Electrical Distribution System Analysis
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Lecture – 12
Models of Distribution Lines and Cables

Students, we have seen how to calculate impedance of the distribution lines in the last class. We have seen that initially we have calculated the primitive impedance matrix where, we have calculated self inductance of the conductor and mutual inductances between pairs of the conductor. And we have seen that the size of this primitive impedance matrix is n by n if there are n number of conductors in your system. And then we have seen the Kron's reduction where we reduce this n by n system of equations or system of matrix into 3 by 3 matrix which is we call it as phase impedance matrix.

So, I will just summarize before going to the Models of Distribution Lines and Cables lecture, let us see what we have seen in the last class. What we did we have seen the equation for transpose line which is actually only one impedance and we are already studied in our B Tech class basically this equation.

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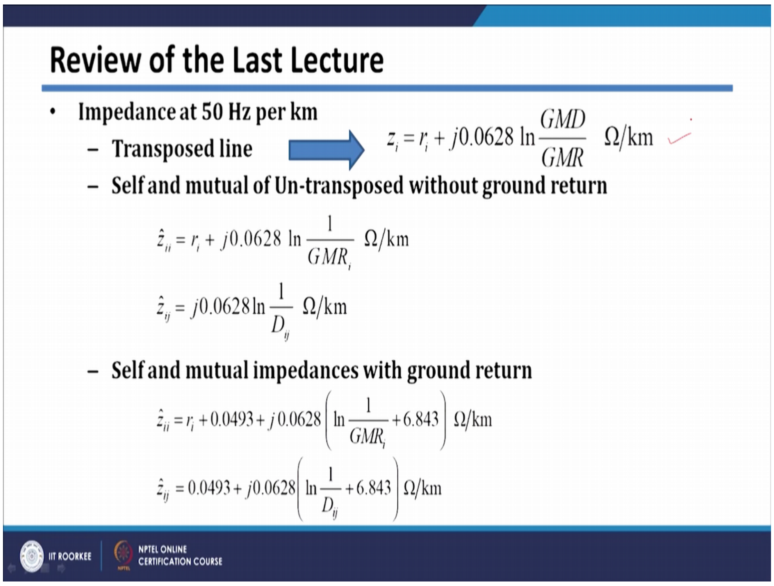

However, in case of un transpose line basically we have seen that in case of distribution system lines are untransposed; because they are short length lines. And if you are considering un transpose line and if their balance means if there is no ground return current if you are considering for calculation, then your equations we have seen that which are given by this Z ii and Z ij where the ground return term is not there.

However if you want to consider ground return path also into in your calculation, then you need to consider the equation which are given by Carson's and we have tried to derive these equations and the terms which are actually given by Carson's ok. So, basically here I added these particular terms, which are getting the effect of your ground impedance that is ground resistance as well as ground inductance into account.

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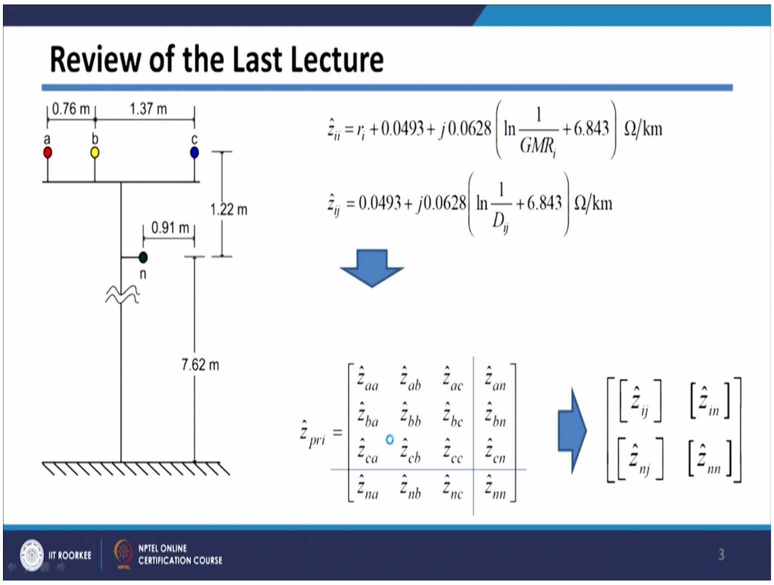

And then using these equations we have seen that how to calculate your primitive impedance matrix by knowing the GMRs; GMRs of various conductors and distance between the various conductors and using this Carson's equation, we have seen that we can get primitive impedance matrix. And this primitive impedance matrix as I told you it will be number of conductor by number of conductor size and if you apply this for the figure which is in the right hand side.

Since there are 4 number of conductors there are 3 phase conductor abc, and one neutral conductor 4 conductors are there. So, 4 conductor your primitive impedance matrix size will be 4 by 4, out of these 3 columns will be for your phase conductor and this fourth column is for your neutral conductor.

Similarly, this row is for your neutral conductor and in your calculations like load flow calculations or short circuit calculations we do not want to take these neutral conductors into account. So, we want to eliminate them; and for eliminating these ground conductors we have seen that we can use what is called as Kron reduction.

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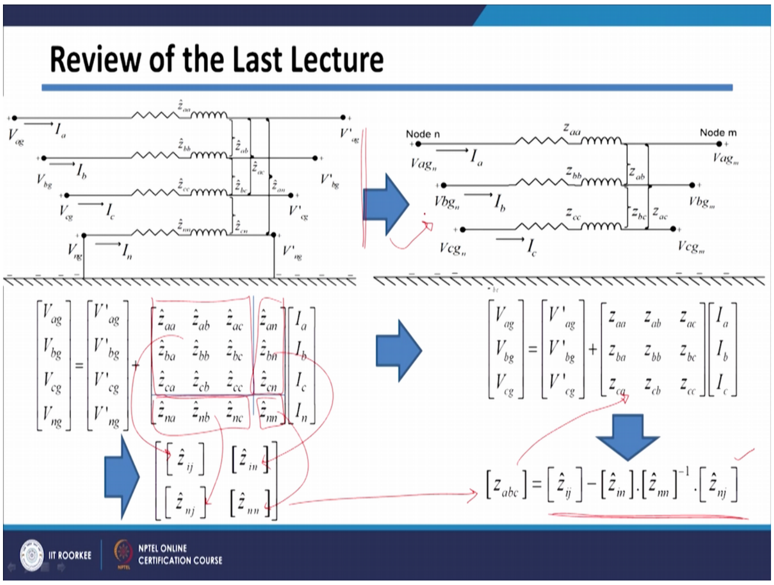

And Kron reduction we have derived one equation which gives.so, first we have seen we have to separate out terms which are related to phase quantities or phase conductors.

So, these are actually phase conductor which are abc, and all the columns and rows which are related to ground conductors they are also separated out ok. Basically this matrix we called it a Z ij this matrix we call it as Z in, this matrix we call Z nj, and this remaining is one term it is Z nn and we have seen that we can apply Kron's reduction which will basically give your 3 by 3 or phase conductor by phase conductor matrix, which is which can be calculated by using this components and we have derived this Kron's reduction technique.

And once you get this you can put this Z abc into account, and in that case your neutral conductor or earth conductors will not come into picture. So, basically what we are doing; we are converting this system of number of conductors to only just 3 phase conductors by eliminating your earth as well as ground conductors. So, this we have seen in the last class. Now let us come to the shunt admittance of the distribution lines. So, till now we have calculated series impedance, now we will calculate shunt admittance.

However, I would like to tell here the shunt admittance of distribution lines is very small and many times it is neglected. However, if the feeder length or distribution line is longer we need to consider shunt admittance for accurate calculations. So, let us see how to calculate it. So, depending upon length of your feeder you can consider or not, that depends upon how much accuracy of calculation you want.

So, before going to the shunt admittance, let us say how to get this capacitance matrix because generally this conductance is neglected because it is very very small value. So, in the all the calculation conductors are neglected. So, admittance will consist of only the capacitive part here. So, let us see how to calculate capacitors or how to get the capacitance matrix for untransposed lines.

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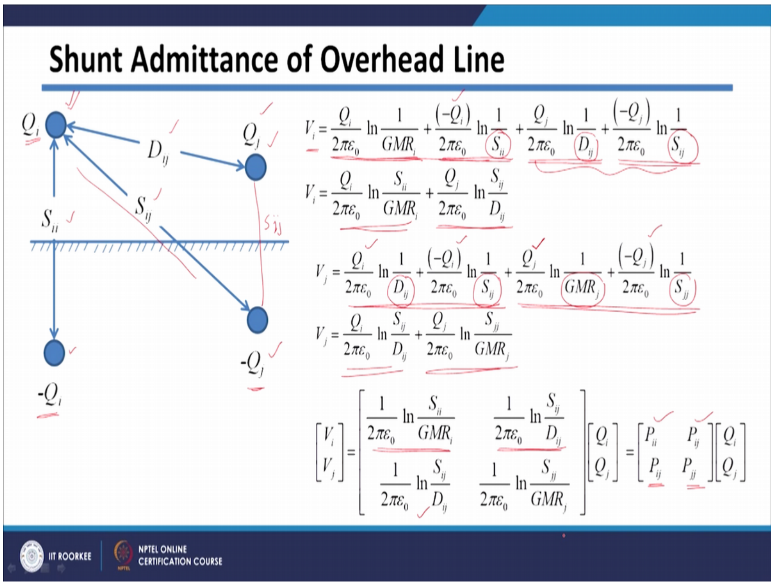

So, here I have consider say 2 conductors say conductor i and this conductor j, and we know that for calculation of the capacitor we need to take ground effect into account and for that we have seen that, we need to use a method of images. So, what we do in method of images, we take the image of the conductor with negative charge. So, image of conductor having charge Qi we have taken this image which is having charge minus Qi.

Similarly, this Qj is having charge minus Qj here and then I shown you the various distances. Now if you write the equation for voltages on this conductor; so, voltages will be due to all 4 charges; conductor charges as well as image charges of the conductor. So, voltage at conductor i will be due to all the 4 charges and it will depend upon distance between those charges.

So, the voltage of conductor i due to its own charge will be given by this term then voltage due to image its own image will be given by this term. Since the distance between image on this conductor is S ii there will be S ii here and charge is minus Qi. Similarly voltage at conductor i due to charge Q j will be given by this term and here again the distance is Dij between the conductor i and conductor j; and this is voltage at conductor i due to image of j means this particular charge, and in this case your distance is Sij. So, I need to put Sij here.

Now if I can take these 2 logarithmics term together, I will get this term here and if I take this 2 logarithmic term together I will get this term here. Exactly similar way I can calculate voltage at conductor j it will be again due to all the 4 charges those are Qi conductor Qi its image, then charge on conductor Qj and its image and corresponding distances this is distance of j with respect to i, this is distance of j with respect to image of i, this is GMR of own conductor because it this j this voltage is due to its own charge, and then Sjj is voltage due to its own image.

So, distance between this will be S jj here. Again if you take logarithmics term together the addition of these 2 terms into logarithmic terms together, you will get this term here and if you take these 2 terms together logarithm terms you will get this term here. Basically in short form I can write this equation; so in the matrix form I can put this equation like this where this term I am calling as a P ii which I have written it here this term I am calling P ij this term and this term is same.

So, this is again it will become P ij and this is Pjj and this P coefficients are called as a potential coefficients of the systems. And it is easy to get potential coefficients of the system because it just depends upon your distance between the conductor and GMRs of the conductor and the permittivity of your or you can say free space that is epsilon 0, because permittivity of arn is free space it almost equal.

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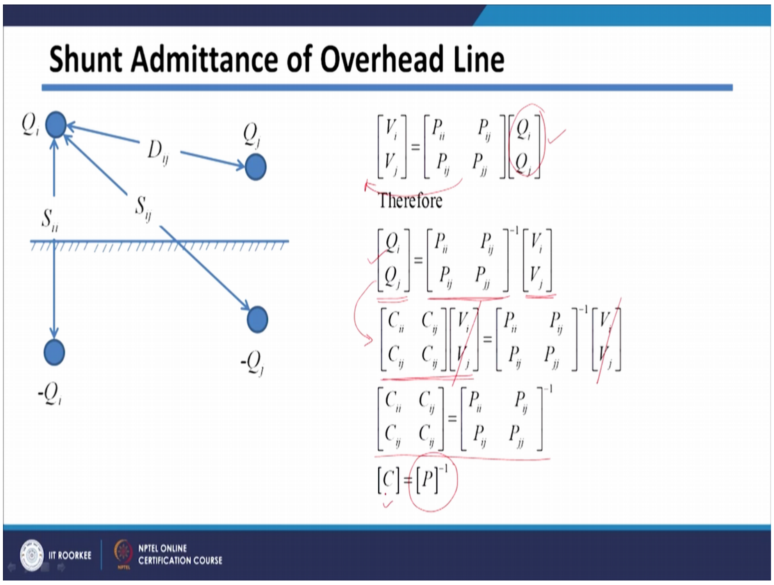

So, in that case we will get this matrix which is called as potential coefficient matrix, and once you get this matrix; you can see that the same matrix I have taken it here, then I can write this Qi Qj term will be just taking an inverse of this and multiplying to the Vij. So, it will Qi Qj will be equal to this matrix of potential coefficient multiplied by V i Vj. Then we know that actually these charges I can write in terms of capacitor and voltages because we know that Q is equal to Cv in this case in there are 2 charges the matrix will be 2 by 2.

So, capacitor and matrix 2 by 2 matrix. So, if after writing Q is equal to Cv you will get this term here. So, instead of Q I am writing Cv here. And in this case if you see the voltages on both the sides will get canceled out these same matrix will get cancel out it with each other. So, because of that I can say, your matrix of capacitance will be just inverse of matrix of potential coefficients and we have seen how to calculate the potential coefficient matrix here.

So, just get the potential coefficient matrix inverse of that matrix will give me matrix of capacitance. Now we have seen for only 2 conductors if there are n number of conductors into the system, and in that case your potential coefficient matrix will be n by n size.

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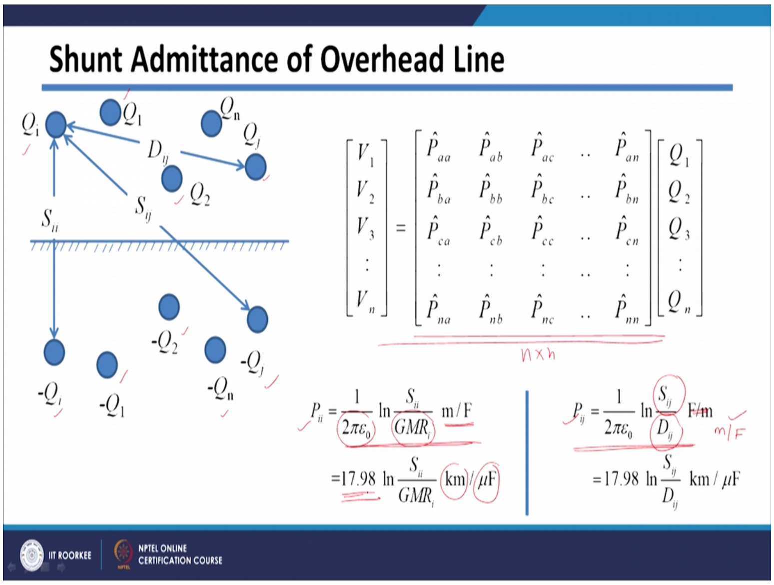

So, in this figure if you see I have shown n number of conductors that is Q 1, Q 2, Qi, Qj and each of them will be having image. So, Q 1 will be having image of minus Q 1 minus Q 2 minus Qn minus Q I minus Qj ok.

So, all will be having images and for this using same philosophy which we have applied earlier we can get this matrix equation, where again this is potential coefficient matrix whose size will be actually n by n; n by n is actually number of conductors by number of conductors. And each of the term into these potential coefficients, we can get it by this equation, the diagonal terms of the potential equations depends upon GMR of the particular conductor; if the same sized conductors are there GMR will be same. And then these non diagonal terms of this perturbation coefficient matrix will be given by this equation; where we have seen that they depends upon actual distance between the conductor and corresponding the image distance.

And once we get this, we can get these potential coefficients and basically this will get in terms of meter per farad here is a mistake, it is meter per farad. Once you get that we can convert them into kilometer per micro farad. So, you need to if you convert them into kilometer and per micro farad; I will get this term 17.98 here; what you have to do is you have to multiply it by 10 raised to divided by 1000 and again divided by 10 raised 2 minus 6.

So, and this is actually 2 pi epsilon 0 here. So, if you calculate that it will come 17.98. So, these equations will give me your potential coefficient value. Now as I told you; you are getting the potential coefficient value now, but it will be n by n matrix and for other analysis purpose as we have done in case of impedance calculation, we need to eliminate the ground conductors. So, in this case also we can eliminate the crown conductor using Kron's reduction techniques which you have seen earlier.

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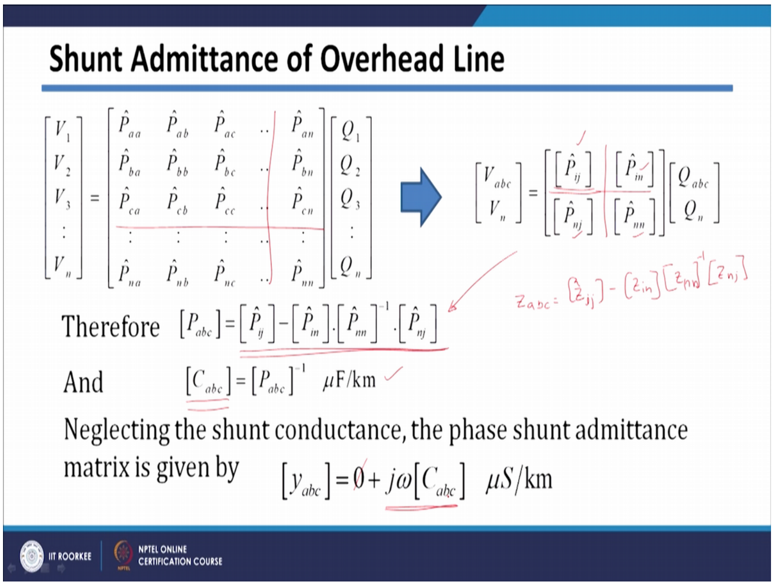

So, in this case also we can divide this matrix by knowing the ground conductor and phase conductor we can separate this matrix into 4 parts. So, these are the 4 parts of matrix and here we know that this P ij's are the coefficient which are related to phase conductors and in this case these all these matrices which basically belongs to the mutual potential coefficient and self potential coefficients of between your phase conductor and neutral conductors.

So, in this case also we can apply Kron's reduction technique exactly similar way like we applied in case of impedance calculation. So, in this case also it will become this equation here, in this in case of impedance we have seen that Z abc will be equal to Z ij minus Z in into Z nn s inverse and Z nj multiply exactly same we can write in terms of potential coefficients here. And we have seen that the capacitance matrix is just inverse of your potential coefficient matrix.

So, we can easily get your capacitance matrix which will be required in the calculation and then once you get the capacitance matrix neglecting shunt conductance, we can just multiplied by j omega. So, j omega into capacitance matrix will give me shunt admittance matrix.

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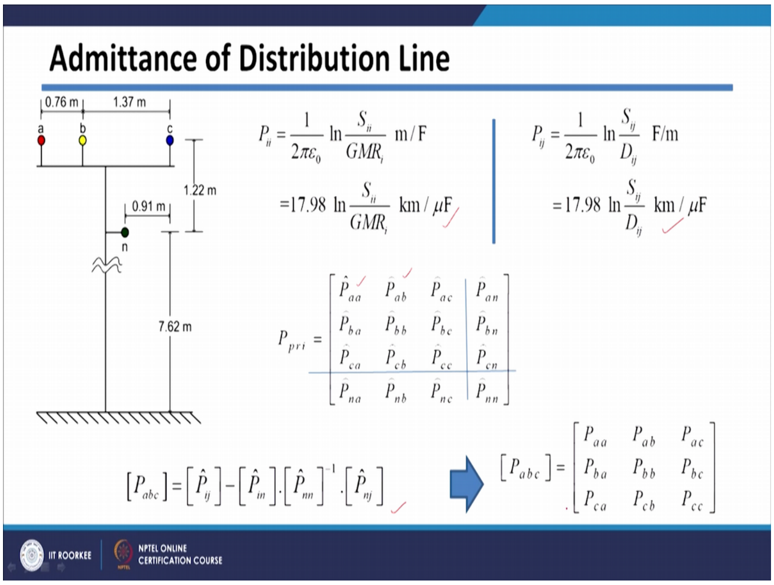

So, let us say again for that example, which you have considered for impedance calculation how can we calculate the potential coefficient matrix.

So, in this case again there are 4 number of conductors, and each potential coefficient will be calculated by using these equations here. So, each of these entry of primitive potential matrix will be calculated using the 2 equations. And once you get this again you can use Kron's reduction to get the phase potential coefficient matrix.

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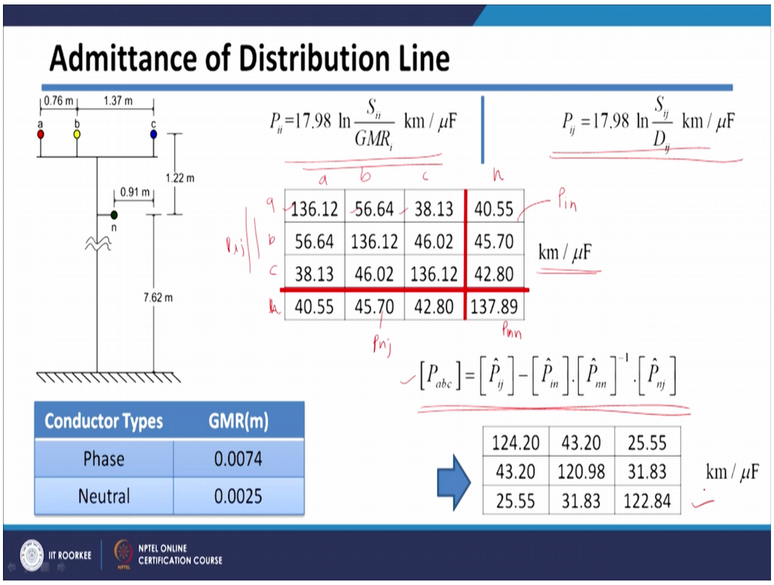

Let us see how we get it. So, by using this equation I can get these potential coefficients, I have calculated for these cases and as you are seen that the unit of these potential coefficient matrix is kilometer per micro farad and then we can use Kron's reduction.

So, if we apply Kron's reduction by dividing into 4. So, this these 3 rows and 3 columns are related to phase conductor and this fourth column is related to your ground wire here so, n. So, we divided into 4 parts; so, this is actually your P ij. So, this part is P ij this matrix is P in this matrix is P nj and this is your P nn and using this metric operation we can get phases phase potential coefficient matrix which is given by this.

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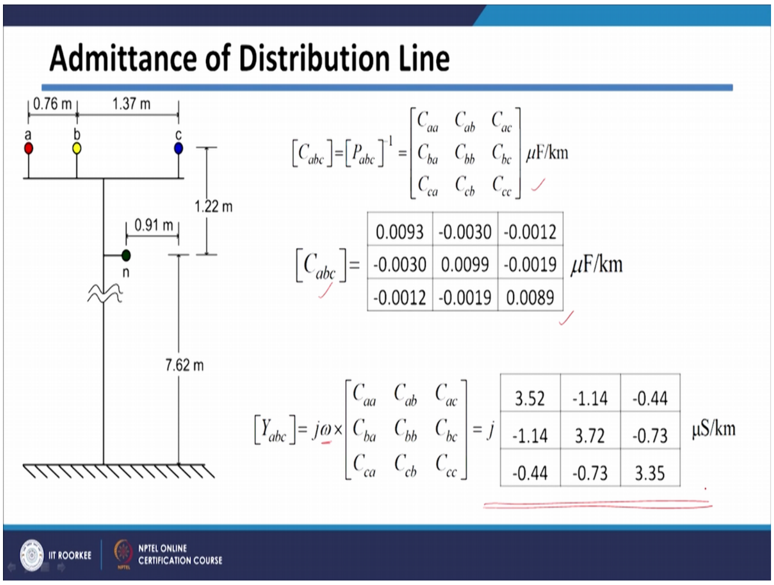

And then you can after getting at the inverse of this potential coefficient matrix is nothing, but capacitance matrix.

So, I just got the inverse of this matrix and then we can calculate shunt admittance by multiplying this capacitance matrix by j omega. So, your shunt admittance matrix will be given by this. So, this is how we calculate admittance matrix of your distribution line, it is also 3 by 3 for 3 phase system. So, till now we have seen how to calculate phase impedance matrix for overhead lines, and then these shunt admittance matrix for overhead lines.

So, let us see how we can use these equations which we have derived for calculation of impedance and admittance of overhead line to underground cables. So, underground cables we know that we already you might have already studied the underground cable chapter. And then in that chapter you might have seen cable consists of inner conductor which is again standard conductor.

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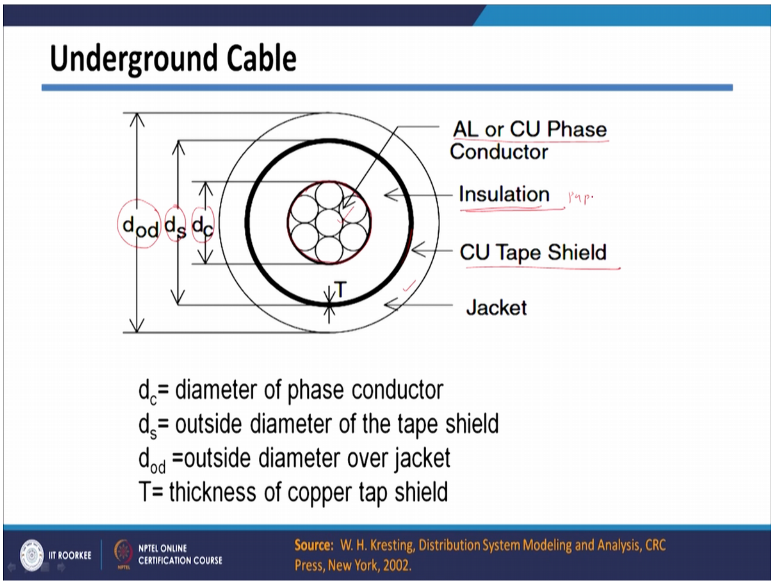

On that standard conductor to equalize the feeders to smoothen the field, we know that there is conductor shield they place it on it, and then there is insulation and after the installation again there will be cooperative shield.

Let us say the diameter of the conductor is dc, then diameter of your copper tape shield is say ds. So, diameter of shield is ds and outer diameter of cable is dod; we know that conductor material again for cable also it might be copper or aluminum, and there are different types of insulations like paper insulation or cross linked polyethylene or PVC or different types of insulation they use for cable.

Let us see how we can use the Carson's equation which we have derived for cable applications. So, let us say there are 3 single phase cables laid down under the ground and the distances between the cables are shown it here.

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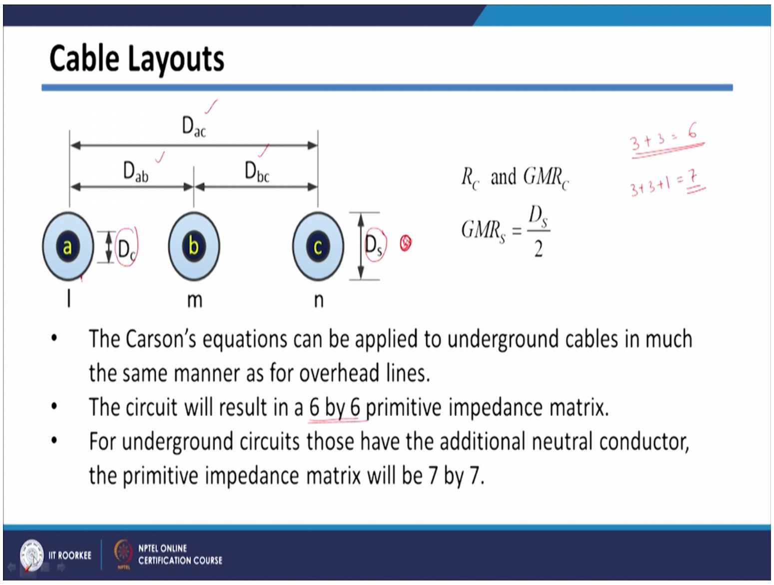

Let us say diameter of your conductor is Dc and diameter of your shield is Ds of for each of the cable. Now if you observe here there are 3 conductors those are abc phase conductors and as I told you your shield is grounded.

So, because of that this each shield we have will be we have to consider each shield as one conductor. So, there are there will be there are 3 shields and 3 conductors. So, there will be 6 by 6 primitive impedance matrix, because number of conductors are now 3 phase conductors and 3 shields. So, total number of conductors are 6. So, if you want to calculate primitive impedance or primitive and matrix for this cable it will be basically 6 by 6 size.

Now, in this case also we can apply Carson's equations which we have derived exactly similar way or similar manner as for overhead lines. So, this exactly similar way we can apply for the cable application also, and in this case you have as I told you your primitive impedance matrix will be 6 by 6 size if there is extra neutral conductor say one more neutral conductor is there which is acting as a neutral or return conductor in that case one extra conductor is getting added. So, 3 plus 3 plus 1 neutral conductor will be there will be same.

So, in that case your my primitive impedance matrix will be 7 by 7, but; however, in for this case it is 6 by 6 let us see how we can calculate it ok.

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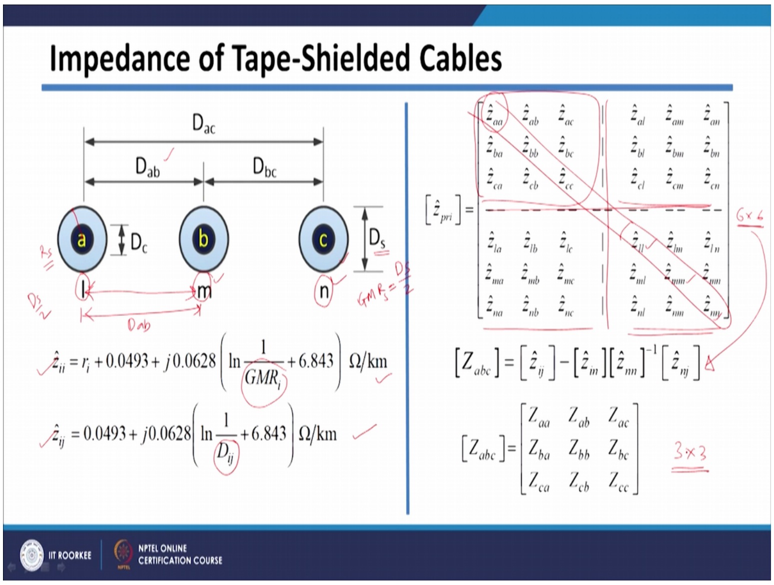

As I told you whatever Carson's equations which you have derived, you can use it for calculation of each entry of your primitive impedance matrix. So, each of the entry in primitive impedance matrix can be calculated by using this one. So, to get the diagonal entries of this primitive impedance matrix basically these entries, we can use this equation and in that case you need to use GMR of the particular conductor.

Generally a same type of stem cell types of a cable you will you will be using in that case your GMR will be same for 3 conductors. Now GMR of the shields; so here the GMR of the shield is required to get this diagonal entries corresponding to shields. So, these are 3 diagonal entries you are basically corresponding to shield to get that GMR of the shield is nothing, but your ah. So, GMR shield I can say GMR shield will be equal to Ds by 2 half of the your diameter of your shield.

Then of diagonal entries of this fed impedance matrix will be calculated by using this equation, and this Dij gives me or we need to know the distance Dij which is between the conductors. So, we can easily get the distances between the conductors and we can get all off diagonal entries of this impedance matrix using this equation. Now conductor between 2 shield conductors we can easily get suppose you want to find out the distance between l and m shields. So, I will given the names for the shields.

So, this shield I am coming ll this shield is m and this shield is n. So, if you want to calculate distance between 2 shields that those are l and m, in that case it will be just center to this center distance between this one means actually Dab in that case also Dab. And if you want to calculate distance between conductor to the shield which will be nothing, but a radius of shield in that case also your Rs which is actually Ds by 2.

So, this is how we can calculate these 6 by 6 matrix which is called as primitive impedance matrix and then again using Kron's reduction where these are the 4 parts of the matrix, because these 3 are belongs to phase conductors and these are corresponding to mutual and self entries of this neutral or earth conductors. In that case we can get then phase impedance matrix which is 3 by 3, which will be used for your different calculations.

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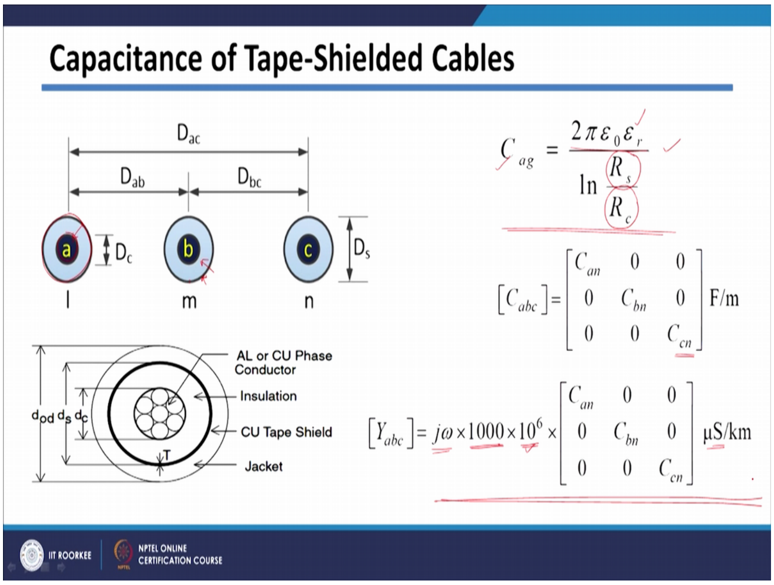

So, this is how we calculate impedance of the cable let us see how we can calculate capacitance of the cable. Now capacitance of the cable will be easier because if you see the electric field distribution due to application of voltage to the conductor, it will be just between your conductor and shield. So, electric field actually will not go out of the cable.

So, in that case your equation for your capacitance will be just 2 cylindrical electrodes and in that is case we know that capacitance between 2 cylindrical electrode is given by this equation where this is nothing, but your radius of shield and this is nothing, but your radius of the conductor; and these are actually constant that is 2 pi epsilon 0 epsilon R where epsilon R is actually relative permittivity of your insulation material.

So, if this is some material it will be having some relative permittivity, and you can easily get the capacitance of phase a with respect to ground and then we can say using same formula we can get capacitance of b phase which respect to ground and c phase with respect to ground. So, in that case your shunt admittance matrix, we can get it by using this one. So, you are just you have to multiply omega then to convert into a kilometer, we need to multiplied by 1000, and to convert it to into farad to microfarad or from Siemens to micro Siemens you need to multiply it by 10 raise to 6.

So, in that case your shunt admittance matrix will be given by this equation. So, this is how we have seen how to get the impedance and shunt admittance for cables and we have seen that we have we can use the same equations, which we have derived for overhead line for the cable also. Now let us see how we can model these distribution lines. So, we have seen how to calculate series impedances and shunt capacitance shunt admittances by using those you can see now how to model these distribution lines.

Now, we can model this lines as pi model, if it is long length line otherwise short model is also is ok because as I told you shunt admittance is very very small in case of distribution line and many times it is neglected. So, in that case your model will be just short line model, but before going to the short line model, let us let us see how the pi model looks ah. So, we already seen pi model in your b tech class. So, in that case we try to derive abcd parameters of pi model of your distribution align sectors.

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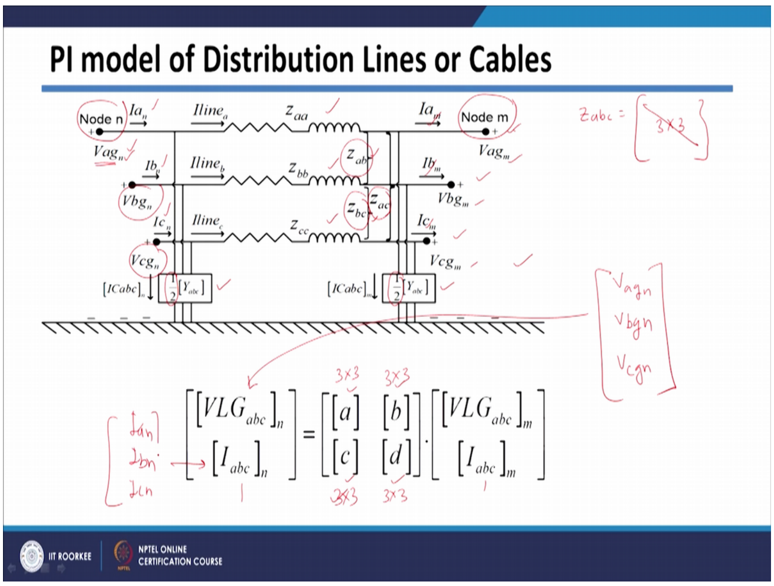

So, here I have shown the 3 phase conductor, because as I told you we need to model since the lines are not transpose as well as there is always unbalance in a distribution system we need to model a distribution system 3 in 3 phase mode. So you need to model each phase; so, these are 3 phase conductor after applying the Kron's reduction and after eliminating your ground conductors.

So, these are actually series impedances of the line and as I told you this is having 3 by 3 matrix, which we have derived Zabc we have seen it is 3 by 3 matrix and which is having mutual impedances also. So, Zaa, Zbb, Zcc are diagonal elements and each will be having mutual impedances that is Zab between a and b, b bc between b and c and ac between a and c and then there will be shunt admittances since we are using pi model half of the admittance is connected at the end, and half of the admittance is connected at the source end.

And then there are actually currents I an and I bn and I cn are actually input currents or source side current, and then I bm I cm and because I am calling this as a node m and this is node n. So, current at node m is I am I bm and I cm and voltages also voltage on n side actually V ag with respect to ground, that is a voltage of a with respect to ground at n node V bgn and V csgn are the voltages on the source side similarly voltages on the load side and you can write them into compact form, which is like this where these are the abcd matrices.

So, we are modeling this pi model and writing in terms of abcd parameters ah. So, in this case each of these abcd matrices will be 3 by 3. So, b also will be 3 by 3, b also will be 3 by 3 as well as 3 will be c will be 3 by 3 and all these currents and voltages there will be 3 by one matrix means if you write this matrix. So, it will be V ag n, V bg n and V cg n and this I it will be actually this I abc matrix will be I an I bn and I cn. So, it will be actually 3 by 1 matrix.

Let us see how we can get this abcd parameter for this pi model.

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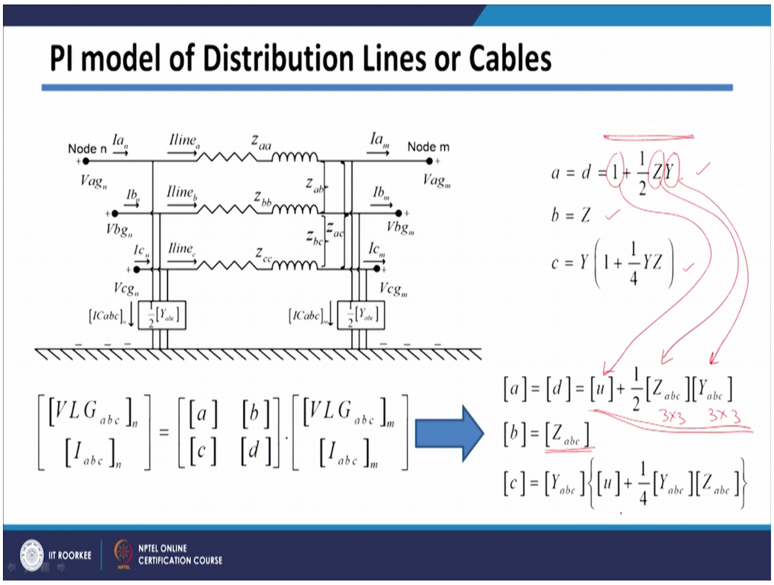

Since we already seen abcd parameters for your transmission line; however, in that case your line will be about just one line, where we are model this where we have taken 2 capacitances like this or admittance by 2, admittance by 2, and then here we are having Z impedance between the line. So, in this case we have seen that your equations for abcd parameter for a single phase line or single phase equivalent model we have seen that it is like this a and d parameters, we have seen it is 1 plus 1 half Z into Y, a b parameter we have seen it is Z, and c parameter we have seen it is y into bracket 1 plus 1 by 4 into YA exactly same way we can write it for this also.

However as I have we have seen that all the 3 conductors are there and each impedance matrix is 3 by 3 matrix. So, what will happen is, all these entries here or all these parameters here they will be 3 by 3 otherwise structure will remains same. So, in this case this one will become u identity matrix of 3 by 3 size, and then this Z will become impedance phase impedance matrix, which is we have seen 3 by 3 in size and this will become y will become your shunt admittance matrix which is again 3 by 3 size.

So, structure is actually you can see that it is remaining same ah; however, size of each of the element is becoming 3 by 3 here. Similarly this Z abc will be 3 by 3 and in this case also all the entries will be 3 by 3 otherwise the structure of the equation will remain same. In case of in case of short line we have seen that these parameters admittance generally we neglect.

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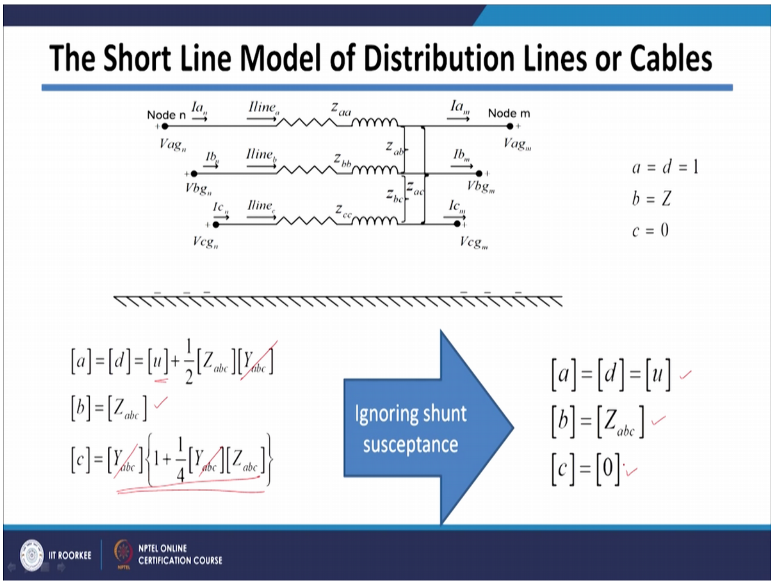

So, if you can neglect the admittance so, this entry and this entry here it will be 0. So, your c will become 0 and your a and d will be just your u matrix and b will be Z abc.

So, if you calculate for short line model your a and d parameters are u, b parameter is Z abc and c parameter will be 0; and as I told you most of the applications of the distribution line this short line model is sufficient to model, the feeders as well as distribution line. However, if the length of the distribution size distribution feeder is longer in that case we need to go for pi model for accurate calculation; however, most of the cases this is a short line model is sufficient ok.

In summary of today's lecture, we have started with shunt admittance and we have seen how we can calculate shunt admittance for overhead distribution lines.

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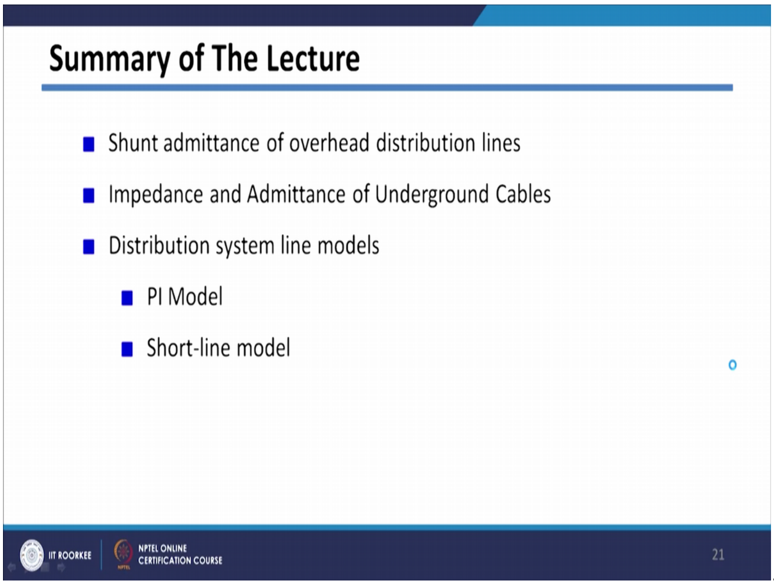

And we have seen that in this case also first we have to get the primitive potential coefficient matrix, we try to derive the equation for each of the potential coefficient and from that we can build these potential coefficient matrix; and since it is n by n in size that is number of conductor by number of conductors we need to again reduce it by eliminating the ground wire.

So, we have seen that we can apply (Refer Time: 35:55) for this elimination, and we can get what is called as shunt admittance matrix from that. So, we have seen that inverse of this potential coefficient matrix is capacitance matrix and from that capacitance matrix you can get shunt admittance matrix. We have also seen that admittance and sorry impedance and admittance of underground cable and we have seen we can use this. So, same equations, which you have used for overhead lines in case of underground cable also.

So, same Carson's equations can be used to get the primitive impedance matrix and then again you can convert into phase impedance matrix. However, admittance of the cable can be calculated by just getting the capacitance of each cable separately. Because in that case we have seen that electric field is confined inside the cable ah. So, charges on one of the conductor in the cable will not affect the charge distribution in other cable. So, in that case you need to calculate the capacitance of each cable separately.

And then we have seen how to model distribution lines initially we have seen the pi model, where we have seen that we can use the same structure of the equation which you have derived in case of transmission line; however, the each entry of that structure we have seen that it is becomes 3 by 3 matrix. And then we have seen short line model, and as I told you most of the cases we need to use this short line model most of the analysis; however, the feeder length is longer you have to go for pi model.

Thank you.