class.

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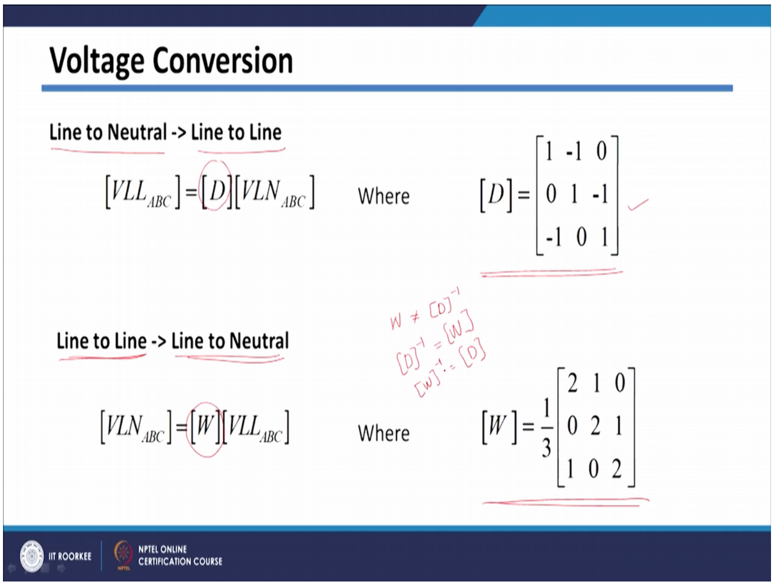

Then we have seen that many times since we are considering three-phase modeling. Many times we need to convert your line to line voltages to line to neutral voltages or delta phase current to the line current and line current to the delta phase current ok. So, we need to know this transformation matrix, so that we can easily convert them.

So, last time we have seen voltage conversion. So, to convert voltages which are basically, line to neutral voltages to line to line voltages, we have got conversion matrix which is called as D and basically which is given by this expressions. So, I have explained you last class how we get this matrix.

Similarly, to convert line to line voltages to the line to neutral voltages, we have got this W matrix we also derived this W matrix in the last class and basically we had got this W matrix here. So, we can easily convert line to neutral voltages to line to line voltages or line to line voltages to line to neutral voltages.

Again I will tell here W is not equal to your D inverse because, inverse of D is nonexistence. However, whenever in expression D inverse or is will be available the in that case we can use it as a W or whenever there is W inverse existing then we can use D matrix. However, these are not exactly the inverse of each other.

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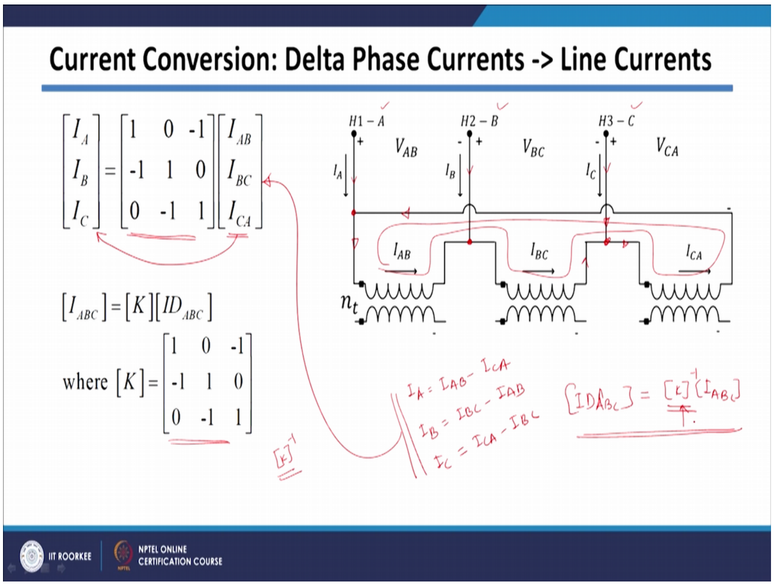

Now, we will see current conversions. In current conversion, basically many times we need to convert delta phase current to line currents or vice versa line means line current to the delta phase currents. One example I have shown it here. Let us see these windings which are connected in delta fashion. We can see that they are connected in loop like this which are actually delta like this. And there are three terminals which are A, B, C are connected to this delta. And then current I A which is coming in, current I B is coming in and then I C is coming in.

So, if you apply KCL at this point you can write I A will be equal to I AB minus I CA. Because, this I CA current is coming which is in this direction I BA is going like this out of this terminal. If you apply KCL at this point, it will be I B will be equal to I BC minus your I AB. And when we apply KCL at this point, it will be I C will be equal to I CA minus I BC ok.

If you put these three equation into matrix form, I will basically get this equation here. And if you observe this equation, it converts your delta currents which are represented here into the line current. So, we can I am calling this matrix k which converts your delta phase current to line current So, this is the conversion for delta phase current to the line current.

Now, if you want to convert line current to the delta phase currents, it will it is little bit tedious task, because as I told you in this case also the K inverse will be infinite. So, this matrix is basically singular matrix. So, if you take inverse of it the inverse is nonexistence means I cannot say ID ABC, ID ABC which is equal to K inverse into I ABC, because K inverse is nonexistence. So, we need to find out this matrix here. So, let us see how we can get this matrix, this is basically K inverse matrix.

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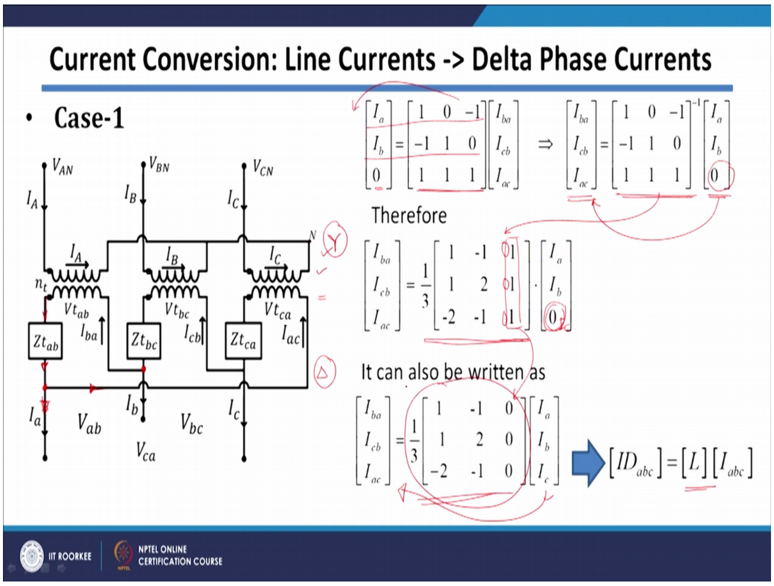

There are two cases. In case 1, if the path for zero sequence current is not available. So, in many cases only positive sequence and negative sequence currents will be present; and there will not be path available for zero sequence current. Like one combination which I have shown it here; in this particular transformer, this connection is actually star connection or Y connection and this is your delta connection. So, this is Y connection in delta connection however this neutral is not grounded. So, this is this Y is having neutral ungrounded system.

Since this neutral is ungrounded, there will not be a zero sequence current into primary side of the transformer. And hence there will not be zero sequence current because for zero sequence current to flow there is should be always balanced in there should be balancing current from the secondary winding. So, whenever zero sequence currents are flowing in a one winding, there should be a zero sequence current flowing in another winding. So, since zero sequence currents are not there in Y here, there will not be zero sequence current in delta.

So, if there is zero sequence current path is not available we can easily write three equations here. So, in this case your ia current will be nothing but your, I ba minus I cb. So, if you see this I a current here in this case this I a current will be, so this will be your I ac current and this is your I ba current.

So, if I apply KCL at this point, so basically I will get this first equation here. Then if you apply KCL at another node, I will get this second equation here. And third equation we can get since addition of all the three delta current, it should be equal to 0, because zero sequence currents are not present there.

So, if you add all the positive sequence current and all the negative sequence currents since they are 120 degree displace with each other addition will be always 0. So, both all these currents are having just negative sequence and positive sequence because of that addition of all this currents will be equal to 0. So, from the addition of all the current that is I ba plus I cb plus I c ac, it will be equal to 0.

So, from that I can easily write by inverting this matrix, your delta phase currents will be equal to your inverse of this particular matrix into this matrix. So, if I am taking this matrix on this side by making the inverse. So, this basically gives me your line currents to the delta currents; however, there is no I C current present here. So, if you calculate inverse of this matrix, the inverse of this matrix will be given by this one you can easily calculate it so and then here we do not have I C current.

So, to get that I C current what we can do since if you have observe this last column is getting multiplied with entry zero. So, what I can do, I can I can make all the entries of this column to be 0 and I can put I C here. It is exactly same because I C is getting multiplied by 0 0 0, so this contribution from that I C is not there, so that is what I have done it here by replace that column with 0 0 0 entry and I replace this 0 with I C

So, now we have got required transformation matrix for the case where zero sequence currents are not present. So, we can easily convert your line currents to the delta phase current using this conversion matrix which I am calling it as a matrix L, which is given by this particular matrix.

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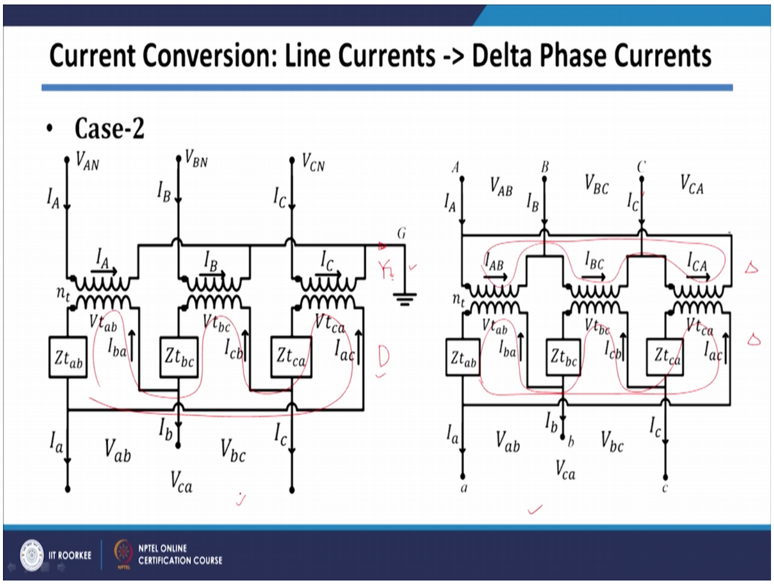

Let us say case two, where zero sequence current will be present. So, I have shown it two cases where there is possibility of presence of zero sequence current. So, in this case, we are having again this y delta transformer. However, unlike your earlier case in this case I am grounding this of y connection.

And because of grounding zero sequence current path will be available, so whenever zero sequence currents are flowing in primary winding, there will be balancing zero sequence current will be flowing in delta winding. However, if you see the secondary terminal there is no neutral terminal available. So, all the zero sequence current will actually only flow through your delta windings. So, balancing zero sequence current will flow only through delta windings.

If you see second example here, here it is a I am considering delta-delta connected transformer. So, both the sides at the line ends, there will not be zero sequence current because I am not using there is no ground terminal at both the end. However, so whenever there is some zero sequence current flow through the delta winding which are basically circulating kind of thing. Then there will be always zero balancing zero sequence current which are flowing another delta. So, it is basically this zero sequence current only flow the delta-delta windings in on the both the side. So, in this case also there is possibility that negative zero sequence current will be present.

So, for these cases, for this kinds of transformer; we need to count this conversion matrices which basically converts line to current to the delta phase current little bit different way.

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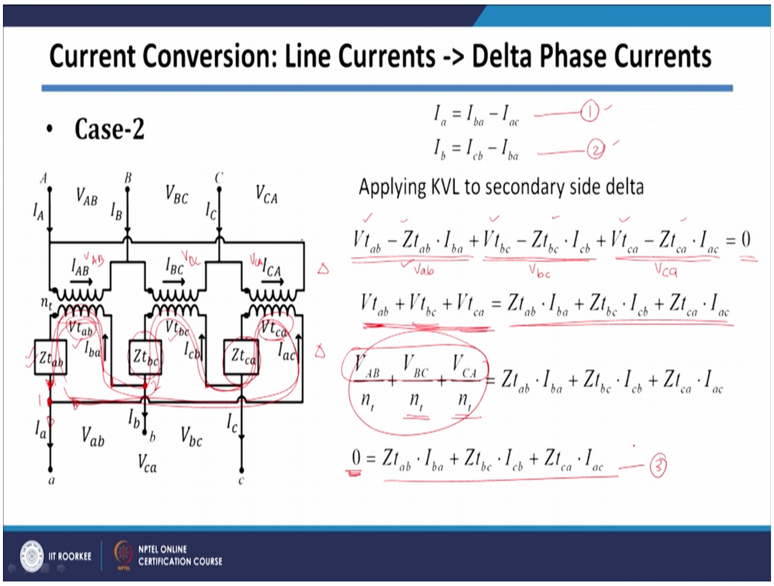

So, let us see how we can calculate them. So, as I told you if you consider this case of say delta-delta winding, where the there is possibility of zero sequence current may present. So, in that case, whenever zero sequence currents are flowing you can see that the zero sequence currents will be basically circulating and the current magnitude will be same. So, in that case your equations will be something like this two equations I can easily get like which we did earlier by applying KCL at this point I will get this first equation. And after applying KCL at this point I will get second equations. So, here basically I a current which is coming out of this terminal; this I ba current is again coming towards the terminal and your, I ac current is going out of the terminal.

These two equations we are getting after applying KCL at this two point that is point one and point two two equations we have got. To get the third equation, we can apply KVL for the loop which I have shown it here. So, we can apply KVL to this whole delta winding loop of secondary side. So, we can easily find out. So, in this case the voltage drop across a phase ab phase will be given by this one which will be just the if you if you say V ab will be nothing but V t ab voltage at this one minus the voltage drop which is happening across the leakage impedance. So, this is nothing but V ab which is voltage at V ab terminals

Similarly, voltage at V bc terminal will be nothing but voltage terminal voltage V t bc minus impedance drop which is happening at across the z impedance. So, this is actually your V bc voltage and third voltage which is V t ca minus voltage of which is happening across this which will be basically V ca voltage. And since the neutrally it is not present addition of all the three voltages will be equal to 0. Now, what we can do the terminal voltages we can take it together means this three terms I am keeping on left hand side and this all these negative terms I have taking on right hand side. So, we will basically get this equation here.

Now, as I told you these terminal voltages or winding voltages across the winding so basically this V ab voltage, so V aba voltage will be across this winding here V bc voltage will be also across this winding and there will be across this winding V ca on primary side. And secondary side it is V t ab, V t bc and V t ca and this V t ab and V ab they will be just related by your turns ratio means I can say V t ab will be equal to V ab divided by n t; V t bc will be just equal to V bc divided by n t; and V t cad will be equal to V cts divided by n t.

And as I told you and this case if you add the three voltages which are basically line to line voltages, they will be always equal to 0, because there is no zero sequence voltages present there, so they will be just having positive sequence and negative signal voltages. And if you add them together they will addition will be always equal to 0. So, because of that your left hand side is becoming zero and only the right hand side is remaining. So, we have got now third equation. So, 1, 2 and there is this third equation. I just take this three equations on next slide.

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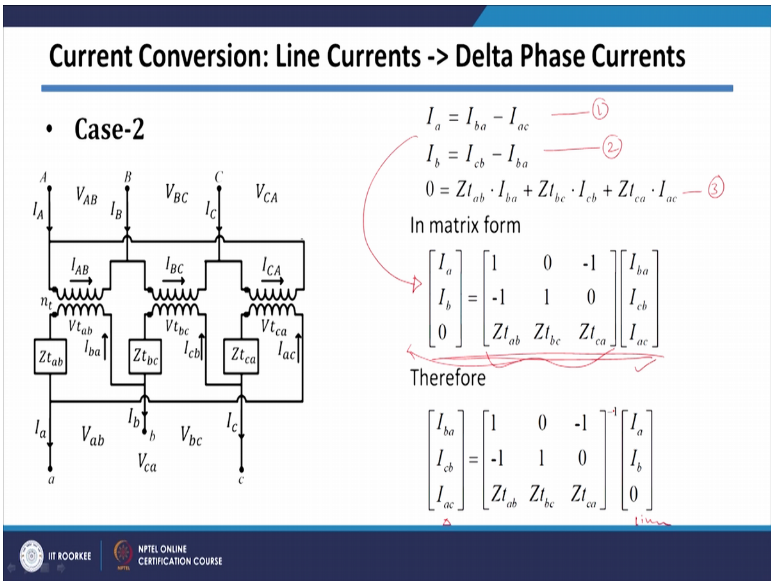

So, let us say this is equation number 1, which I have got; the equation number 2 and then equation number 3 which we have got on last slide I am taking it here. We can put them into matrix form. So, if you put these three equations into matrix form I will get this term here, so this matrix (Refer Time: 21:29) equation.

Then I can take the inverse of this matrix and multiply it to this side, I will get this basically this matrix which is basically delta phase currents. And these are actually line currents. So, we need to take the inverse of this matrix here.

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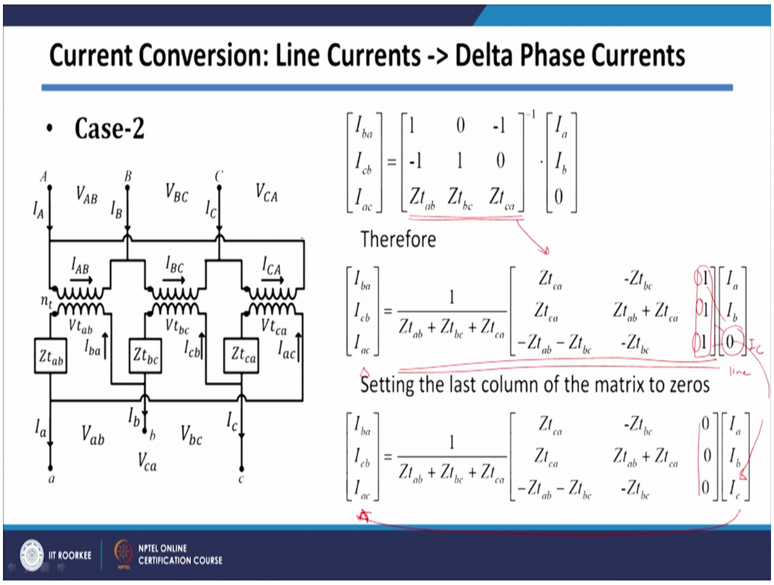

So, to get delta phase currents, I need to take the inverse of this matrix. And inverse of this matrix can be calculated which comes this one. So, this is basically this inverse here. In this case also, we are now getting required currents; here we are getting delta currents and here we are getting line currents. However, this one term I C is missing.

So, in this case also we can use same trick which we are we have used earlier that is I can make this column equal to 0, because this column is getting multiplied with zero every time. So, I can make this column equal to 0 here and I can put I C here. So, anyway I C is getting multiplied by 0 0. So, it will not change anything with respect to equations.

So, instead of 1 1 1, I am putting it 0 here. And instead of 0, I am putting I C. So, again we have got the required matrix here which converts your delta phase currents sorry line currents into your delta phase currents. In case of if there is zero sequence path is available.

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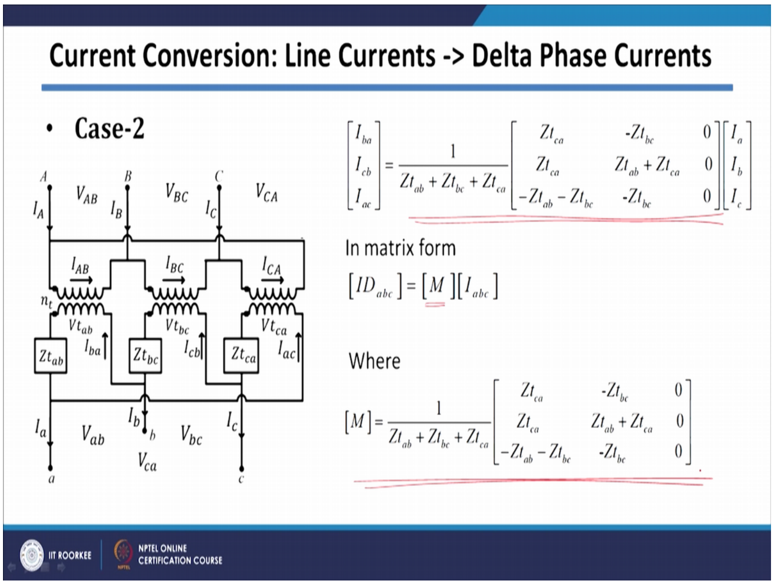

And I am calling this matrix which we have got here I am calling matrix M. And this matrix M is actually basically represented by this one.

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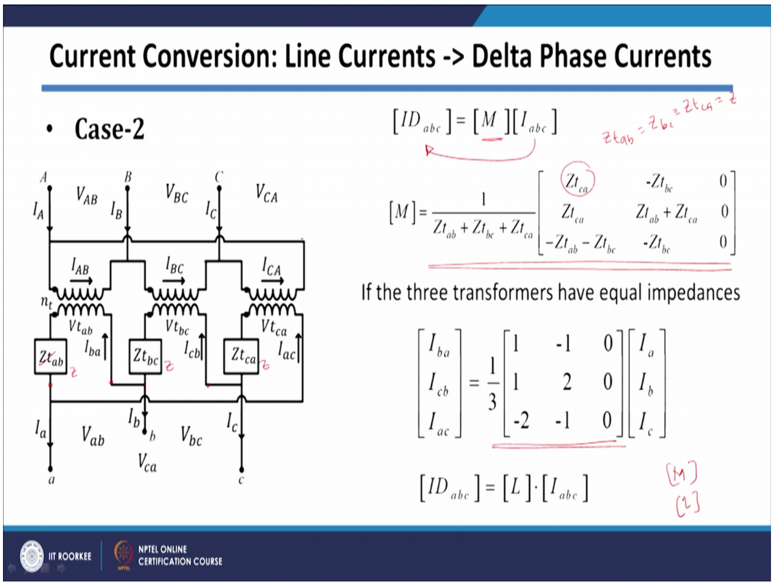

So, we have got required transformations M we have got this M matrix which is basically converts your line currents. So, delta phase currents. We need to use it, because we are always operating on unbalance system. So, they will not be just related by root 3, so there are that is why actually we need to go for this complicated matrices here, because these currents are unbalanced currents even though angels will not be 120 degree displaced with each other as well as the magnitudes will not be same.

In this case, as I told you when impedance says are unequal we are getting unequal drops across the terminal. However, when these impedances are equal say this is just z, this is just z this is just z in that case impedance this voltage drop across each winding it will become equal. So, in that case when these all impedances are equal means Z t ab equal to Z t this is equal to Z t ca ca eqaul to z in that case all these entries will get cancelled out each other; and in that case your matrix will be this one. So, whenever equal impedances are there, this M matrix and L matrix they will be always equal.

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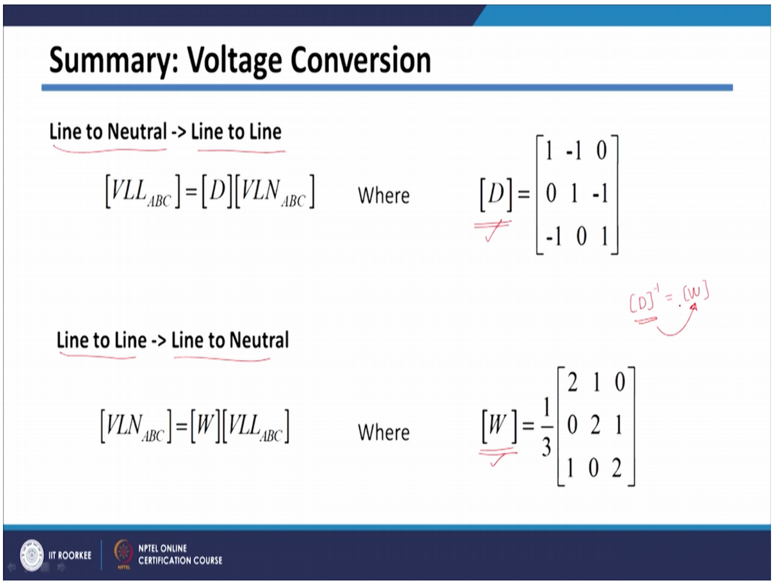

So, this gives me line current to the delta phase current conversion. So, in summary of this lecture or last two lectures, we have seen various conversion matrices which are required, because as I told you or most of the transformer which are in distribution system mostly they will be three-phase transformer and they will be unbalanced and you need to model actual three-phase transformer. So, whenever modeling actual three-phase transformer, we need all these conversions.

So, in summary we have got this W D matrix we have got this W matrix which are basically used for voltage conversions. So, in case of la conversion of line to neutral to line to line voltages D matrix will be helpful; when converting line to line voltages to line to neutral voltages your W matrix will be helpful. And as I told you whenever we are getting d inverse matrix we need to use W matrix even though inverse in is non-existing. So, whenever expression we are getting d inverse we need to replace by W.

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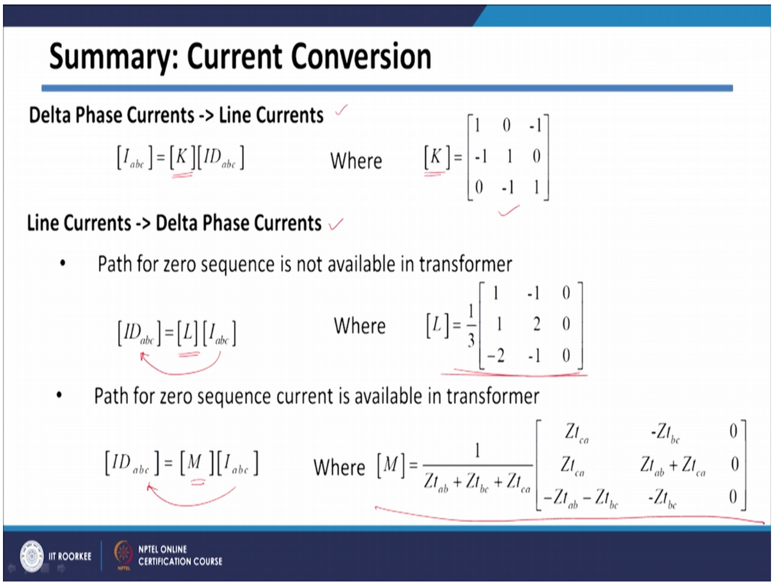

Similarly, we have got for current conversions. And two types of current conversions will required that is we need to convert delta phase current to the line currents or line currents to the delta phase current. So, two types of conversions are required. Converting delta phase current to the line current is easy; and for that we have seen that we can use this K matrix we have derived also

However when we are converting line current to the delta phase current, there are two cases exists. One is whenever there is no path available for zero sequence current and whenever there is path available for zero sequence current, so we have seen that. Whenever path is not available for zero sequence current we have got this conversion matrix, which convert line current to the delta phase currents the conversion matrix is l. And when zero sequence current path is available then your conversion matrix which converts line current to the delta phase current is M and m is given by this particular expression.

However, we have seen that if impedances in the all the three-phases if they are same. Generally, in three-phase transformer this condition becomes true that is imp this impedances are equal approximately equal, then this M and L matrices they are identical or they are same.

Thank you.