Electrical Distribution System Analysis
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Lecture - 13
Modeling of Single-Phase and Three-Phase Transformers

Students, welcome to this lecture number 13; and it is on Modeling of Single-Phase and Three-Phase Transformers. Before going to the transformer, let us see what we have seen in last class and then we will go for transformer. In the last class, we have seen how to model the distribution lines, and we have seen the pi model and short line models of the distribution lines. And we have seen that the structure of this equations of pi model, they are same as single-phase equivalent, which we have which we have use in case of transmission line modeling.

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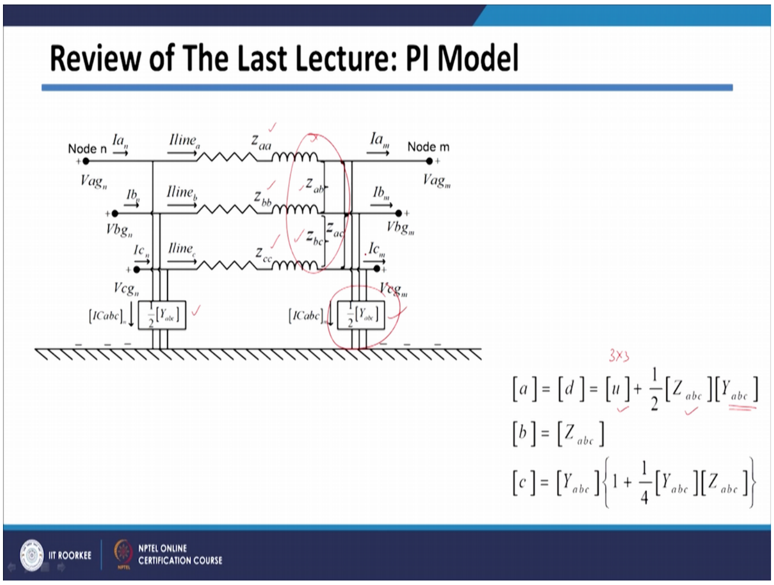

However here since there are all three-phases as well as their shunt admittances of all the three-phases are existing each of these entries they become 3 by 3 matrix; So, here this unity matrix which is actually 3 by 3 unity matrix. Z abc is matrix of 3 by 3, which consists of your all the line impedances and the mutual impedances between the line.

Similarly, admittance matrix is nothing but the 3 3 3 by 3 admittance matrix; and we have seen how to get this admittance matrix also in the last class.

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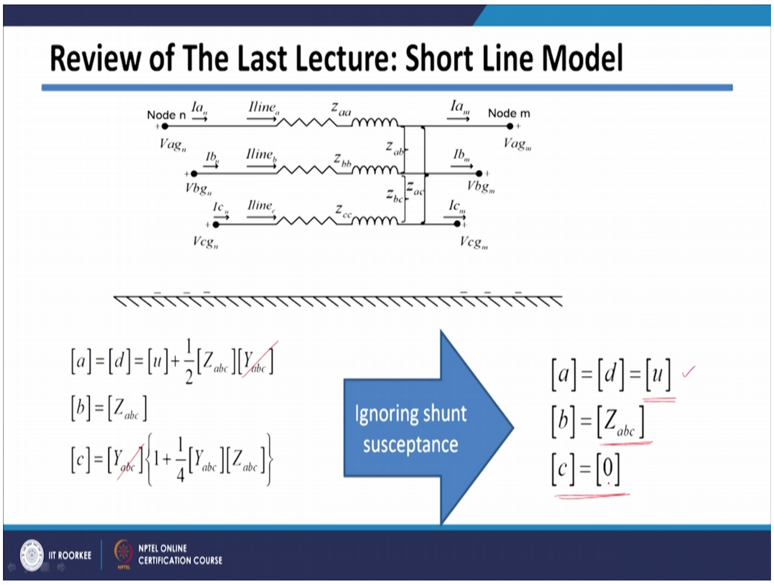

And, in case of short line, we have seen that a, b, c, d mata mater you can get it by neglecting your shunt admittance, in that case, you need to make this admittances to be equal to 0. In that case, your short line model will be just like this, where a and d parameter will be just unity matrix of size 3 by 3; b parameter will be 3 by 3 matrix of your shunt sorry series impedances. And, c matrix will be 3 by 3 matrix of 0’s. And, as I told you, most of the cases we will be using short line model, because length of line is length of distribution line is shorter. And in that case, your shunt admittance will be negligible.

Let us come back to your transformers. We know that transformers are used to step up or step down the voltages. And in case of distribution system, generally it will be step down. So, we know that the sub transmission level voltage, we need to step it down to say initial voltage, that is 11 kV, which we say HT line.

And then, again it is further step down to your LT, that is 440 volt or 400 volt three-phase or 230 volt single-phase. So, for this transformation, we need transformer and transformer is very very important component. And we have seen that generally distribution system need to be analysed in three-phase because most of the cases it is unbalanced and three-phase analysis is required.

So, the connections between the phases or connection between the winding also need to be considered while modelling these transformer. But, before going to the actual three-phase connections, let us see how we can model single-phase transformer. So we already seen this single-phase two-winding transformer equivalent circuit we already seen it. Just I want to model it by converting those equations which you are derived for single phase transformer and want to put them into what is called as your a, b, c, d parameter format.

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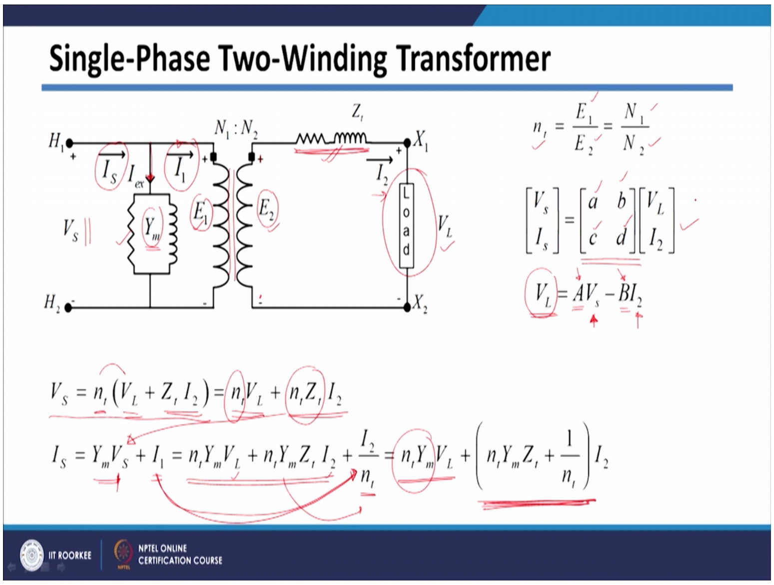

So, to put that let us see this transformer single-phase transformer two-winding transformer. We know that there is magnetizing branch since it is shunt branch; I have shown it using admittance Y m. Then, there is ideal transformer, which is having the internal voltages E 1 and E 2, load voltage V l V l and then load current I 2.

And the leakage impedance of the transformer refer to the secondary side is shown it here. This is applied voltage or source voltage V S, I S primary current. This I 1 is primary current after your excitation current or current to the excitation branch. Then n t, we know that E 1 by E 2. In this case, these are the primary voltage by secondary voltage.

Anyway we can take either way also, but generally, I am considering in step down configuration. I am considering, it will be voltage on primary side divided by voltage on secondary side so again it will be N 1 by N 2 here. And, we are interested in calculating these 4 parameters a, b, c, d small a, b, c, d and then, there are two more parameters I am interested in that is capital A and capital B, because many times the source voltage is known and we are interested in calculating your load voltages. So, in that case, your load voltage need to be written in terms of source voltage and load current.

So, in that case this capital A parameter and capital B parameter will be useful, so we will try to derive the capital A and B capital B also ok, so let us write the equation. So, in this case whatever voltage, which is coming across E 2. If you multiplied by a number of tons, I will get the voltage E 1. And, voltage E 1 and V S they are same, because that same voltage is coming V S, so E 1 and V S they will be same.

The voltage across these terminals, which is E 2, so E 2 will be equal to voltage at load end plus drop, which is happening across this impedance; So, it will be real plus drop, which is happening across these leakage impedance Z t into I 2 multiplied by number of tons will give me E 1 and E 1 will be equal to V S, so we have this equation here.

From the first principle if you take this n t inside the bracket, so it will become n t multiplied by V l and then n t into Z t into I 2. Then if you want to write the equation for I S, so if you see this I S, I S is having two parts. The this excitation current part which will be voltage multiplied by this admittance, which is given by this one and then remaining current which is going into primary winding which is say I 1.

And, I 1 and I 2 they will be related so they will be just n t into I 1 divided by n t so sorry I 2 divided by n t. So, I can just right I 1 is equal to I 2 divided by n t and whatever equation for V S we have got we can put here to dependant this V S.

So if you put this V S equation into this equation, I am getting this 2 terms here. And, I am up to writing I 1 and I 2, they will be related by your transformation ratios. So, I 2 divided by n t will be your I 1 so in that case separating out the terms, which are V L and I l together. So, there is only one term which we related to V S, I am taking this one and there is two terms related to I 2 so if you take them together I will get this term here.

So here, we can easily see that I am writing 2 equations V S, source side voltage and source side current. In terms of, load side voltage and load side current. So basically these parameters, which your getting it here, that is n t, n t into Z t and this n t into Y m and this particular bracketed equation. These are nothing but your a, b, c, d parameter, because you are getting in this in our required form.

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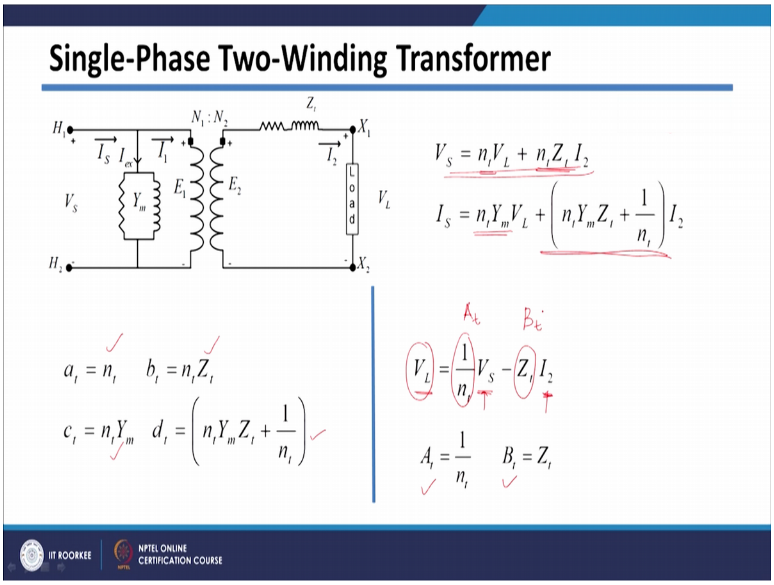

So, once you get into required form. So, this will become your a parameter, that is n t here, this will come your b parameter n t into Z t, n t into Y m will be your c parameter and these bracketed term n t Y m Z t plus 1 by n t is your d parameter.

And as I told you, we are also interested in calculating the second term here, which is capital A parameter and capital B parameter to which relates source voltage and your load currents. Basically, these two terms are known. To get that, you can just write rewrite this equation in terms of V L so in this case so V S divided by n t minus Z t into I 2.

So, basically this will be your required parameter A and this will be your required parameter capital B, which relates your source voltage and load current into the load voltage; Because as I told you many times we need to calculate load voltage where your source voltage and load current basically is known. So, this gives me your capital A. So, since it is for transformer I am writing A t and B t so this is related to the single-phase two-winding transformer.

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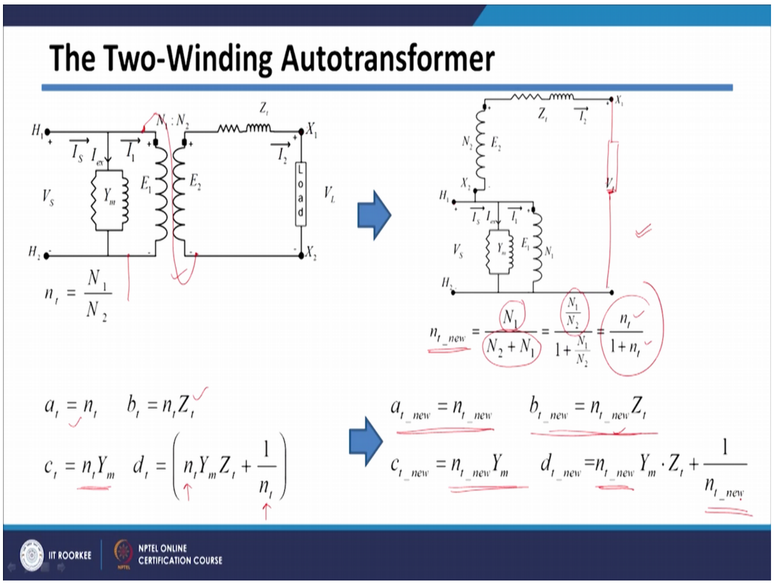

Sometimes, this single-phase two-winding transformers may be having a auto transformer kind of configuration and we can easily derive the a, b, c, d parameters for a auto transformer kind of configuration by knowing the equations for single-phase transformer. So, in this case say let us see, the same suppose this same single-phase transformer is connected like auto transformer configuration, which is shown like this um. Basically, this terminal here is connected, basically connected here, these terminals connected here and then that case configuration will become something like this.

So, in that case your new turns ratio so primary turns will be again N 1, because the same primary voltage is getting applied across this one and secondary terminal we are taking x 1 and load is we are connecting between your x 1 and H 2. So, basically load is getting connected between this terminal and this terminal here.

So, if you are connecting like this, then total turns which are coming across load side will be this N 1 turns plus your N 2 turns. So, in that case, your turns if primary turns are N 1 and secondary turns are actually N 1 plus N 2, so you can just divide denominator and numerator by N 2. So, this will become your turns ratio n t of same transform of having two two-winding configuration so and this will become 1 plus n t.

So, this is my new turns ratio if you connect the same two-winding transformer into auto transformer configuration. Now, you can easily calculate a, b, c, d parameters for this transformer. Just, what you have to do is in this case, it is a parameter is just n t it will become a new a parameter will be n t new, which is basically this term here; b parameter we have seen it is n t into Z t in this case it is n t new into Z t; c parameter is n t into Y m, here in this case n t new into Y m.

And here also, this n t will get replaced by your n t new. So, this is how we can get the auto a, b, c, d parameter for auto transformer kind of configuration, so this is about single-phase transformer. Now, let us go to the three-phase transformers. So, there will be most famous configuration which is grounded by grounded connection.

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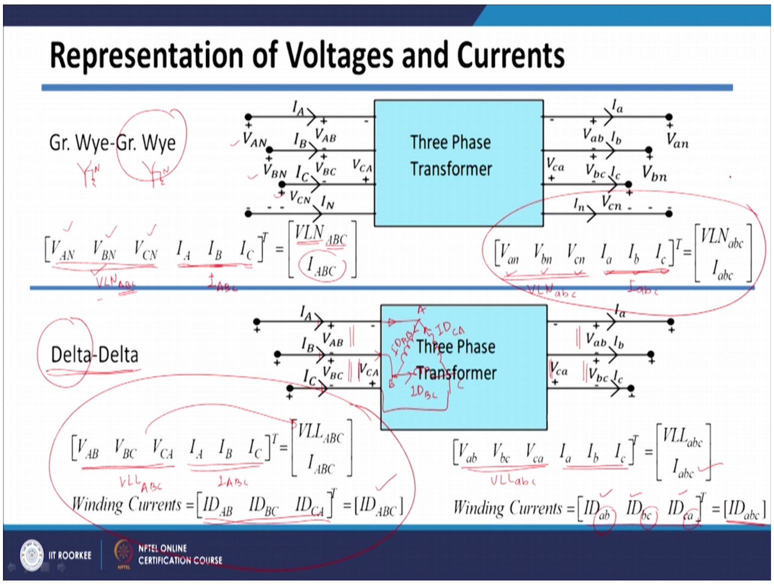

So, Wye-Wye connection which is having both side ground connection. Now, this I am representing it or you can say this neutral is grounded here. Then primary voltages basically phased to ground voltages or phased to neutral voltages they are V AN, V BN and V CN which are represented heat here and then there are line currents I A, I B and I C I written it here.

However, to write them in compact form since, these voltages are line to neutral voltages. I am writing them as all the 3 voltages as voltages line to neutral and all the 3 means a, b, c so that is why this three terms, I am just writing VLN a b c and capital terms I am using, because they are primary side and all these 3 currents I am writing I a b c, so this basically this all the 3 current represent your I a b c.

Similarly, on your secondary side the again it is since it is ground connection the voltages I can write them in terms with respective neutral so it they will be V AN, V BN, V CN. I A, I B, I C will be line current and collectively. I can write this will line to neutral voltages and for all the 3 pages so I am writing V abc.

So, these three times will be written as VLN abc and this 3 currents will be just I ABC. So, this is actually representation of voltages and currents if there is star connections on primary and secondary side or may be primary side or secondary side. Let us say there is delta connection for this case I am considering delta delta winding. So, there will be no neutral.

So, in that case we can represent them using line to line voltages. So, there will be V AB, V BC and V CA and then there will be line currents I A, I B and I C and secondary side also it is since it is delta there will be V ab, V bc and V ca voltages. So, these three voltages line to line voltages and your line currents now this line to line voltages I am writing VLL earlier I have written VLN so in this case VLL.

So, all the three voltages are VLL capital ABC, since they are line currents so this will be just I ABC and here also, this is VLL small abc all the three voltages and then this is your I abc line currents. Since, this is delta-delta connected kind of transformer. So, there will be delta currents means, these currents will be get divide into delta windings which will be like this. So, say this phase connected here and then this C phase is connected here

So, let us say the delta currents are flowing like this. So, this will be is your ID AB, this will be your ID BC current and this will be your ID CA. So, these currents are 3 currents are like this and collectively these 3 currents are written as ID ABC basically, they represent your delta currents. Similarly on the secondary side, ID abc they are basically 3 delta current so which are represented by these 3 terms and this small letters ab, bc and ca. As I told you they represent your as secondary side and capital letter represent your primary side.

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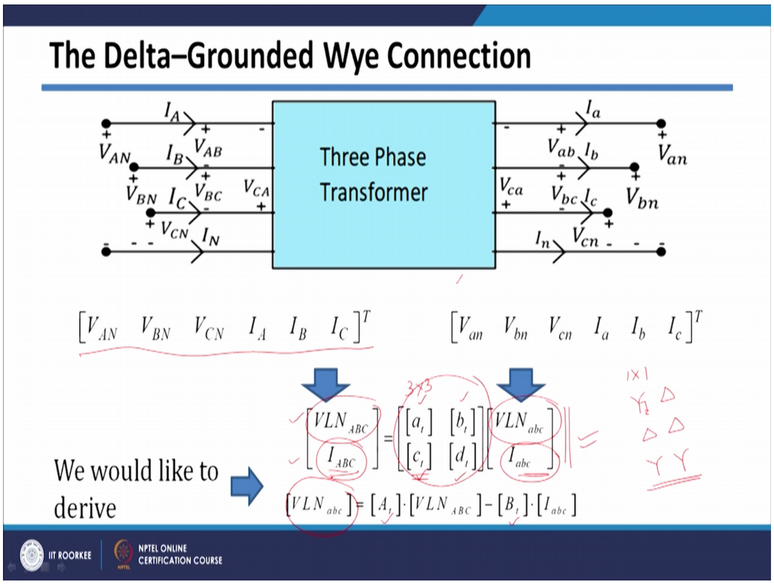

So, this is actually represented, representation of various voltages and currents in star connections and delta connections. The winding is a delta and grounded Wye. So, some transformer says primary side is delta and secondary side is grounded Wye. Then your terms will be, these terms will be on primary side and these terms will be on the secondary side so this is actually representation.

Now, in this case in three-phase transformer also, we are interested in calculating these a, b, c, d parameter. So, as I told you once you know a, b, c, d parameter, you can write them in any terms or it is easy to get whatever quantity, which you want either any side means source side or load sides quantities can be easily calculated, if you know the a, b, c, d parameters of your component.

So, in this case also we need to put this three-phase transformer into a, b, c, d parameter format. So, in this case, since in single-phase we have seen that, there is one by one entries. However, in three-phase transformer, these all these a, b, c, d parameter they will be 3 by 3 size matrices.

So, these line to neutral voltages and line to currents of primary side and line to neutral voltages and line current on secondary side are actually linked by your a, b, c, d parameter. So, all the for all the configurations like we know that there are many configurations, so there is there might be grounded Wye delta transformer or delta-delta transformer or just Wye Wye transformer so there are many configurations.

So, for each of the configuration, we need to bring those equation into this format. So, that we can use them in any analysis like short secret or load flow analysis or state estimation; And one more thing is, that we also find out this capital A t and capital B t matrices, because many times we need to know load voltages, in terms of source voltages and load currents.

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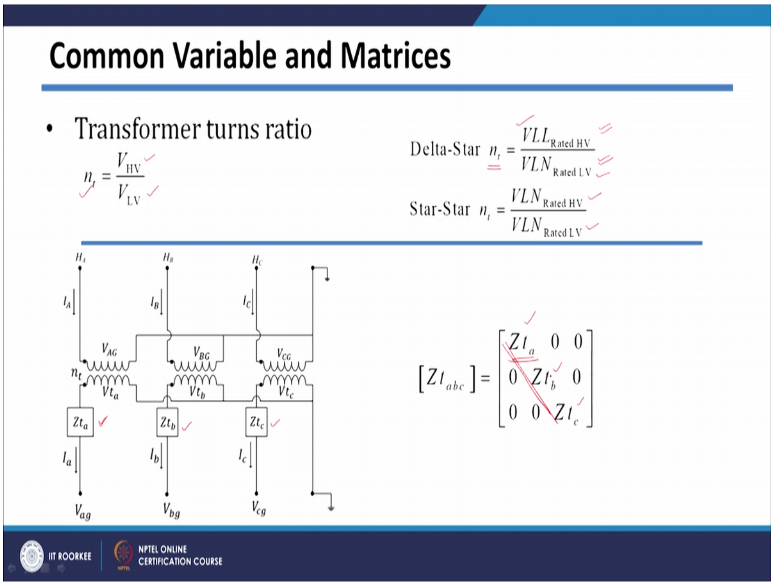

Let us see some terms, which are related to different derivations for different configurations. Before going to this configuration, let define some terms so here I defining turns ratio so turns ratio is defined. We are defining or we are using the terminology that voltage rated voltage on HV side divided by rated voltage on LV side, I am calling as a turns ratio.

So, in case of say delta- star transformer, the voltage across primary winding will be line to line voltage and voltage across secondary winding will be line to line neutral voltage. So, while calculating n t for delta- star configuration, we need to take the ratio of line to line voltage on HV side divided by line to neutral voltage on LV side. However if you see star-star connection, we can take line to neutral voltage ratio on HV side and line to neutral voltage on LV side. So, this is your turns ratio we define n t.

Then in all the configurations I am referring, these leakage impedance of your transformer on secondary side; So, I am taking on the secondary side or voltage side and connecting it together and the matrix of it is actually Zt a, Zt b and Zt c are the diagonal entries, which are corresponding to a phase, b phase and c phase. If it is delta connection, this impedance may be between 2 phases. In that case, I can say Zt ab, Zt bc and Zt ca. So, wherever particular configuration is coming I will explain that. So, these are the 3 impedances of 3 transformers in three phases.

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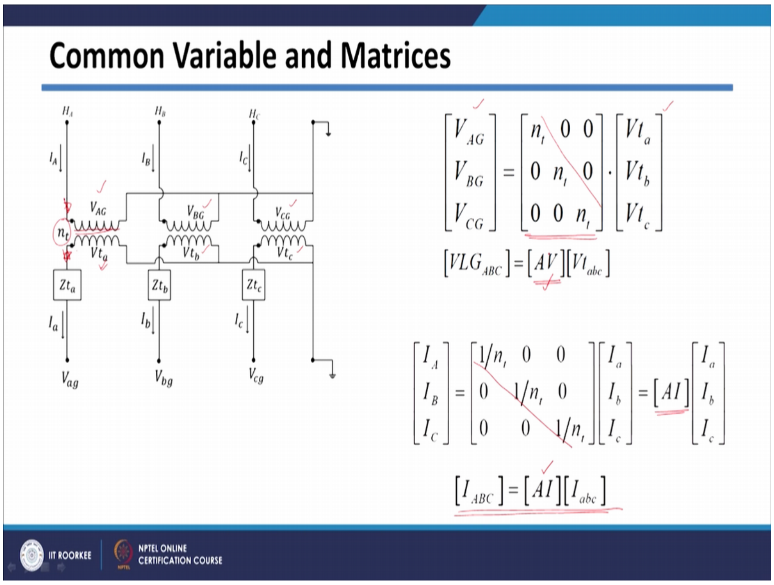

Then, the winding voltage ratio so, basically the voltages or cross the windings or always related by your turns ratio. So, the voltage V AG here and Vt a they will be always related by your turns ratio.

Similarly, V BG and Vt b and V CG by and Vt c, they will be related by turns ratio. So, this turns ratio will be again 3 by 3 matrix, which is having configuration like this, where n t is your diagonal entry and this particular matrix I am calling matrix AV voltage ratio which basically let terminal voltage of secondary to the primary voltages. Basically, this is related to voltages across the winding.

Similarly, winding currents we will also be related to turns ratio means current, which is going inside winding and then which is going an opposite I can see that. So current, which is going in and which going out of the winding, they will be also related by turns ratio. However, it is in opposite direction. So, primary current will be just 1 by t 1 by n t times I a. So, in that case we are defining this matrix AI, which is which are which is having diagonal entries 1 by n t and in that case your equation will be like this.

So, this AI and AV basically we can call them as a transformation matrix, which basically transforms the voltages oh and current. So AV, I am using for transforming the voltages, terminal voltages and AIM I am using for transforming winding currents. Now many times in all those configuration, what we need to do is in case of voltages many times, we need to control, we need to convert line to neutral voltages to line to line voltages and vice versa means line to line voltages to line to neutral voltages.

So, those transformation matrix also need to be known. It is since it is not balance system. They will not be just related by root 3 so that is why in this case we need to get those matrices.

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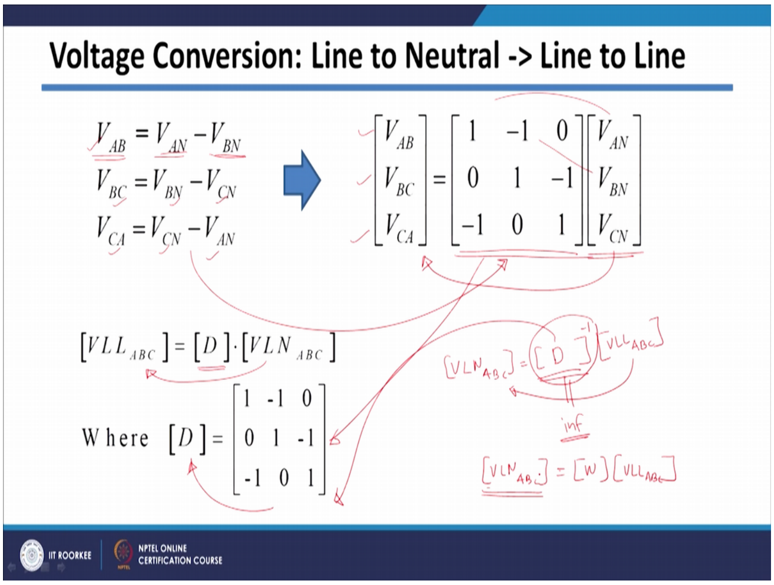

Since, it is we are considering totally unbalanced kind of system, which is having all the types of current for positive sequence 0 sequence negative sequence. So, let us see first conversion that is line to neutral to line to line voltages. So, if you want to convert line to neutral to line to line voltages so we know that line to line voltages V AB. We can write as V AN minus V BN. Similarly, V BC V BN minus V CN, V CA will be V CN minus V AN.

And if you write them in matrix format on this side, it will be all these line voltages and on this side there are line to neutral voltages. So, these line to neutral voltages, we want to convert into line to line voltages. And in that case your conversion matrix will be this one. So, basically all these 3 equations, I have just written into matrix form means, V AB will be just V AN V AN minus V BN like this.

So, as I told you, your line to neutral voltages will be converted in to line to line voltage using this D matrix, which I have calculated at here. Now, suppose you want to calculate line to neutral voltages and line to line voltages are known. Means, I want to calculate VLN ABC and then it will be equal to some matrix here, multiplied by VLL ABC.

So, I want to convert line to line voltages to the line to neural voltages. So, actually you may thing that, A it should be D inverse so if I have put D inverse, it will automatically convert. However, this D inverse if you see, it is non existence and if you take the inverse of this d matrix, it actually comes infinite because it is singular matrix. So, we cannot do that way because in this D inverse is non existence which will come infinite.

So, in that case we need to find out how we can convert means, what will be this matrix here. I am just I am calling this as a W matrix and we want to find out, what will be this W matrix, which converts line to line voltages, unbalanced line to line voltages to line to neutral, unbalanced line to neutral voltages. So, to convert your line to line voltages to line to neutral voltages we have seen this W matrix.

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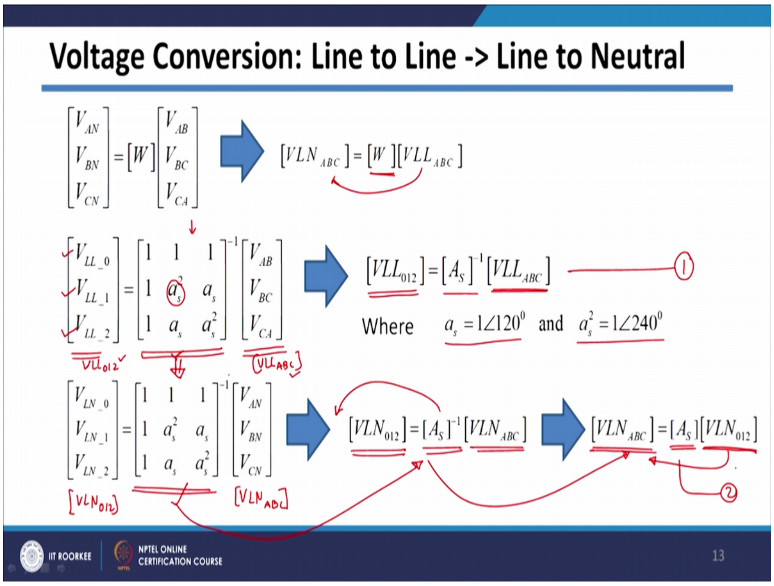

However, we want to get this W matrix here. So, we can get this W by matrix from this following relations. So, we you have to use sequence component theory to give get this

W matrix. So, we know that according to sequence component theory, we can convert these phase quantities into sequence quantities using this transformation matrix here. So, phase voltages V AB, V BC and V CA will be converted into VLL because these are line to line voltages. They will be converted into line to line 0 sequence voltage, line to line positive sequence voltage and line to line negative sequence voltage.

And since these are nothing but, line to line voltages. I can in short form. I can write VLL 012 and this will be VLL ABC. So, these are the sequence components, these are nothing but phase components of the voltages and this is your transformation matrix. So, in short form VLL 012 will be nothing but A S inverse, where this is your A S matrix multiplied by VLL ABC so this is nothing but phase domain voltages. And, we know that this A S in transformation matrix is given by 1 angle 120 degree and then that case you A S inverse will be 1 angle 240 degree.

Similarly, we can get sequence components of line to neutral voltage, so your this line to neutral voltages in phase domain will be written like this so these are the line to neutral voltages in phase domain. These are the line to neutral voltages in sequence domain so 012. And these are nothing but your convergent matrix, which is given by again it will be same matrix here. So, I can write these equation into short format, which is VLN 012 will be equal to A S inverse this is your an as inverse matrix and VLN ABC so this expression in short form I can write it like this.

We can actually again write this equation like by taking this A S inverse on this side. So, in this case VLN ABC face quantities will be nothing but, A S matrix multiplied by VLN 012 so here, we are converting your sequence quantities into phase quantities. So, we have got this these two relations here, I can say this is relation number 1, this is a equation number 2.

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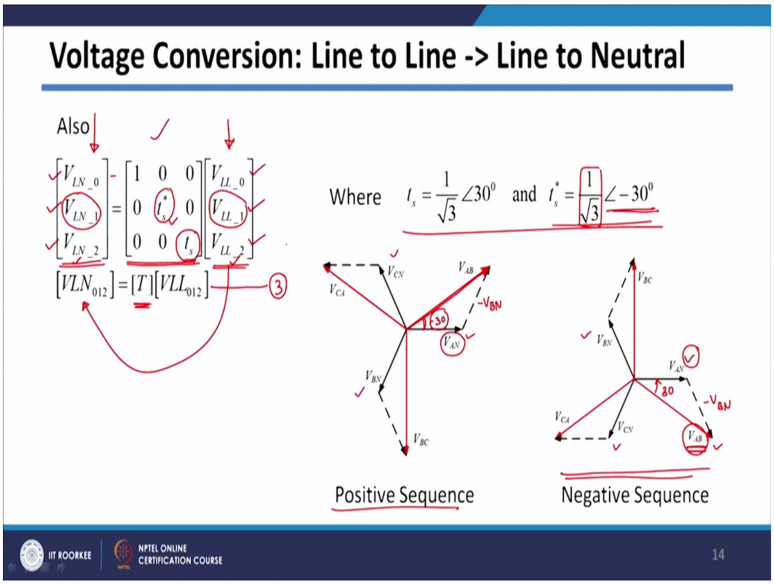

And third equation, we can get it from relation between line to line sequence component and line to neutral sequence voltages. So, this is nothing but line to line sequence voltages and line to neutral sequence voltages. So, this is line to neutral 0 sequence voltage, line to neutral positive sequence voltage, line to neutral negative sequence voltage.

Similarly, this is line to line 0 sequence voltage, line to line positive sequence and line to line negative sequence. And these are related by this transformation matrix, which I am calling T. So, if you see these entries here, 0 sequence components in line to neutral as well line to line they are same.

If you see line to neutral voltage and line to line voltage in positive sequence, they are related by this t s star, which is represented by this and this can be easily understood by this positive sequence figure here. So, you can see that in positive sequence this is your V AN, V BN, V CN so it is our positive sequence.

And if you calculate line to line quantities so V AB will be nothing but, V AN minus V BN. So, this will be the nothing but minus V BN, which will be write it like this. And so if you add them together, it will be nothing but V AB here and if you see, the phase angle difference and magnitude difference between these 2 quantities.

So, in positive sequence your line to neutral quantities will be 1 by root 3 times of line to line quantities and phase angle will be lagging with respect to line to line quantities. So, that why here we are getting t s star, which indicate the line to neutral quantities are lagging behind line to line quantities by 30 degree.

Similar way, in case of negative sequence, you can see that V AN here, V BN here, because it is rotating in opposite direction and V CN is here. So, in this case if we calculate V AB, which is basically V AN minus V BN, so this is minus V BN here. And if you add them together, it will be your V AB.

So, in this case in case of the negative sequence, you can see your line to line quantities are lagging behind your line to neutral quantities or we can say line to neutral quantities are leading with angle 30 degree with respect to line to line quantities, so that is why to represent this leading 30 angle of line to neutral quantities with respect to line to line quantities, we represented by t s.

So, transformation matrix, which convert your line to line sequence quantities to the line to neutral sequence quantity quantities is given by this matrix T here. And this matrix T is nothing but this matrix here, whose t s and t s star is given by these relations. And I am calling this relation as expression number 3. And these three relations to on last slide; and this one I am taking on this slide.

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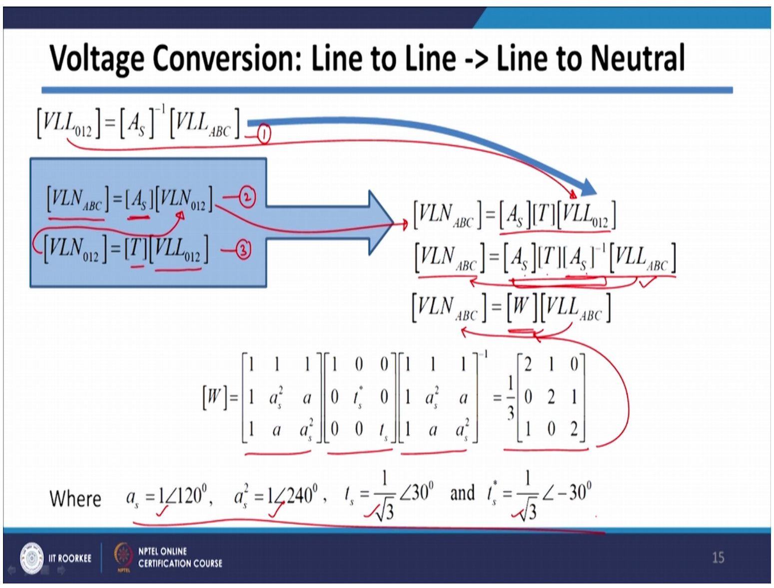

So, this is your expression 1, this was your expression 2 and this was you expression 3 which we have got on last two slides. And from, these two relations by putting this VLN 012 here, and writing this expression here, I can get VLN ABC will be equal to A S multiplied by T multiplied by VLL 012, so A S multiplied by T multiplied by 012. Then, this VLL 012, I can put here from this expression number 1. And in that case it will be A S inverse multiplied by V line to line ABC, so VL ABC.

So, here we get line to line quantities phase quantities on right hand side; and line to neutral phase quantities on left hand side. So, we have converted this these line to line quantities into line to neutral quantities using this matrix transformation here and that is represented by W. So, W will be nothing but A S multiplied by T multiplied by A S inverse. And if you explicitly calculate that A S multiplied by T multiplied by A S inverse, you will get this matrix.

So, this W matrix we will be given by this expression here which converts to line to line voltages to the line to neutral voltages where a s, A S square, t s and t s star are given by this expression.

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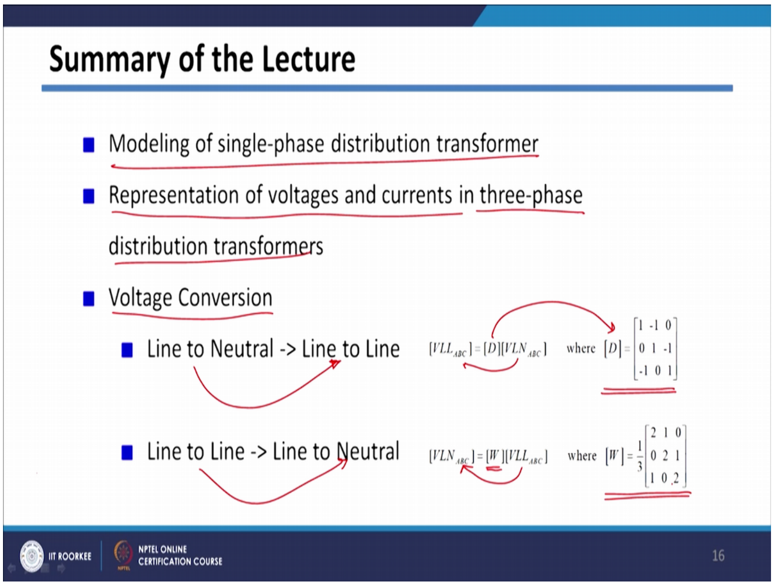

So, in summary of today's lecture, we have seen modeling of single-phase distribution transformer, then we have seen how we represent voltages and currents during modeling of three-phase distribution transformers. And for that, we need various convergence, two convergence in case of voltage we have seen that is line to neutral voltages to the line to line voltages and line to line voltages to the line to neutral voltages.

And we have seen that in case of line to neutral voltages to the line to line voltages, we have this matrix D and which you derived D matrix is given by this expression here. And to convert line to line voltages to the line to neutral voltages, we have this W matrix; and W matrix will be given by this expression here. In the next class, we will see current conversion and few transformer models.

Thank you.