Electrical Distribution System Analysis
Dr. Ganesh Kumbhar
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture – 10
Series Impedance of Distribution Lines and Feeders Part I

Dear students, we have completed our chapters 2 in the last lecture it was approximate methods of analysis and this time we are going to start new chapter that is modeling of distribution system components. In this particular chapter, we will see the modeling of various components in the distribution system starting from distribution feeders and then transformers, regulators, capacitor bands and loads and distributive generator also.

So, in today's lecture that is lecture number 10 we will start with Series Impedance of Distribution Lines and Feeders. So, you how to calculate series impedance that particular thing we will see in this particular lecture. Before going to this lecture just revise what we have seen in chapter 2 as I told you chapter 2 is basically on aprox method by approximate methods of analysis of distribution system.

In that chapter particularly we have seen 2 factors.

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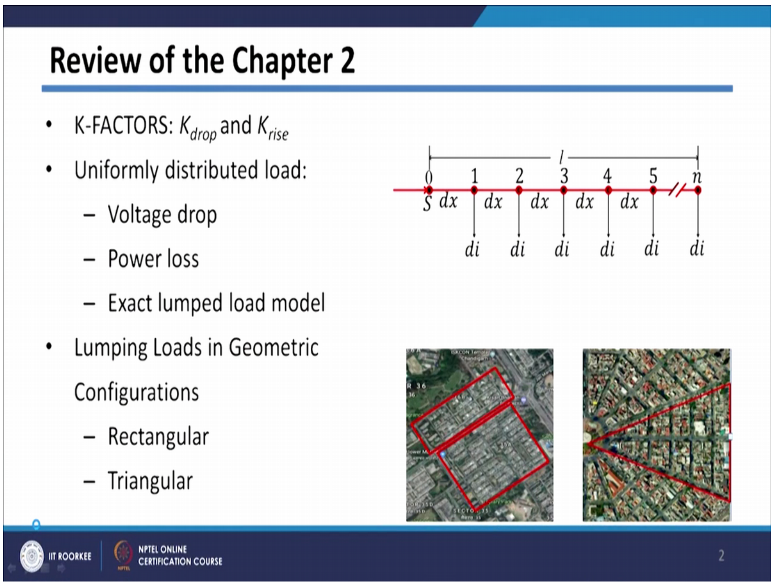

Those are K drop factor and K rise factor and we have seen that K K drop factor is nothing, but a percent voltage drop per kVA per kilometer and it is basically used to find out a limit of loading of your distribution system or to find out voltage drops in the distribution system or we can say if you want to find out the conductor size the K drop factor will be useful.

Similarly, we have seen that seen the K rise factor K rise factor is basically percent voltage rise per kVR per kilometer. So, this is basically useful for placement of capacitor that is nothing, but finding the location of the capacitor or size of the capacitor in distribution system. Then we have seen uniformly distributed load and their modeling means if there is uniformly distributed load how we can find lumped model of them to find out the voltage drop and power loss these are the approximate values of voltage drops and power loss.

Then we have seen exact lumped model of uniformly distributed loads after that we have seen the lumped model for various geometrical configurations; particularly a rectangular configuration and triangular configurations. Now let us start with this new chapter which is basically calculation of series impedance of your overhead conductors or feeders. So, we know that the different types of materials are used for conductor those are basically copper and aluminum.

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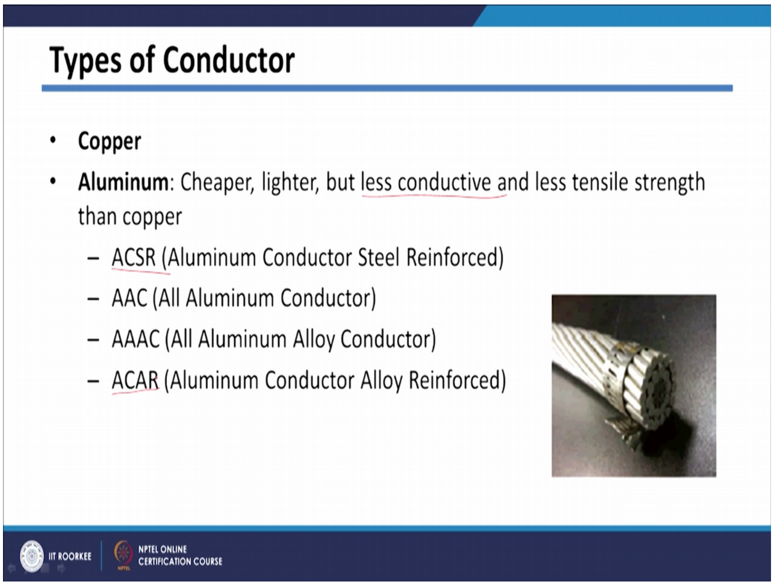

And nowadays since aluminum is cheaper and lighter it is widely used; however aluminum as I shown it here aluminum is less conductive and it is having a less tensile strength. Means long conductor will be requiring a higher tensile strength because we are hanging them between the tower to increase the tensile strength we use ACSR conductor those are nothing, but aluminum conductor with steel reinforcement which I written there.

Then there is AAC, AAC conductor which is all aluminum conductors then triple AC conductor those are all aluminum alloy conductors and then there are a aluminum conductor with alloy reinforcement. So, with these alloy reinforcement is basically to increase the tensile strength of the material. Out of this there are other types also, but out of this all types ACSR are widely used. So, structure of ACSR conductor is like there are aluminum strands and steel strand.

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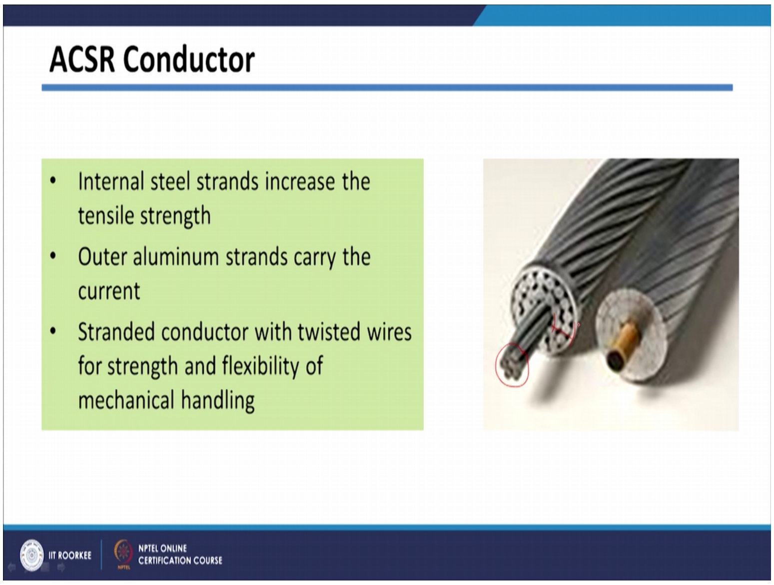

So, steel strand will be at the middle which are basically give to increase the tensile strength. And the aluminum conductor, which are at the outer part they are basically to carry the load current ok. So, as you know resistance is calculated by using this formula.

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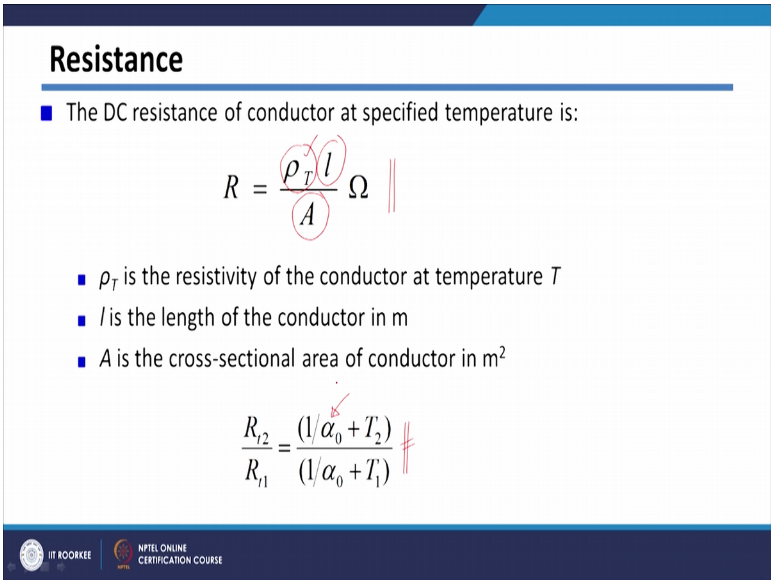

Where this rho T is actually resistivity of the conductor material this is length of the conductor and area of cross section of the conductor. And as you know that resistivity of the material is temperature dependent, so resistance will also change whenever temperature is changing.

So, if you can want to calculate resistance at different temperatures you can use this formula. So, if you know the resistance at one temperature resistance at another temperature can be calculated using this formula, where this alpha 0 is known as temperature coefficient of a resistance; if that is known resistance can be calculated at any temperature.

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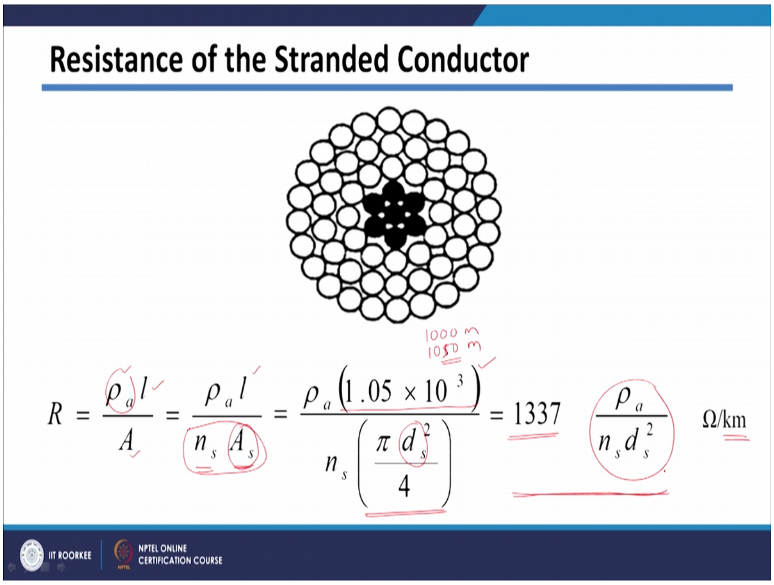

However, we have seen that our actual conductor is not a single conductor earlier formula was applicable for only the single conductor.

However, if there are many strands in a conductor like I shown it here we can calculate the resistance like this I shown it here. So, resistance is given by a again rho l by I a same formula resistivity length of the conductor cross sectional area. Now since there are many strands of aluminum let us say there are n S number of strand and each strand has a cross sectional area of a S. So, overall cross section area of aluminum will be given by n S multiplied by A S.

Now this cross sectional area of one strand will be given by this formula where d S is diameter of one strand. So, this is nothing, but overall cross sectional area of one strand of aluminum conductor and this is nothing, but I am taking for a length of the conductor. So, I am considering 1 kilometer length; so actually I have to take 1000 meter because this formula is valid for 1000. So, length is 1000 meters instead of 1000 meter, I just taken 1050 meters because we have seen that in case of stranded conductor we twist it or in case of ACSR conductor, there is twisting of conductor and because of the twisting length of the conductor is increase.

So, approximately to accommodate that increased length we have taken this 50 input are extra for considering the twisting. So, if we can take all these constants together I will get 1337 here and this these are the remaining variables which will give me resistance of the ACSR or stranded conductor for different strands; however, generally data of all these types of conductor; that is a ACSR or AC or triple a data sheets will be available and data sheets will give me giving you lot of data.

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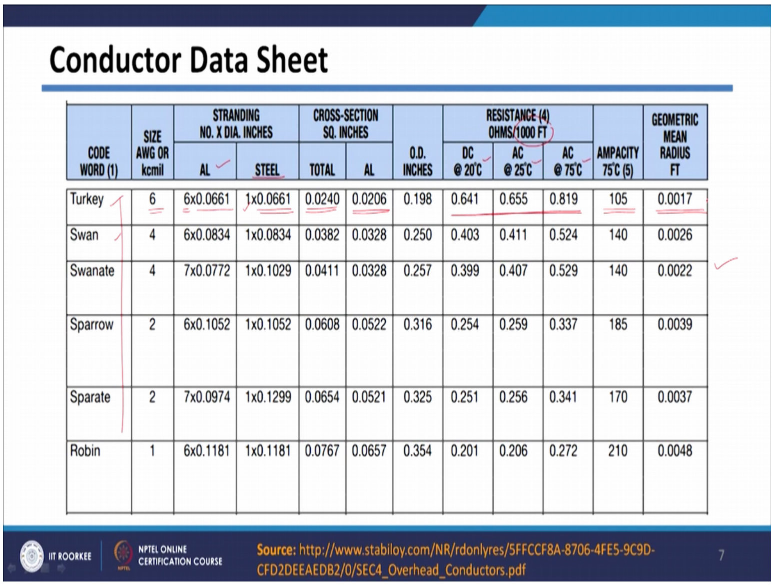

CODE WORD (1)	SIZE AWG OR kcmil	STRANDING NO. X DIA. INCHES		CROSS-SECTION SQ. INCHES		O.D. INCHES	RESISTANCE (4) OHMS/1000 FT			AMPACITY 75°C (5)	GEOMETRIC MEAN RADIUS FT
		AL	STEEL	TOTAL	AL		DC @ 20°C	AC @ 25°C	AC @ 75°C		
Turkey	6	6x0.0661	1x0.0661	0.0240	0.0206	0.198	0.641	0.655	0.819	105	0.0017
Swan	4	6x0.0834	1x0.0834	0.0382	0.0328	0.250	0.403	0.411	0.524	140	0.0026
Swanate	4	7x0.0772	1x0.1029	0.0411	0.0328	0.257	0.399	0.407	0.529	140	0.0022
Sparrow	2	6x0.1052	1x0.1052	0.0608	0.0522	0.316	0.254	0.259	0.337	185	0.0039
Sparate	2	7x0.0974	1x0.1299	0.0654	0.0521	0.325	0.251	0.256	0.341	170	0.0037
Robin	1	6x0.1181	1x0.1181	0.0767	0.0657	0.354	0.201	0.206	0.272	210	0.0048

So, one data sheets one of the data sheets I shown it here where this first column shows the codeword for the conductor and these codes were codewords are generally given the names of birds or animals. So, these are your the codewords of your conductor and then required a data of each conductor is given here.

So, you can see that this gives size in AWG or kilo circular mills and then there are actually our aluminum strands; these are the number of strands and diameter of each strand then steel there is a only one strand and this is diameter of steel strand. Then total cross sectional area of the conductor out of this aluminum area then this gives me a resistance values of this conductor at different temperatures for 1000 feet. So, we can easily calculate for 1 kilometer at various temperatures of the conductor.

Then this gives the ampacity of the conductor and this gives you geometric mean radius of that particular conductor. And whenever geometric main conduct areas of the conductor will be required for while calculating the inductance of the conductor; so, resistance will be available from data sheets.

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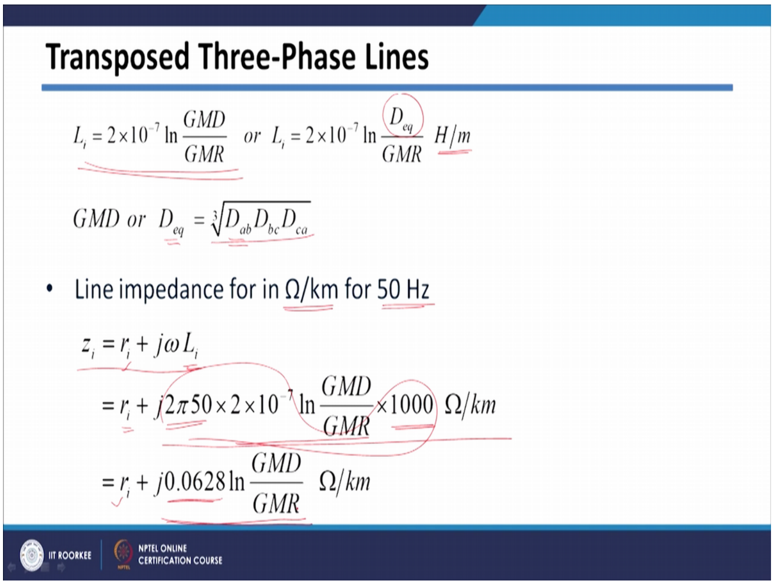

Let us see how we can calculate inductance of the line and we are already seen the inductance formula for transpose line. So, we know that this is your formula which is 2 into 10 raised to minus 7 natural log of GMD by GMR or we say a instead of GMD sometimes; they say the equivalent and this GMD or D equivalent is given by cube root of D ab plus D b D ab multiplied by D bc multiplied by D ca and these are nothing, but a distance between the conductors.

Ah In that case line impedance for 1 kilometer in ohms at 50 hertz can be calculated like this. So, it will be your total impedance will be the resistance plus j omega into inductance ah; resistance we can get it from the data sheets and inductance can be calculated from this part or reactance can be calculated from. This part this is your omega, this is your inductance and since I am considering per kilometer I have to multiply it by 1000 because this formula is gives me inductance in Henry per meter. So, to convert into kilometer we have to multiply it by 1000.

Now, if we take this coefficient together; it comes out to be 0.0628 natural log of GMD by GMR. So, resistance we get from datasheets and we can get the reactance part using this equation.

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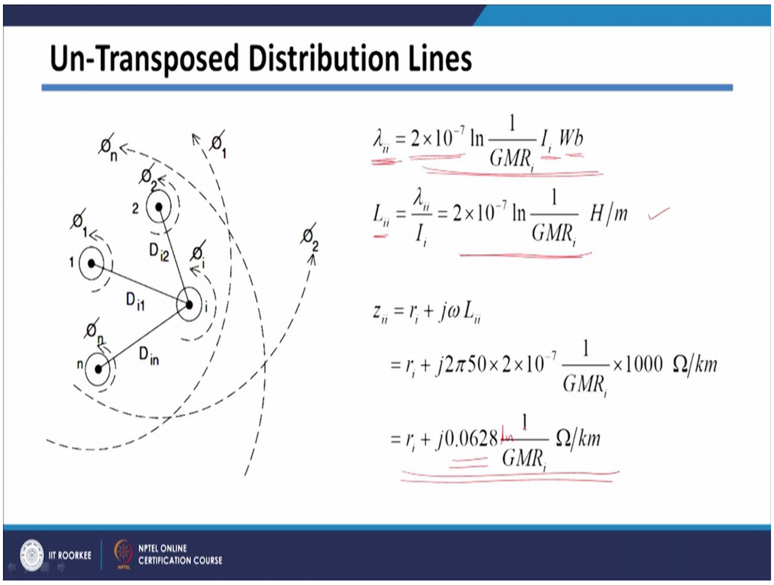

However, we have seen that our distribution lines are generally un transposed because they are short length line and transposition only done the length of the length is length of the line is longer. So, in case of distribution feeders they are generally un transposed; so, if there are un transpose feeders and if there are n conductors in the system, then there will be n by n matrix because there will be a self inductance mutual inductances we have to take them separately.

If they are transposed we mentioned it together. So, in case of un transpose line we need to calculate self inductance and mutual inductance. Let us see how we can calculate self inductance of the conductor and that self inductance we can calculate from the self linkages self flip linkages. So, flip linkages lambda ii are nothing, but flip linkages of conductor I due to its own current i.

So, due to current ii and this is given by 2 into 10 raised to minus 7 natural log of 1 divided by GMR of that particular conductor which will be in webers. And if you calculate inductance which you know that plate linkages divided by current will give me inductance. And if you divide this formula by ii current we will get this equation that is 2 into 10 raised to minus 7 natural log of one divided by GMR i. And this will give me self inductance of that conductor and then can we can we easily calculate yourself impedance of that conductor by again same formula here.

So, here also you will get this same constant that is 0.0628; natural log of 1 divided by GMR i self impedance of the conductor will be given by this formula.

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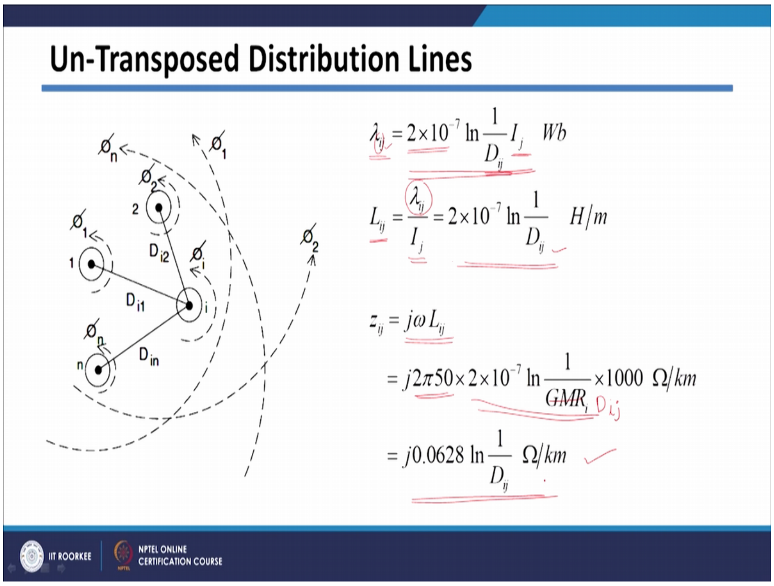

Let us see how to calculate mutual inductance. So, for mutual inductance I am considering here flip linkages due to current ij; so, flip linkages of conductor I due to current in conductor j. So, any conductor j the current is ij here and flip linkages of conductor i due to current j is given by this formula which is 2 into 10 raised to minus 7 natural log of 1 divided by D ij; D ij is nothing, but distance between conductor i and j.

Ah Then inductance mutual inductance between these 2 conductors will be given by this flip linkages divided by your current. So, if you divide this equation by i i j, you will get 2 into 10 raised to minus 7 natural log of 1 divided by D ij, the distance D ij is distance between the 2 conductors. Then we can calculate the mutual impedance in this case there will resistance will not be there.

So, it will be just j omega L ij so, this is your omega this is your L ij which you have derived yeah in this case here also be this is D ij into 1000 ohm per kilometer. So, finally, you will have this formula here which will give me mutual impedance between their 2 conductors.

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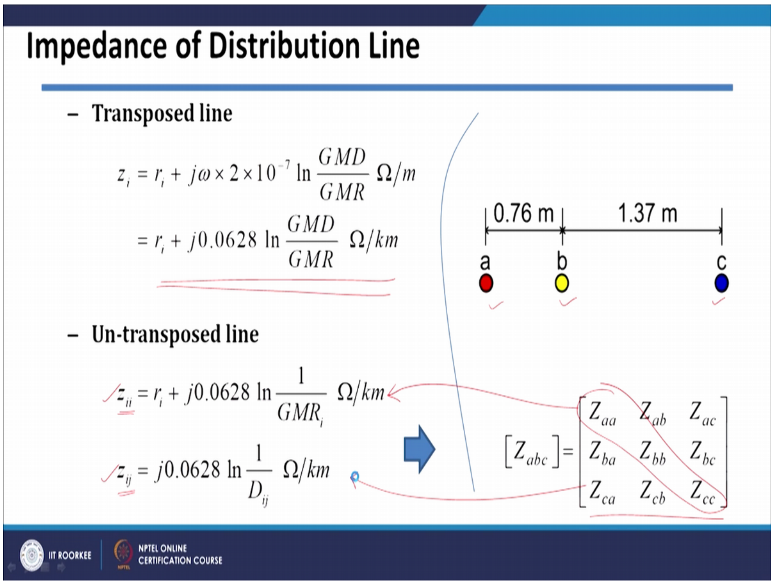

So, in summary we have seen that in case of transpose line we have only one impedance value which is given by this situation here. And in case of un transpose line as I told you will be having n by n matrix if there are n number of conductors into the system and this n entries of n by n matrix will be given by this formula, where this z ii will give you a self impedances of those conductor and z ij will give you mutual impedances between the conductors.

Like one system which I shown it here where suppose there are 3 conductors then there will be 3 by 3 matrix for un transpose line and entries of those 3 by 3 matrices will be given by this one. So, this Z aa, Z bb and Z cc will be calculated using this formula here which is given Z ii and the remaining entries are calculated by using this formula z ij.

Till now we have seen that we can calculate the impedance of distribution line if they are transposed or un transposed in case transpose who are getting one impedance only, in case of un transpose we are getting matrix of n by n size if there are n number of conductors and we also seen that how to calculate those entries; however, this formulas are applicable if there is no ground return current ok.

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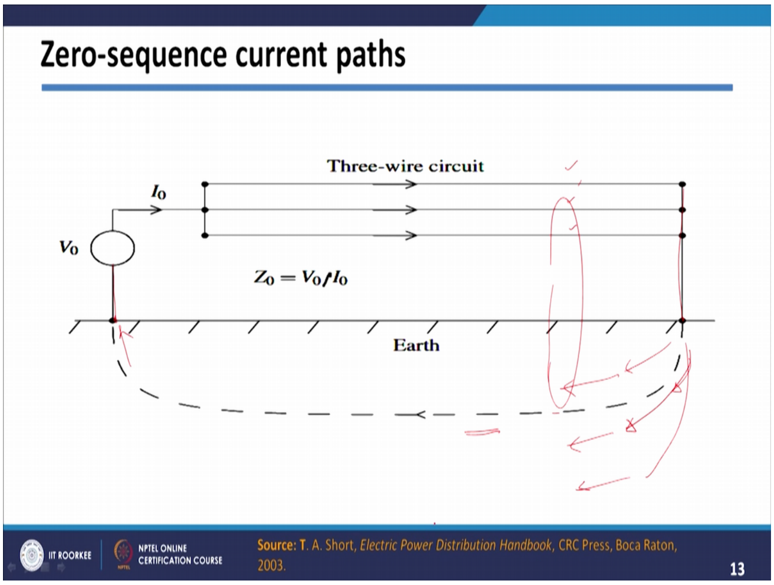

Basically if you see this particular figure here generally what happens whenever there is unbalance into current that unbalance current will flow through the earth. So, if there is no need to the earth 2 3 different conditions; so, in this condition we are considering 3 phase 3 wire system, where there is no return path through the neutral wire in that case if the grounding system is here and then substation grounding is here in that case your unbalance current 0 sequence current will actually flow through the earth path and it will come here.

Now, when this current is flowing through the earth a earth will act as a one conductor and already there are these 3 conductors. And because of earth current there will be mutual impedances between this earth current and all this current. Now column here that we do not know what is the resistivity of the earth because it will not be uniform everywhere and whenever current is flowing through the earth the path of the current is also not definite. So, current will take any path which is available let us see what affect it happens before that we will just see 2 different system.

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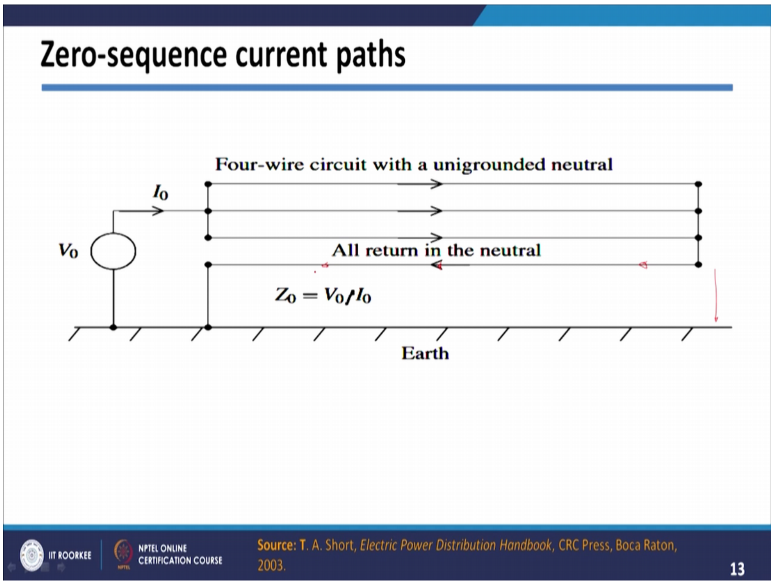

Let us say this is another system in this case there is multi grounded neutral. So, it is 3 phase 4 wire system; however, grounding is done at different places. So, this is your 3 phases and this is neutral conductor; so, unbalanced current will take path through your neutral wire as well as some current will also flow through the ground because this is having multi grounded neutral. So, the current which is 0 sequence current will take path through; this both this conductor that is neutral conductor as well as ground conductor.

In another case, there is a possibility that the grounding is not there multiple grounding not there you; only single grounding is there. So, in that case whole unbalanced current will actually flow only through the neutral because we are not grounding the system at this place. So, whole unbalance current will actually flow through the system.

So, in that case there will not be earth current, but earlier 2 cases there will be some earth current and because earth will act as a conductor it will be having mutual impedances with all phase conductances; conductors. And considering those mutual impedances or calculating those mutual impedances will be very difficult task. So, we will see how we can consider the effect of current which is flowing through the ground.

Let us say you are having these 2 conductors.

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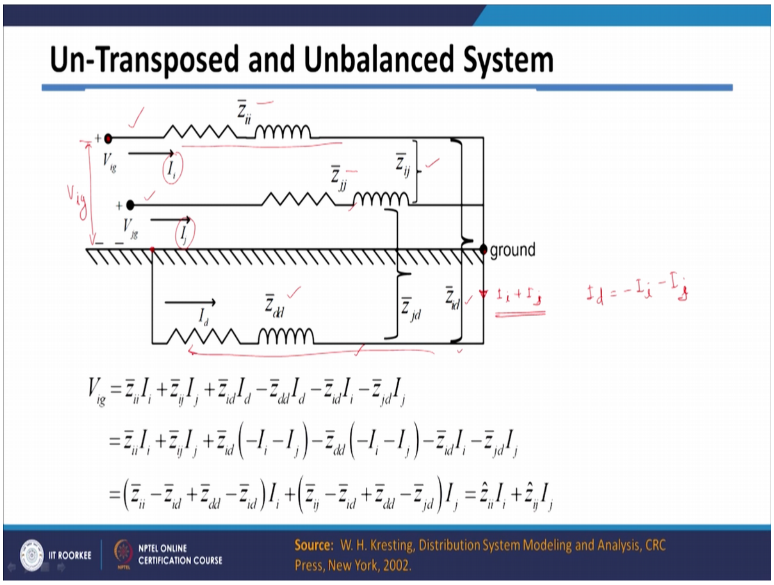

This is a one conductor I another conductor is say j Z ii is self impedance of conductor I Z jj is self impedance of conductor j and Z ij is mutual impedance between them. Now I am considering the ground also as a one conductor because it is carrying your unbalance neutral current in that case earth will act as a another conductors.

So, there will be 3 conductors and Z they Z dd will be your; self impedance of earth, Z jd will be mutual impedance between conductor j and earth and Z id will be mutual impedance between conductor I and earth. So, all the mutual impedances and self impedances are shown there; let us say this conductor I is carrying current ii and conductor j is carrying current I j and then the I ii plus your I j current will be flowing through this.

It will get either at this point and then ii plus I j current will be flowing through the ground. And I am considering this I d current which is flowing through ground means I can say your I d current will be equal to minus ii minus I j minus I j. Let us say I want to calculate voltage V ig that is nothing, but voltage of this terminal with respect to ground terminal at this place. So, I am interested in calculating this voltage drop here which is V ig. So, this volt voltage drop will consist voltage drop across this then voltage drop across this branch.

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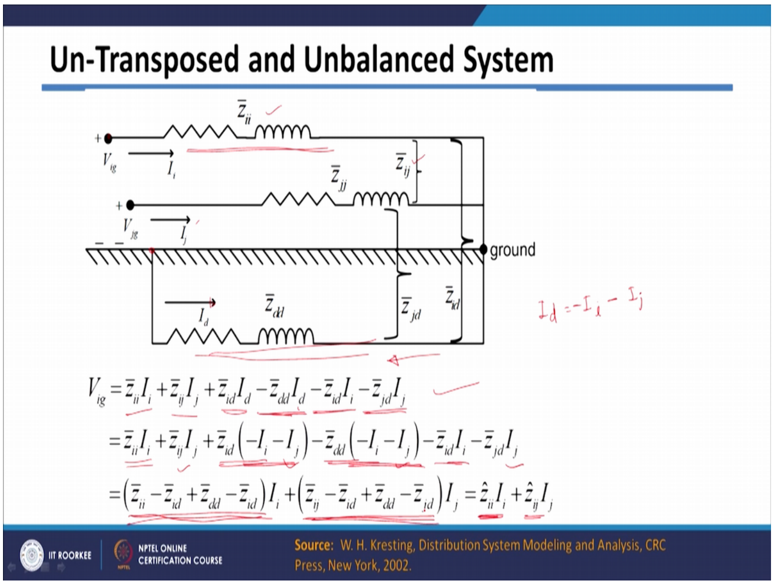

And if you see this voltage drop across this branch will be due to all the 3 currents. So, it will be Z ii into ii this is a shell drop.

Then drop in Z ij which is due to current I j; so, it will be zij into I j plus drop due to current I d which is Z id into id. So, this is nothing, but drop which is happening in into this due to all the 3 currents then there will be a drop which is happening in this branch due to all the 3 currents. So, it will be minus because this drop we are taken in this direction and I d current is flowing in oppose direction there will be minus sign here. So, it will be Z dd into I d which is due to self current and self impedance Z id into ii due to current I j Z jd into I j due to current ij. So, total drop across these 2 points will be given by this equation.

Now, as I told you your I d current will be just ii current minus ii current minus your I j current. So, it will be minus ii minus ij; so instead of I d, I can put minus ii minus I j here also there is I d instead of that I can put minus ii minus I j here and then I have taken all the terms which are related to ii and I j together. So, the terms which are related to ii there are 4 terms which are related to ii I have taken it together. So, this I will get this term and if I take current terms to be related to I j together I will get this term.

And this term I am calling Z ii cap; so, this total term of 4 impedance adding I am calling Z ii cap and here I am calling Z i I j cap which is a addition of all this 4 impedances.

Now let us see what is the meaning of all this 4 impedances, which you are adding; so, let us take only this term which is Z ii cap.

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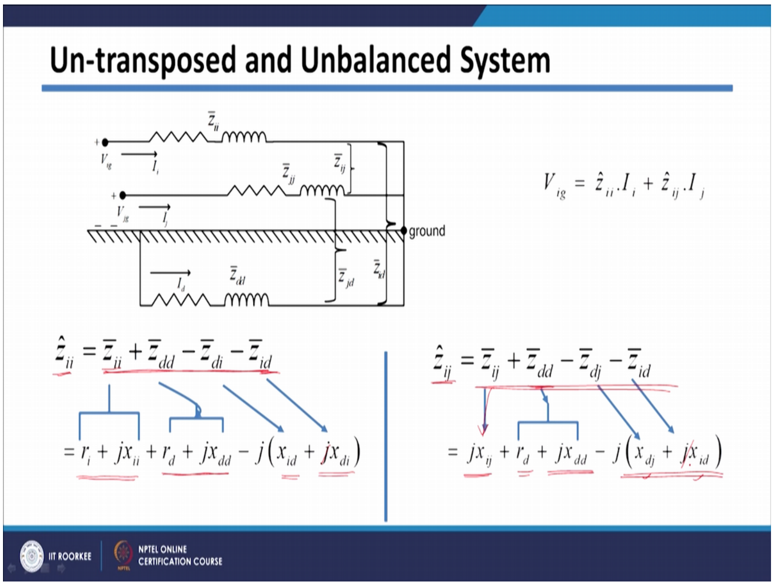

So, if I take this term here Z ii cap which will be consisting of 4 impedances which I have shown in that figure. Now Z ii will be consisting of resistance of the conductor plus reactance of the conductor, Z dd will be nothing, but resistance of the earth and a reactance of the earth Z di will not be having resistive parts. So, it will be having only reactance and Z id also will be having since it is mutual it will be having only reactance Z um. So, this j will not be there; so x id plus j xdi in this case also there will not be j.

Similarly, another that another term Z ij will be having this 4 terms which I explained in last slide in this case also this Z ij will be having only reactance term. Z dd will be having resistance of the earth plus reactance of the earth and this is nothing, but your mutual impedances that is why only reactance terms are there.

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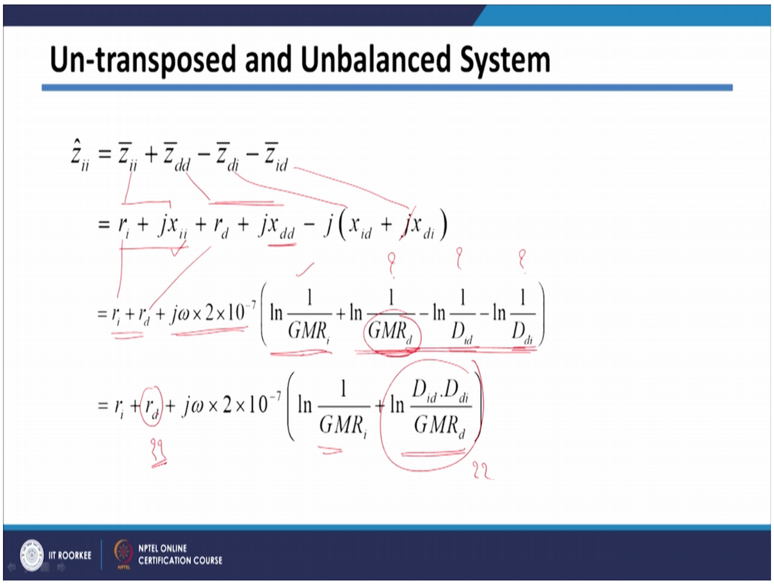

Now, let us expand this first term Z ii as explained explained you are having this is your Z ii 2 terms this is your 2 terms related to Z dd this is term related to Z di and this is related to Z id this j will not be there.

So, I just taken resistance term together there is ri and rd and then reactances term together which are basically this one. And in this case xii we already seen that we can get the inductance of the conductors using the GMR information of the conductor. And you can get the reactance of the earth by using the GMR information of the earth and then this is nothing, but distance between earth conductor to the conductor; i, this is nothing, but again distance between the earth conductor the to conductor i and just.

In this case this term will be easily available because GMR of the conductor will be available. However if you see this 3 terms GMR of earth it is very difficult to get GMR of the earth; we will see that why it is? Because as I told you when the current is flowing through the earth it is taking many different path and this earth is not uniform. So, whenever we are considering a earth as a conductor which is having very enormous dimensions and because of that GMR of earth is very difficult to get. So, we know do not know this quantity.

Similarly, since we do not know the conductor as a earth as a conductor where it is lying inside the earth, we do not know what is the distance between earth conductor to the conductor i. So, this term is also not known this term is also not known ok. So, what I did here I have taken the terms which are unknown together and the term which is known here. So, basically this term is unknown here; similarly we do not know this term similarly resistance of the earth is also difficult to get exact value; s this is difficult

So, other things we can calculate so; however, this 2 terms it is difficult to get if there is earth return current.

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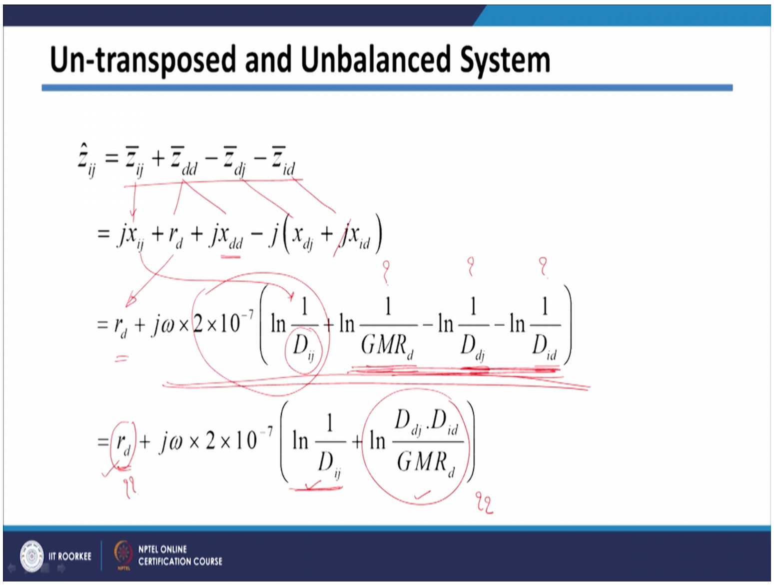

Also now consider another term which is Z ij which we are written there are again 4 terms which is take down in the last slide. In this case also Z ij will be having only reactance Z dd will be having resistance of the earth as well as reactance of the earth. And this 2 terms will be again since they are mutual they will be having reactances only.

Then again we can take the resistance out of this and then this is real part and then this is your imaginary part and this represent your xij which you can told you this nothing, but mutual impedance between the lines which we have seen that we can get it from the distance between the 2 conductors; that is 1 upon dij into 2 2 into 10 raised to minus 7 which gives me a reactance or we can say self impedance of the conductor.

Ah This is nothing, but again your GMR of ground this is nothing, but distance between ground to conductor j and this is again distance between conductor i to ground conductor. As I told you since the earth is having enormous dimensions getting this GMR of earth or distance between earth conductor to the conductor i conductor j will be difficult. So, again this 3 terms will be unknown to us. So, in this case also this known terms I have taken together. So, this is your known term; however, this unknown term by integrate integrating this logarithms together this is your unknown term.

And in this case also this rd which is unknown; one more thing you can observe that zij is nothing, but mutual impedance between the conductor while we are considering earth return also taken into account. In that case we can see that in mutual impedance terms also getting small resistive term here ok. And as I told you it is very difficult to get these unknown quantities which I explained in this slide directly.

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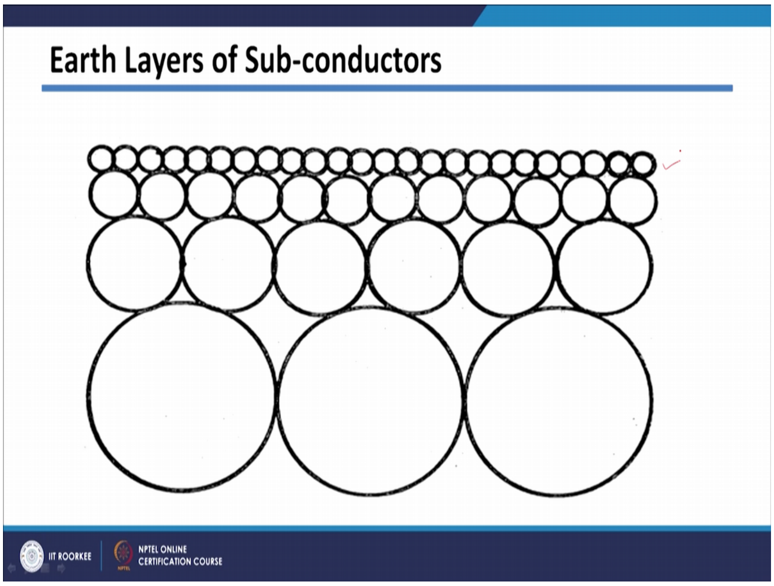

And which is very very kind of tedious task because earth will be having you can say layers of the conductor.

So, there will be lots of lots of conductors in the below the earth surface which can be modeled conductors like this which will be in. In fact, might in numbers; so, that is why getting GMR of the conductor or getting the distance between earth conductor to the phase conductors will be a difficult task.

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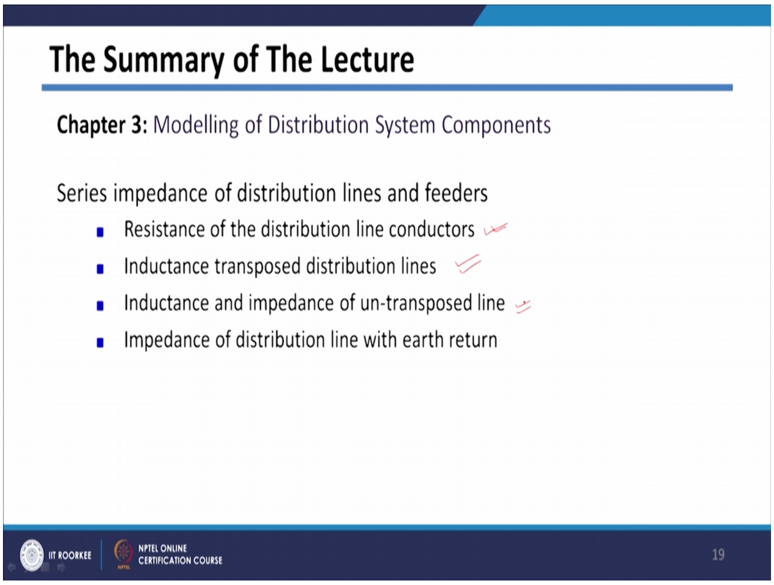

So, in summary we have started the chapter number 3 which is modeling of distribution system components. And we have started with first component that is distribution feeders and distribution lines and we have started with calculation of impedance of this distribution lines or feeders.

And then we have seen this different types of conductors, we have seen how to get the resistance of the conductor how to calculate it and then we also seen that how to how can get this resistance value from the data sheets. Then we have seen the inductance of transpose distribution line impedance also we have seen and then inductance in impedance of un transpose line which will be having if there are n number of conductor it will be having n by n matrix; however, in case of transpose line there will be one only one entry or only one number.

Then we have seen impedance of distribution line with earth return. So, we have seen that in case of earth return some current flows through the ground. And because of that ground current they at earth will be acting as a one, one of the conductor and there will be mutual coupling between the phase conductors to these earth conductor and it will make calculation of your impedance little bit tedious. So, in next class we will see that how we can take this earth return into account.

Thank you.