

Basics of software-defined radios & practical applications
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Lecture - 09
Distortions Parameters: Nonlinear Distortion

So, in the series of basics of software defined radios and particular applications today we are discussing the topic of distortion parameters and within this topic we are discussing non-linear distortion.

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Distortion introducing elements in receiver

Effective noise figure of ADC:

$$NF_{ADC,dB} = 10 \log \left(\frac{V_{ADC,rms}^2}{Z_{in} 10^{-3}} \right) - SNR_{ADC} - 10 \log \left(\frac{f_s}{2B} \right) - 10 \log \left(\frac{kTB}{10^{-3}} \right)$$

$V_{ADC,rms}$ is the rms value of the ADC input voltage range.
 f_s is the sampling frequency (in hertz).
 Z_{in} is the converter input impedance.
 SNR_{ADC} is the signal-to-noise ratio of the converter.
 B is the bandwidth (in hertz).
 T is the system temperature (in Kelvin).
 k is Boltzmanns constant (1.38×10^{-23} J/K).

Calculate: noise figure of an ADC for 100MHz sampling rate, $B=1$ Hz, $Z_{in}=200\Omega$, 1 Volt peak voltage input to ADC, S/N of 76 dBc.
Solution: 24 dB

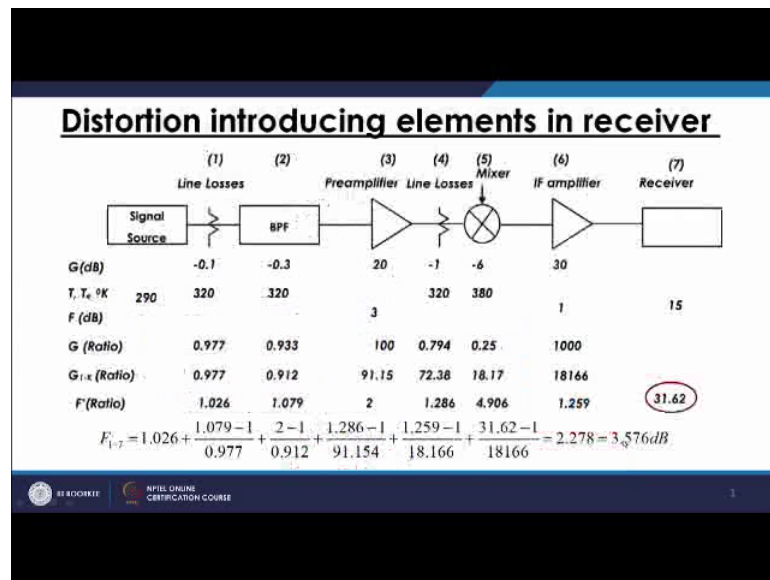
***Noise figure is quite high leading to noise factor of 15.84!**

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So, while talking about the distortion introducing elements and receiver, we were talking about noise figure of the ADC. In the last lecture, we were discussing the noise figure of an ADC and as an assignment, it was given to the participant that the sampling rate is 100 megahertz, the bandwidth considered is 1 hertz, input impedance is of 200 ohm and applied voltage input is 1 volt peak. It is to be noticed it is not peak to peak; it is peak. And signal to noise ratio of the system is given as 76 dBc.

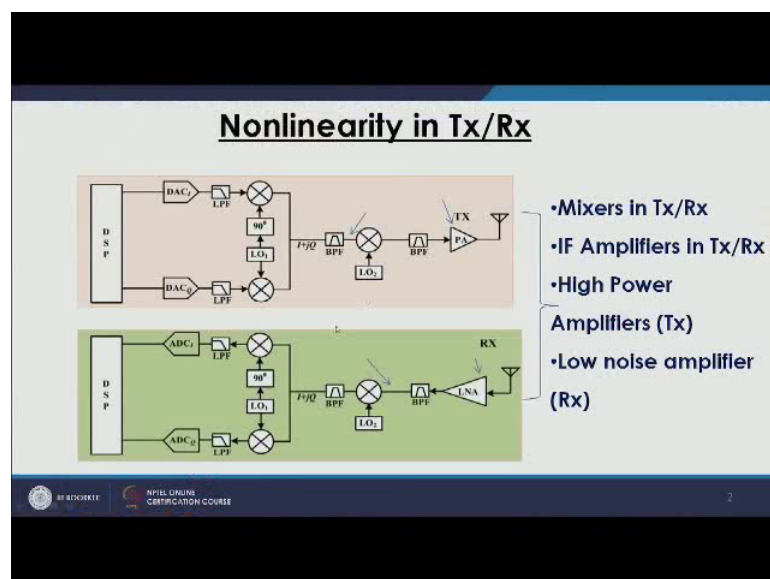
Now, if you put these values in this equation, we will get our solution as 24 dB. Now, it is quite high value and if you calculate the corresponding noise factor by converting this dB back into the absolute form, you will get 15.84.

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Now, if you remember a previous example, when we were calculating the noise figure of the whole system, if you see here, the receiver portion was showing exceptionally high noise factor. So, we can say that what we are achieving here is actually matching with this data and receiver because of the ADC is having very high noise factor. So now, moving on to the current distortion introducing elements in receiver, we will continue with the different architectures.

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So, if you remember this is the heterodyne structure for our transmitter and receiver, in the transmitter receiver, we have discussed the noise scenario, noise due to phase and I q imbalance DC offset and all these parameters and how they are related to different topologies. Now, let us have a look at the nullity in the both of the transmitter and receiver paths.

So, the nullity in the path is mostly seen in the mixers which are used for the up and down conversion. And more than that we have amplifiers, so that we can send our signal to a high level; so, those are called high power amplifiers. Apart from that to maintain the level at the mixer input; so, that it is at the proper range of power, we have IF amplifiers in both transmitter and in receiver.

Receiver side also we also have low noise amplifier, so that whenever we get the signal the power which we have lost due to attenuation that can be recovered and this amplified signal is sent to the band pass filter. In filtering and all those application, again some attenuation happens and our signal level drops. So, just before our ADC and DAC; we have to control this level again and some kind of amplifiers are used here. As long as the nonlinearity in the mixer is concerned, it leads to up conversion, down conversion process; so, it is beneficial in this case. We have some unwanted components like harmonics, but that we can ignore easily because we are using filters before and after this component.

Now, let us discuss the component of the amplifier which is IF amplifier or high-power amplifiers or low noise amplifiers. These amplifiers when they have nonlinearity; it is not in desired effect we want our amplification to be linear.

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Software Defined Radios

Limitations of direct-conversion transmitters
➔ Motivation for digital front end!

Concept of pure digital RF front-ends »

DSP → D/A → [Antenna] ← [Antenna] ← A/D → DSP

Limitations: (1) Requirement of very high speed D/A and A/D due to Nyquist Sampling Criterion for very high carrier frequency.
(2) High power to transmit signals to long distances. **(RF Amplifier cannot be avoided)**

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So, this undesirable thing, we cannot actually completely avoid. Why is that? If you remember, when we started our discussion initially on the software defined radios, we have proposed the concept of pure digital RF front ends. And in this, we have proposed that, it is desirable that all the functionalities can be ported back to the DSP domain. And after that if first of all, we can directly do the D to A and A to D converter and it will be a very small portable device and it will be a very re-configurable, but what was the limitation? First limitation was A to D and D to A speed, but we have seen that we have methods like R F sampling where, we can directly do the sampling at the RF speed and in that case, we can actually avoid all these components like mixers and filters and additional filters.

So, this first limitation is the current the modern scenario we are able to actually reach into a stage where we can actually avoid this problem, but the second one the high power to transmit signal to long distances, we cannot avoid RF amplifier, whenever we have to amplify any signal we have to have amplifier there. So, the power amplifiers the amplifiers which are actually used to power up the signal, so that they can travel to a long distance; they are of two types first is solid state.

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Power Amplifiers

• Solid State Power Amplifiers: SSPAs	• Travelling Tube Wave Amplifiers: TWTAs
•Transistors.	•Vacuum Tube based Electronic Device.
•Medium nonlinearity, memory effects.	•High nonlinearity, no memory effects
•Compact in structure	•Bulky in structure
•Wireless Communication frequency range	•High frequency (mm-wave applications)
•Medium efficiency and power.	•High efficiency and power.
•Mobile communication.	•Long distance, satellite communication.

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Power amplifiers short form is SSPA and their traveling to a wave amplifiers T W Ts if we have a look at these 2, the solid state power amplifiers are using transistors, they have medium nonlinearity, but they have memory effects. Memory effects means whenever you apply any signal the output is not only the function of input, but the previous value of input also.

So, that is why hence the name of memory effects. The additional advantage is that it is compact it is nature. Wireless communication frequency range is it is range of operation and most of the mobile etcetera, they use this SSPA's because of their compact nature, efficiency is medium and also power level after power level is medium and it depends on different classes.

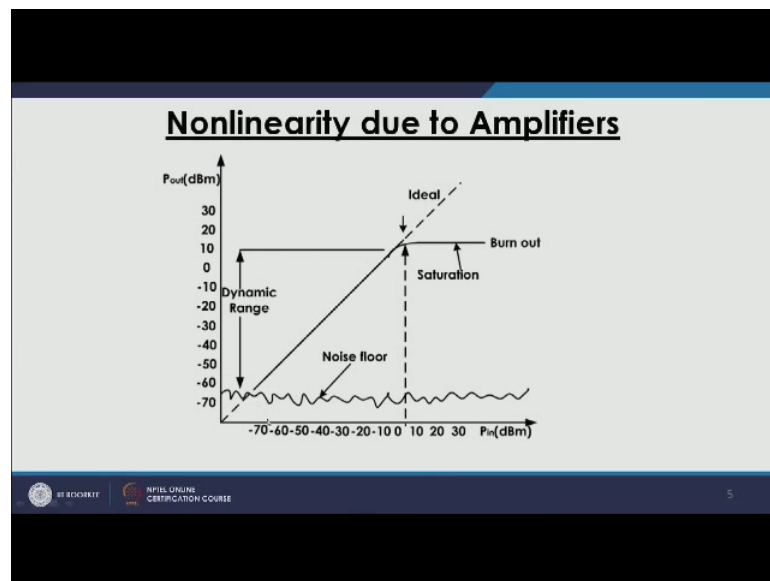
So, solid state power amplifiers have different classes and according to them, it can be operated. It is mostly used for the mobile communication. Now, if we have a look at their travelling tube wave amplifiers, they are vacuum tube based electronic devices, they have much higher nonlinearity as compared to their SSPA counterparts, they does not have any memory effects, they are bulky in nature.

But they have advantages such as they can be used for very high frequency. So, millimeter wave applications normally they can be used. They have higher efficiency with respect to the SSPA counter parts and they can provide much higher power. Now I/ want to clear here that even in the field of SSPA, their research is going on, so that high

efficiency and power can be achieved, but T W Ts are established in this field they are mostly used for the long distance and satellite communication, because they can send very high power. So, it can go very far. So, it was just a comparison based on you know outlook of these 2 amplifiers.

Now, let us have a look at the nonlinearity due to amplifier.

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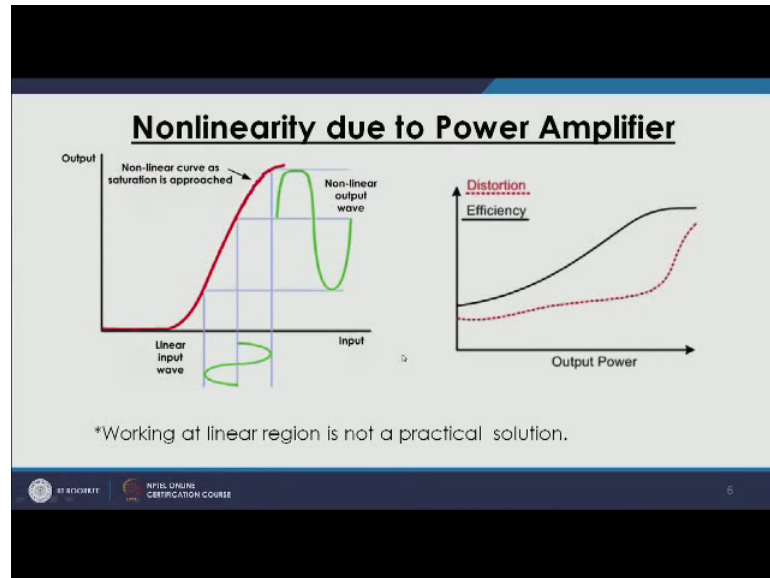
Whenever we apply any input power we want our output power which is proportional to the input power with some gain. So, if you look at this diagram whenever we are applying minus 70 dBm power in the input at the output we are getting almost or minus 60 dBm output power.

So, there is a gain of 10 and between these 2 values 10 dB gain this power amplifier is giving to us. Now it is a good thing if it is given 10 dB amplification then, whenever we apply minus 60 we should be able to get one is 50 as we can see from where for minus forty we should be at minus 30 and as long as it keeps doing this, it is working in a linear region and we say that amplification is happening properly.

But when we are going to very high input power level, after some time our output is not proportional to the input and it becomes saturated; it becomes flat, it is being shown here. If we still keep increasing, is if we still keep giving it enough power then, eventually it will burn out. So, when you start seeing the saturation power, we know that it is working

at it is peak frequency. Now, what is the problem in this scenario? As long as it is working in the linear region there is no problem, but when it starts working in the non-linear region here, then we have problems.

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Now here we are showing non-linear system where we are applying a signal at the input and the output shown in the y axis in this form. So, we can see that input wave, when it is a sinusoidal signal in the nature, when it is working in the linear range the output is the amplified version and it is sinusoidal in the nature, but whenever the waveform is going towards the saturation region then there is some clipping.

So, for this region, when it is becoming saturated, it is becoming flat here because, it not go beyond that it is clipped there and because of that our output becomes non-linear. It is not a linear function of the input anymore. Moreover, as we can see from the picture, we are losing some power here. If it would have been taken able to take the full swing, we would have taken have gotten this waveform.

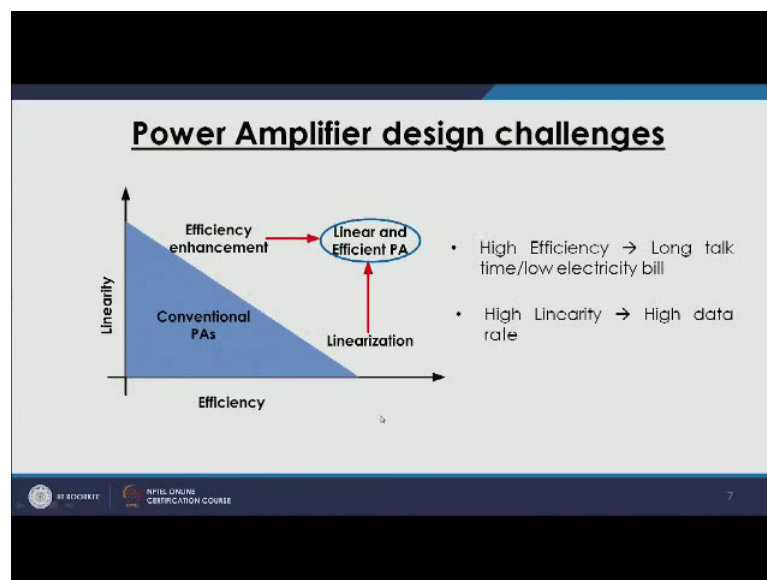
So, we are losing this power. So, where does this waveform power goes? So, we will discuss that soon. Now if you look at this diagram and the previous diagram of the P A characteristic P in versus P out, we can say let us just work in the linear range why we have to work at the saturation region at all.

Now, this is the limitation; we cannot work at the linear region because, when we see the curve of the efficiency with respect to the output power taken by the power amplifier, we can see that distortion keep increasing with the output power. So, distortion is minimum here and at this region it might be possible it is only because of the noise and on the nonlinearity.

But, if you look at the efficiency of the power amplifier, it is also increasing with the output power and it is at it is best at the maximum output power. So, efficiency means whenever you applied some power, some portion of that power is going into the output power. So, ratio of that is called the gain and their relation is defining the efficiency that how much power is converted into actually is full power.

So, of course, if our efficiency is very low, then we cannot have full utilization of the signal. We are only getting one person or some percentage of the power which we could have received. So, of course, we want our efficiency to be very high, but at very high region then, we have the nonlinearity there. So, it becomes a compromise between 2 things.

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If you plot our efficiency of power amplifier versus linearity of the power amplifier, we can see this kind of curve, which stepping in nature. It means, when we have efficiency very low then linearity is very high, but as we keep increasing efficiency, linearity goes low. So, we have to choose somewhere in between.

So, one method can be that we have to do the compromise either we lose efficiency or either we lose linearity. Now, if we lose linearity for the sake of efficiency, what is the problem there? Our data is getting distorted because, it is not the replica of the signal what we have proposed originally. So, the data rate it will fall down because, the data which is being received at the receiver, it is not the perfect data. Most of the data bits are distorted and they are not useful they are not giving the useful information. So, of course, we need high linearity.

In that case, to maintain this high linearity, we have to work in the lower region and because of that, we have to send apply very high energy at the input and it will lead to long talk time and during longer talk time it will take more and more power and the efficiency will be lower. It will lead to your high bill eventually or high lower time of the battery uses. So, battery will expire soon because, you are always taking more and more energy from that. So, we want to have good efficiency as well as linearity. So, let us have a look at the distortion parameters how they actually work.

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Distortion Parameters

Assuming a simple third-order approximation of the PA transfer characteristic given as:



$$y(t) = k_1 \cdot x(t) + k_2 \cdot x^2(t) + k_3 \cdot x^3(t) \quad \text{(A)}$$

where $x(t)$ and $y(t)$ are input and output of PA. The coefficients of PA model are given by k_1 , k_2 and k_3 .

If we apply single carrier signal: $x(t) = A \cos(\omega t)$

$$\cos^3 x = \frac{3 \cos x + \cos(3x)}{4}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

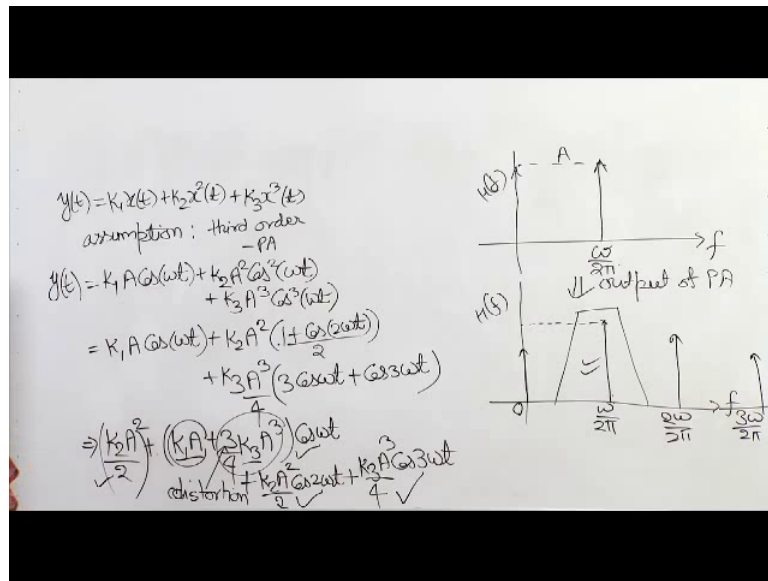
Let us assume that our power amplifier which is a non-linear device. It is actually third order transfer function. So, y is a function of x and x is the input signal.

This k_1 , k_2 , k_3 are representing the coefficients attached with the particular kernel, particular nonlinearity order which are according to any particular P A. So, what we do

in the real life that we when we have the input and output data, we fit our k_1, k_2, k_3 , so that it is resembling; the actual y of our practical power amplifier.

So, these k 's are the P M module coefficients according to that. So, let us say, a see one example, when we are applying single carrier signal. So, our $x(t)$ is actually just a sinusoidal signal which have one amplitude and ah frequency is ω and it is being applied here.

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So, when we do that; so, third order P A it is our assumption here. Our $x(t)$ is a cosine function. So, let us put that value in here. So, our $y(t)$ will become $k_1 k_2 e$ square \cos square ωt plus $k_3 a$ cube \cos cube ωt . Now, if we expand this relations, and so, the expression becomes this. So, if we collect our terms together for the D C frequency and ωt frequency in the higher frequency, you will have D C terms as. So, there is no frequency in this component it is coming from here and it is just coming from there sorry.

So, only this is the D C term. Then, we can collect the coefficient which as connected with the our original frequency and then, we can collect the high order terms. So, we have 2 higher order terms twice of the frequency which is $k_2 a$ square divided by 2 $\cos 2 \omega t$ and 3 not 3 this $k_3 a$ cube divided by 4 ωt .

So, if you look at this expression, we will have 4 components from here. If we apply single sinusoidal signal, one will be at D C, one will be at our original signal frequency, one will be at second harmonic and one will be at the third harmonic.

So, we apply a signal which was at and the in-frequency domain, we had applied a signal, which was at ω upon 2π frequency and at the output because of the nonlinearity, we are getting a signal which is having a component at 0 frequency. A component here, which does not have the same amplitude. It has different amplitude than this one and ω upon 2π .

Then, it has some component at 2ω upon 2π as well as 3ω upon 2π right. So, for one signal in the frequency domain, we will see more than one components. Now these components are very far. So, you can say we will filter it out and we can take this signal, but still if you look at this our output is not exact replica of our input. Basically, it has some extra terms here. So, we wanted actually just this term, but this is the distortion term, this is term which is coming from the nonlinearity this one.

So, this was the case of the single tone we can from our eyes, we can visualize that what will happen to different terms there. Now, most of the advanced communication system waveforms they are multi carrier. So, it is more significant. So, take care of total test we have to see that is mutual interaction between the different tones here.

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Two-tone Intermodulation

A two-tone test may be performed, to understand the individual and mutual interaction in the nonlinear PA.

The two tone signal are situated at frequencies f_1 and f_2 ($f_1 < f_2$) with amplitudes X_1 and X_2 respectively:

$$\begin{aligned} x(t) &= X_1 \cos(2\pi f_1 t) + X_2 \cos(2\pi f_2 t) \\ &= X_1 \cos(\omega_1 t) + X_2 \cos(\omega_2 t) \end{aligned} \quad \text{(B)}$$

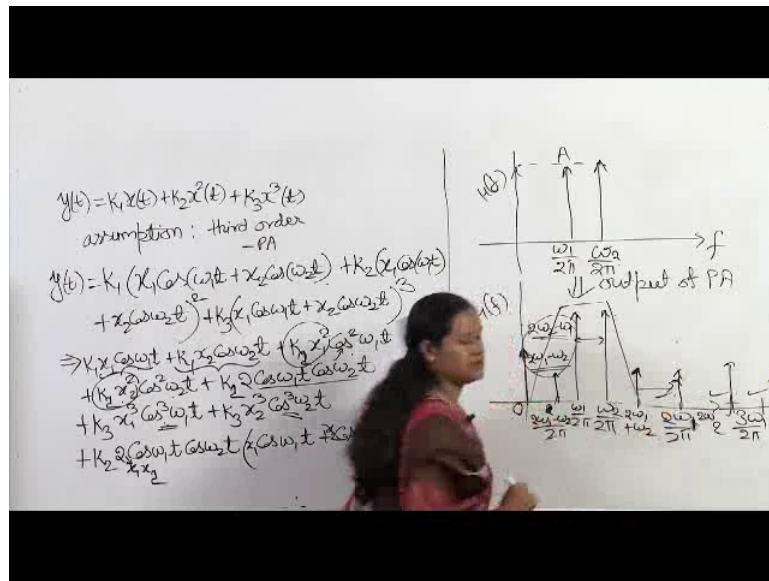
From equation (A) and (B), the output signal becomes:

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So, let us assume that, we have 2 tones and they are situated at the frequencies f_1 and f_2 and we are assuming that f_1 is at a frequency which is lower than the f_2 . Now, we have 2 amplitudes one is x_1 and the second is x_2 and this is what we are representing here $2\pi f_1$ we are representing as ω .

So now, instead of this, we apply this 2-tone signal here. So, when we apply the signal I still we are using this model of the third order; so, $y(t)$.

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K_1 is given and instead of $x(t)$, we are using our new signal which has the amplitude $x_1 \cos \omega_1 t + x_2 \cos \omega_2 t$. Now, the second term is the same, but with the square term. So, we will be opening the expression for this one. So, this part will be intact. So, it will be $k_1 x_1 \cos \omega_1 t + k_1 x_2 \cos \omega_2 t$.

Now, in this expression it will have expression such as $k_2 x_1^2 \cos^2 \omega_1 t + k_2 x_2^2 \cos^2 \omega_2 t + k_2 x_1 x_2 \cos \omega_1 t \cos \omega_2 t$ it is $k_2 k_2 k_2 k_2$ now for the third expression, k_3 will be there and it will be $x_1^3 \cos^3 \omega_1 t + k_3 x_2^3 \cos^3 \omega_2 t + k_3 x_1^2 \cos^2 \omega_1 t \cos \omega_2 t + k_3 x_1 \cos \omega_1 t \cos^2 \omega_2 t$ and the bracket.

So, it has x_1 here and x_2 and similarly, they also have $x_1 x_2$ here multiplied. So, we will have all this terms there. So, we can simplify this. From here, we can see that right the original frequency, we are going to have these components. From here similar to the

previous case, we will have 1 plus cos 2 omega 1 t. So, we will have one D C coefficient from here and one at 2 omega 1 term. Similarly, here will be one component at 2 omega 2 and one D C component. This kind of term you always know that when we are multiplying 2 sinusoidal signals, they give of the frequency of omega 1 minus omega 2 and omega 2 minus omega 1.

So, it will be difference of these frequencies. This will this terms will give you this single tone term which will give the distortion and the term at the 3 omega 1. Similarly, the term at the 3 omega 2 and one term which will be added to this term; in this term, you will be multiplying them together and then you will have the terms, where you will have 2 omega 1 minus omega 2 and 2 omega 2 minus omega 1 terms. So, we are summarizing these terms in this slide and the output is shown here. So, D C term is basically k 2 x 1 square plus k 2 x 2 square which is coming from here, if we open this expression.

It will give you k 2 x square and k 2 x 2 square. Now, at our first frequency omega 1 frequency we are having component which are actually multiplication of x

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$$\begin{aligned}
 y(t) = & \frac{k_2}{2} X_1^2 + \frac{k_2}{2} X_2^2 + X_1 \cdot \left[k_1 + \frac{3}{4} k_3 X_1^2 + \frac{3}{2} k_3 X_2^2 \right] \cos(\omega_1 t) + \\
 & X_2 \cdot \left[k_1 + \frac{3}{4} k_3 X_2^2 + \frac{3}{2} k_3 X_1^2 \right] \cos(\omega_2 t) + X_1^2 \frac{k_2}{2} \cos(2\omega_1 t) + X_2^2 \frac{k_2}{2} \cos(2\omega_2 t) + \\
 & X_1 X_2 k_2 \{ \cos[\omega_2 - \omega_1] t + \cos[\omega_2 + \omega_1] t \} + X_1^3 \frac{k_2}{4} \cos(3\omega_1 t) + X_2^3 \frac{k_2}{4} \cos(3\omega_2 t) + \\
 & \frac{3}{4} k_3 X_1^2 X_2 \{ \cos[2\omega_1 + \omega_2] t + \cos[2\omega_1 - \omega_2] t \} + \frac{3}{4} k_3 X_1 X_2^2 \{ \cos[2\omega_2 + \omega_1] t + \cos[2\omega_2 - \omega_1] t \}
 \end{aligned}$$

1 and x 2 both; so linear oppression of x 1 second order of the amplitude of the first frequency signal and the multiplication of the 2 amplitudes. Similarly, at the second frequency also, the amplitude is a function of the amplitude of the both the signals x 2 as well as x 1. Apart from these tones, we will be having our harmonics at 2 omega 1 2 omega 2 like previous cases.

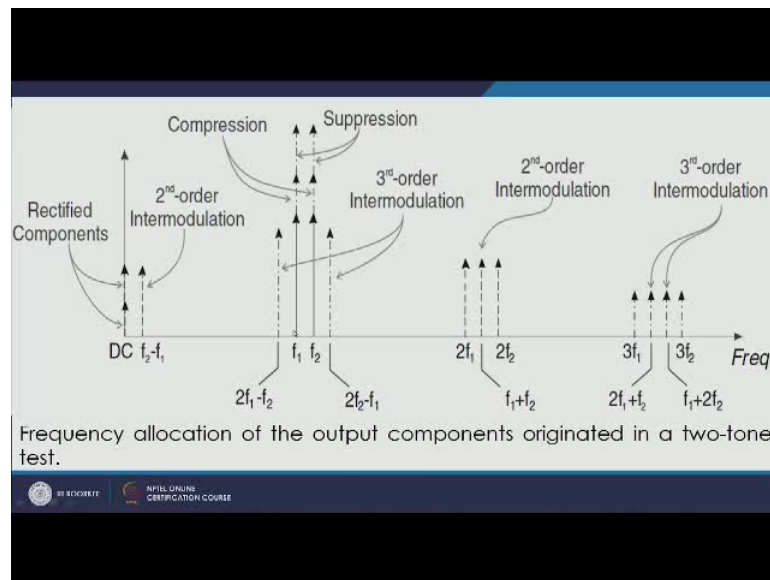
Apart from that, we will be having our first order function at $\omega_2 - \omega_1$ and $\omega_2 + \omega_1$. Because it is third order, we are also seeing the third harmonic here and apart from that we will be having $2\omega_1 + \omega_2$, $2\omega_1 - \omega_2$ like components at the output and we can combine them together and they will have this amplitude which is again the function of amplitude of both the frequencies.

So, in the frequency domain, if we have a look at this signal itself, let us say it was ω_2 by 2π and it was ω_1 upon 2π it was 2 tone signal, we have seen from there we will have some signal at D C, we will have some signal at the original frequency ω_1 and ω_2 and we will have $2\omega_2 - \omega_1$.

So, it will have $\omega_1 - 2\omega_2$ divided by 2π and some addition term is also there very though $\omega_2 - \omega_1$ are being added together. So, it is it will be somewhere here, something like this. In fact, it will be much farther away. And then, you will have $2\omega_2$ terms $2\omega_1$ will be before that and $2\omega_1$ and $3\omega_2$ terms all those terms will be appearing here.

So, as you can see, total signal has more higher number of distortion products here. Again, $2\omega_2 - \omega_1$ term and $2\omega_2 - \omega_1$ term it will be very much near to this one because, there will be very small distance with respect to each other here. If we see $\omega_1 - \omega_2$ components summation, it will fall much further here. So, basically if you put a filter here, then this third component has a tendency to come in the filter range and the harmonies etcetera we can easily filter out all right.

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So, this is how it will look in the frequency range f_1 f_2 , because of per amplified this should have some gain, but because some of the power has leaked into some other domains.

So, at $2f_2 - f_1$ $2f_2 - 2f_1 - f_2$ the nearby components are these and because, the addition of these terms it is 2 times here and one times here it is called third order intermodulation. The frequency terms which are twice of the original frequencies, they are called second order intermodulation. So, basically f_1 plus f_2 terms 2 times f_1 terms and 2 times f_2 terms they are called single second order intermodulation. The terms which are in addition, but their summation of 2 times f_1 and f_2 it is again third order intermodulation the third harmonic also part of this one.

The DC and $f_2 - f_1$, they will be very near at the end. So, as I/ was saying if you put a filter here, it is difficult to filter these components. These components can be easily filtered. So, normally whenever you are here about the power amplifier nonlinearity, you will see that people are mostly talking about third order intermodulation and not much about other kind of modulation because, they are much further there. So, if you look at the amplitudes here at

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PA output for two tone test signal			
Term	Frequencies	Amplitude	Classification
$x(t)$	f_1, f_2	X_1, X_2	Linear term
	$2f_1, 2f_2$	X_1^2, X_2^2	Second harmonic
	dc (from f_1), dc (from f_2)	X_1^2, X_2^2	Rectified component
$x^2(t)$	$f_1 - f_2$	$X_1 \cdot X_2$	Second-order intermodulation
	$f_1 + f_2$	$X_1 \cdot X_2$	Second-order intermodulation
	f_1, f_2	X_1^3, X_2^3	Compression
	f_1, f_2	$X_1 \cdot X_2^2, X_1^2 \cdot X_2$	Suppression
	$3f_1, 3f_2$	X_1^3, X_2^3	Third harmonic
	$x^3(t)$	$2f_1 - f_2, 2f_2 - f_1$	$X_1^2 \cdot X_2, X_1 \cdot X_2^2$
$2f_1 + f_2, 2f_2 + f_1$		$X_1^2 \cdot X_2, X_1 \cdot X_2^2$	Third-order intermodulation

The original frequencies f_1 and f_2 if our amplitudes for x_1 and x_2 are at twice of the original frequencies we get the x^2 function second harmonic. At the near the D C and level from the first and second, we are having the x_1^2 and x_2^2 component.

At the first order sorry second order because the summation of f_1 and f_2 , their 2 frequencies second order intermodulation are if their of the this nature $f_1 - f_2$ then, they are just simply the multiplication of these 2 amplitudes, whether it is negative it is subtraction or whether it is addition it is just the multiplication of 2 amplitudes. The compression basically is seen in terms of q function, x_1^3 and x_2^3 and separation is actually intermodulation of these 2 terms which is x_1 into x_2^2 and x_1^2 into x_2 . So, just at particular frequency you can imagine you can have you know intimate you can have a guess that what should be the amplitude order if you know the original amplitudes.

At the third harmonic, it is simply the cube of the original amplitudes. Now, look at the third order intermodulation which are very important intermodulation products at the negative frequency. Whichever frequency is coming earlier in multiplication with the 2, it is having the square term and the other term is multiplied this in the amplitude of that is just simply multiplied without any power there. Similarly, in this case $2f_1 - f_2$ minus $2f_2 - f_1$ should be there and it is f_2^2 as for x_2^2 , which is related to the second harmonic.

Similarly, here also whenever you have multiple multiplication in the frequency domain in the amplitude you can see the power of that frequency. So, the n th order intermodulation component for


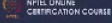
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PA output for two tone test signal

n th-order intermodulation components, for n odd, will have amplitudes given by

$$\frac{1}{2^{n-1}} \cdot \left(\frac{n}{n+1} \right) \cdot k_n \cdot X^n$$

For equal amplitude of both tones, there is a linear relationship between IMD powers and input signal power.

n equal to odd will have amplitudes which are given by this formula. So, how we can use it? Suppose, we want to calculate for the n equal to 3 because it is some important IMD term. So, let put an equal to 3 here 1 upon.

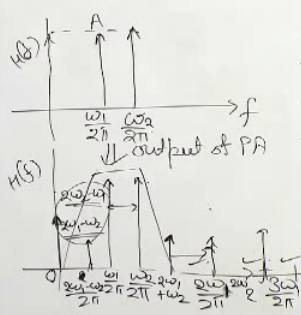
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$$\frac{1}{2^2} \cdot \frac{3}{2} \cdot k_3 X^3$$

$$\Rightarrow \frac{1}{4} \cdot \frac{3}{2} \cdot k_3 X^3$$

$$\Rightarrow \frac{1}{4} \times \frac{3 \times 2 \times 1}{2 \times 1 \times 1} \cdot k_3 X^3$$

$$\Rightarrow \frac{3}{4} k_3 X^3$$



The diagram illustrates the frequency spectrum of a two-tone test signal. The input tones are at frequencies $\frac{\omega_1}{2\pi}$ and $\frac{\omega_2}{2\pi}$. The output spectrum shows the original tones and third-order intermodulation products at $\frac{\omega_1 + \omega_2}{2\pi}$ and $\frac{\omega_2 + \omega_1}{2\pi}$. The amplitude of the intermodulation products is shown to be $\frac{3}{4}$ of the input signal amplitude A .

2 to the power n minus 1. So, it will be 2 here, then $3 C 2$ which is 3 plus 1 divided by 2 and K^3 and X^3 .

So, this will be the power of that. So, if you simplify this it will become factorial 3 factorial 2 factor 1. So, then $K^3 X^3$ factorial we are opening. So, the amplitude of third order intermodulation product should be $\frac{3}{4} K^3 X^3$ and this is what we are seeing this expression also. Third order on intermodulation product whether they are the third harmonic or they are third order intermodulation they are using the coefficient of $\frac{3}{4}$ always.

So, with this we are finishing this lecture and in the next lecture, we will continue with this more specification which are based on this analysis.

Thank you.