

Basics of software-defined radios & practical applications
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Lecture - 10
Distortion Parameters: Nonlinearity Specifications

So, in the series of Basics of Software Defined radios and practical applications, we have discussing distortion parameters and, we have seen the distortion due to power amplifier nonlinearity. And now we will see this specifications which defines the impact of such distortion on the signal quality.

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PA output for two tone test signal

n th-order intermodulation components, for n odd, will have amplitudes given by

$$\frac{1}{2^{n-1}} \cdot \binom{n}{\frac{n+1}{2}} \cdot k_n \cdot X^n$$

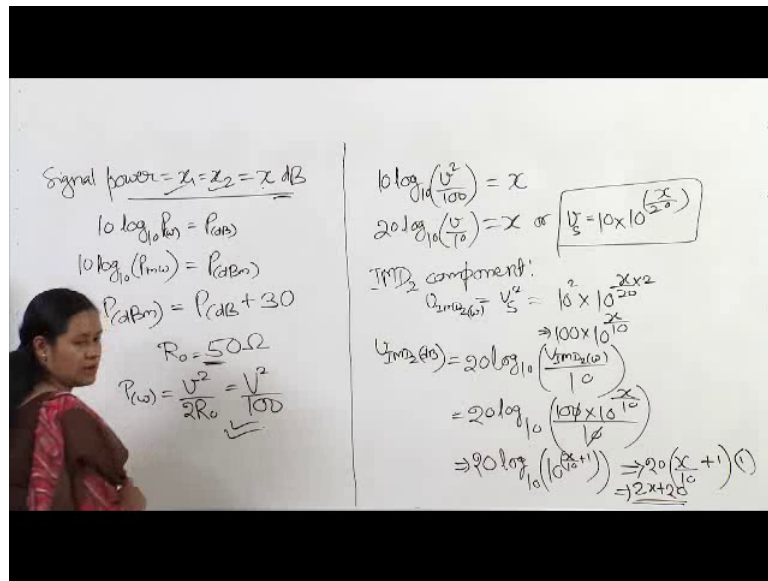
For equal amplitude of both tones, there is a linear relationship between IMD powers and input signal power.

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So, this was the expression which we had seen for the amplitude of the IMD terms, when this is the n th order IMD and if n is the odd number. We have discussed that the IMD 3 and the odd ordered IMDs they come near to the actual signal and they are more important.

So, let us have a look, if this IMDs have any relation with the input power, for this analysis purpose, let us assume that the amplitude of both the tones are equal.

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So, signal power for both the tones x_1 is equal to x_2 is equal to x . Now, let us say it is in dB and we, want to calculate the respective voltage for this thing. We have discussed earlier that, when you want to convert from watt to dB it is $10 \log_{10} P_w$ means watt and it becomes $P_{in} \text{ dB}$, if we do $10 \log_{10}$ in milliwatt.

Then it is $P \text{ dB m}$ and $P \text{ dB m}$ is actually your power in dB, plus 30 because, in milliwatt it is 10 to the power 3. So, that becomes plus 30 here; so, this relation we have to keep in mind, moreover we have discussed earlier that, whenever we are working with the RF circuit our cables, they have to be matched they have the impedance of 50 ohm. So, R_{naught} is always 50-ohm and. We match our components which are to be added to that cable to the 50 ohm. So, all the power calculations happen with respect to this 50 ohm.

So, basically if you want to convert a voltage, to power it will be P_w will be V^2 square upon $2 R_{naught}$ or, it will be V^2 square upon 100. So, we will keep this in mind and we will do our calculations. So, we were starting with the point that our signal power at both the tones is $x \text{ dB}$, now we want it to convert into voltage.

So, what will be the relation $10 \log_{10} V^2$ upon 100, should be equal to x . We can also write it $20 \log_{10} V$ upon 10 equal to x or, we can say our voltage is actually 10 to the power x upon 20 and, multiplied with 10 right. So, x upon 20, 10 to the power and multiplied with this thing. So, it is for the signal, main signal as a representing our signal voltage.

Now, if this our signal voltage, we had seen from our expression that, for the second order intermodulation, it is always x_1 into x_2 or x_1 square x_2 square. Now we have assumed that x_1 and x_2 , they have the same power of x . So, for any of the second order distortion, it becomes x square x square again, x square x square.

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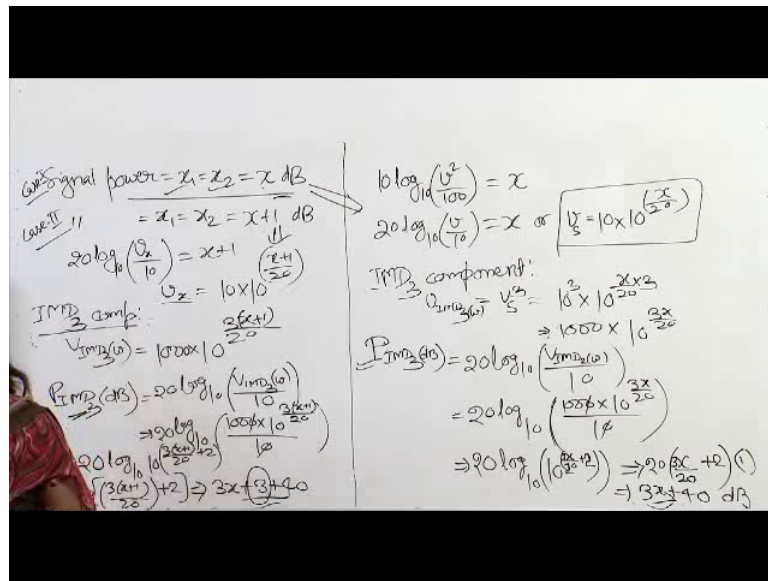
PA output for two tone test signal			
Term	Frequencies	Amplitude	Classification
$x(t)$	f_1, f_2	X_1, X_2	Linear term
	$2f_1, 2f_2$	X_1^2, X_2^2	Second harmonic
	dc (from f_1), dc (from f_2)	X_1^2, X_2^2	Rectified component
$x^2(t)$	$f_1 - f_2$	$X_1 \cdot X_2$	Second-order intermodulation
	$f_1 + f_2$	$X_1 \cdot X_2$	Second-order intermodulation
	f_1, f_2	X_1^2, X_2^2	Compression
	f_1, f_2	$X_1 \cdot X_2^2$	Suppression
		$X_1^2 \cdot X_2$	
	$3f_1, 3f_2$	X_1^3, X_2^3	Third harmonic
$x^3(t)$	$2f_1 - f_2, 2f_1 - f_2$	$X_1^2 \cdot X_2$	Third-order intermodulation
	$2f_1 + f_2, 2f_1 + f_2$	$X_1^2 \cdot X_2$ $X_1 \cdot X_2^2$	Third-order intermodulation

So, any kind of second order intermodulation, we can say that our voltage is becoming double, please keep in mind in this table, this x are representing your values in voltage in amplitude, in my assumption this $x_1 x_2$ I am representing power. So, please bared that in mind.

Now, our IMD 2 component, let us call it $V_{IMD 2}$, it should be square of this term V_s square. So, it will become 10 square into, 10 into x upon 20 into 2 or, I can say 100 into 10 to the power x upon 10. So, this is the voltage here, now if I want to convert it back into dB. So, it is in watt $V_{IMD 2}$ and dB will be what? Again $20 \log 10 V_{IMD 2}$ in watt, divide by 10, by using this relation again right. So, we are getting our power in dB here

So, let us simplify this, it becomes 100 into 10 to the power x upon 10, divide by 10 it will cancel out one of the data, it will become $20 \log 10$ and, it will be 10 to the power x upon 10, plus 1 now they have the same base. So, if we take it before this, it becomes 20 times x upon 10, plus 1 and this term will be $\log 10$, 10 means 1. So, final expression becomes $2x$ plus 20, let us keep this in mind.

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Now, if single power is increased by plus 1 dB. So, x_1 is x_2 , but now the power has been increased by 1 dB. So, x plus 1 dB. So, what should be voltage corresponding to this, again they can do the calculation, like we had done the calculation here $20 \log_{10} V_x$ upon 10 is x plus 1. So, we will have our expression like that, instead of x we will have x plus 1.

So, new V_x will be 10 into 10 to the power x plus 1 upon 20. So, we have increased case 2. So, case 1 was when we have power x , case 2 is when we have increased power by 1 dB now V_x is there. So, what will be the IMD 2 component for this? $V_{IMD 2}$ in watt will be this square of this term. So, again 100 into 10 to the power x plus 1 upon 10.

So, we had done all these calculations, instead of x we are using x plus 1 here right. So, it is for the power when 1, power increase by 1 dB. Now let us calculate the power, power for this one actually this should be power also, because they calculating power here right. So, $P_{IMD 2}$ dB should be $20 \log_{10}$ and $V_{IMD 2}$, which should be converted in back into our power divided by 10.

So, it becomes $20 \log_{10}$ and, 10 to the power. So, it is 100 into 10 to the power x plus 1 upon 10 here (Refer Time: 10:01) divided by 10. So, this will go away and the expression becomes $20 \log_{10}$, 10 to the power x plus 1 upon 10 plus 1. So, we take it further, it becomes $20 \times$ plus 1 upon 10. Plus 1 and 10 if there the same base it will be 1. So, if we simplify it will be $2x$ plus 2 plus 20.

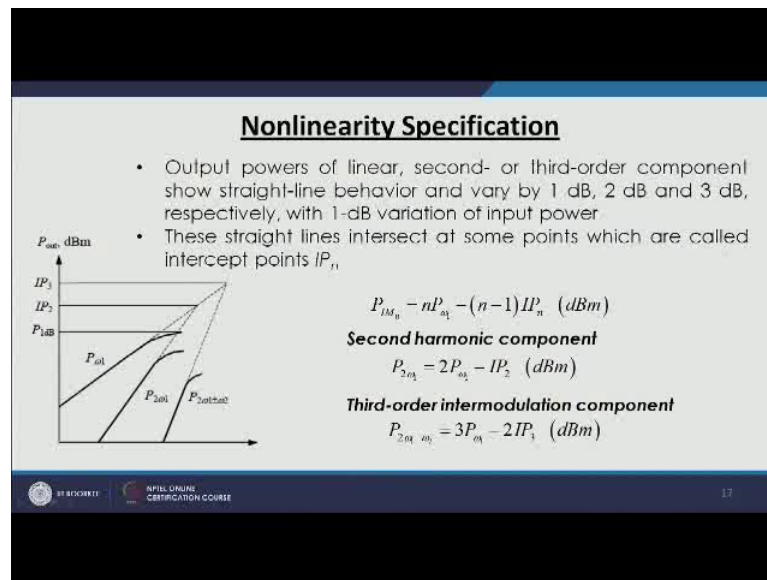
So, initially when it was not increase in 1 dB, it was $2x + 20$. Now it is $2x + 2 + 20$. So, this the impact on the second harmonic. So, if we increased our input single power by 1 dB the second distortion terms, which can be harmonic or the distortion terms, they increase by 2 dB, now let us see what will happen for the third inter modulation product.

So, whole calculation they have to be done for the third inter mode, third inter mode. So, it will become cube. So, it will be cube and it will be 3. So, it will be thousand into $3x$ upon 20 and again the calculation here, will have the same factor it will be thousand into 10 to the power $3x$ upon 20, divide by 10 it will be canceled out and, it will become $3x$ upon 20 plus 2. And the output individually will become $3x$ divide by 20 plus 2. So, it will become $3x + 40$.

So, for the third IMD product it will become $3x + 40$ dB. So, it is IMD 3 similarly, if I increase our signal power by 1 dB then, I am calculating for the third inter modulation product then, what will happen? That we can take the calculation from there it will be 1000 into 10 to the power $3x$ upon 20 and, we can put that value here, because if third power and instead of x we should use $x + 1$ because we have increased by 1. So now let us do this calculation for IMD 3. So, 1000 into 10 to the power 3 times $x + 1$ divided by 20 divided by 10.

So, this goes here. So, $20 \log 10$ and it becomes, 10 to the power $3x + 1$ divide by 20 plus 2 and, if we simplify further 20 times, $3x + 1$ divide by 20 plus 2 and the output becomes $3x + 3 + 40$. So now, for the third order distortion if you compare this term and, this term by increasing the signal power by 1 dB we have increased the signal power IMD power for the third inter mode by 3 dB.

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So, by increasing the signal power by 1 dB; we are getting the variation of 1 dB 2 dB 3 dB for the inter modulation product, for the first order it is 1 dB, for the second order increase it 2 dB and, for the third order the increase will be 3 dB.

So, it represent a straight line because, the variation is only incremental there and, it is shown in this way, if we are plotting in the logarithmic domain. So, if we plot our input versus output and this x axis input power in dB m, output power in dB m then, they have a linear relation, with respect to each other. Because of this relation, we can have the intercept of these lines, with the original signal line and this signal points are called intercept point or IPn.

So, this n is representing the order of the IMD. So, if it is second order. So, it will be IP2, if it is third order it will be IP3. So, in this diagram we are showing the diagram that for P omega 1 is the actual signal, which we should receive. So, this is the original signal and because of that, the second order distortion their power it is being plotted here and, for the third order it is being plotted here.

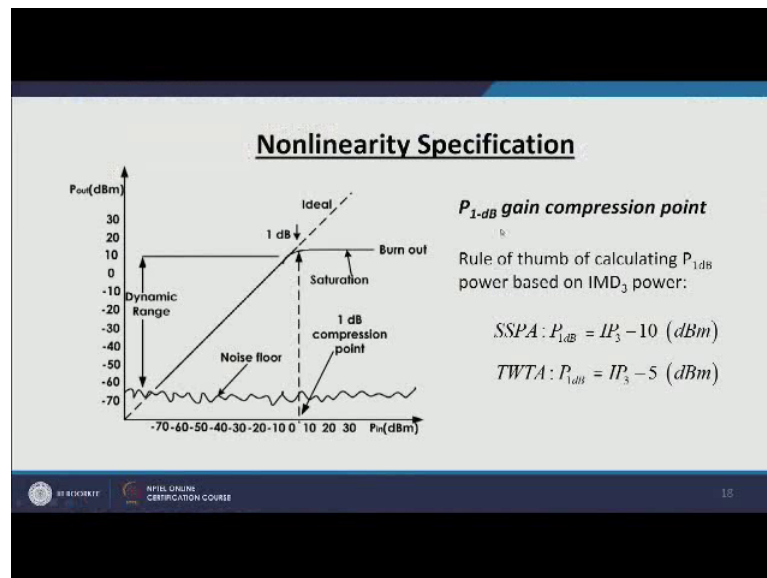
And if you see in this diagram, if we keep increasing the trajectory in it is original path, they will intersect at some point. So, these points where they are actually intersecting, with the original signal power path they are called IPn. So, this the power of this can be calculated based on the, equation of line and this is given by the formula, n times the original signal power, minus n minus 1 IPn point where it is intersecting the P omega 1

point, if we know these 2 points these 2 values. We can calculate the IMn product for example, for second harmonic component for the 2 omega 1, n will be equal to 2. So, 2 P omega 1 and, P omega 1 is the power of the original signals and n is 2. So, n minus 1 is 1 n IP2 value, you can put their.

So, by if you know these 2 values IP2 point and, you know the P omega 1 values at that point, then you can calculate your power at 2 omega 1, that is that is the power of second harmonic similarly, for the third order inter modulation which is very important it is near to the our signal. So, it is third order because 2 times of 1 frequency and 1 type of other frequency. So, 2 plus 1 third inter modulation.

So, n equal to 3, 3 times P omega 1 minus n minus 1, 2 times IPn is equal to 3. So, by doing these 3 values we can calculate the distortion component and vice versa, if you are able to measure the IMD terms by some measurement instruments, then you can define this IP3 point, that where it will intercept the path of the original signal. So, it is one of the definition which is used a lot in the communication system.

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Another definition is P1 dB gain compression point. So, as I told you initially that, when you plot your Pout versus Pin, it is straight line, but after some time it becomes saturated and, it becomes almost constant right. So, the point where it is 1 dB compressed, with respect to the original path it is called 1 dB point; means if we keep increasing, if we keep drawing this original path. The path when it deviates 1 dB with respect to the

measure path it is called 1 dB point. So, this dotted line which we are reading in the Pin, input power will be called one dB input compression point where the data is compressed by 1 dB with respect to the original path.

So, this is a very important parameter most of the commercial sheet, they provide you the P1 dB gain. So, you can have some idea if this P1 dB gain is very high, you can assume that your device is quite linear for a long range, if it is low it means it will be saturated very early. So, this is how it is provided in the commercial datasheets, now the rule of thumb for calculating P1 dB power, based on the IMD 3 power is given by the formula for solid state power amplifier P1 dB can be calculated by it is IP3 power, third intercept point minus 10, it is a rule of thumb which has been seen by measurement people have measured so many times for the different solid state power amplifier and, they found that normally they follow this rule.

Similarly, for TWT a the rule is IP3 minus 5. So, what do we see from here? If we have same IP3, we have inter section point which is similar, whenever P1 dB point comes quite early as compared to the SSPA. So, we can say from here that about TWT a is more non-linear, with respect to SSPA. Because our P1 dB point, is coming earlier as compared to the SSPA point. So, it is saturating earlier as compared to the SSPA point.

Again, it is important to note, if we are able to have our IP3 calculation and, we have seen that IP3 calculation we can do, if we know the signal power and, if we are able to measure the intermodulation power, IP3 value we can calculate easily. And once we have IP3 value, we can actually define our P1 dB point, with this P1 dB point we can actually decide, which power amplifier to purchase, by looking at the data sheet of the power amplifier.

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Carrier to Intermodulation Ratio

Defined as the ratio between useful output power and IMD power, usually measured, using logarithmic units, in decibels below the carrier (dBc):

$$C/I = \frac{P_{out}}{P_{IMD}}$$

$$C/I = \frac{P_{out} \left[f_{n(m)} \right]}{P_{IMD} \left[2f_{2n(2m)} - f_{m(n)} \right]} \quad n, m = 1, 2$$

Third harmonic is most prominent near band of interest

$$P_{IMD_3} = 3 \cdot P_{out, dBm} - 2 \cdot IP3_{out}$$

$$\longrightarrow \left(\frac{C}{I} \right)_{dBc} = 2 \cdot (IP3_{out} - P_{out, dBm})$$

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Now, one of the important parameter is, carrier to inter modulation ratio. So, it is defined as the ratio between the useful output power and, the IMD power which is normally given in terms of logarithmic units. So, it is in dB c so.

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Handwritten derivations for Carrier to Intermodulation Ratio (C/I) in dBc:

Left side (Spectrum Diagram):

- Carrier power: $P_{out} (dB)$
- Intermodulation power: $IMD_3 (dB)$
- Carrier to Intermodulation Ratio: $C/I = \frac{P_{out}}{P_{IMD_3}}$
- Carrier to Intermodulation Ratio in dBc: $C/I (dBc) = P_{out} (dB) - P_{IMD_3} (dB)$
- Carrier to Intermodulation Ratio in dBc: $C/I (dBc) = P_{out, dBm} - (3P_{out, dBm} - 2IP3)$
- Carrier to Intermodulation Ratio in dBc: $C/I (dBc) = 2(IP3 - P_{out, dBm})$

Right side (Algebraic Derivation):

- $10 \log_{10} \left(\frac{V^2}{100} \right) = X$
- $20 \log_{10} \left(\frac{V}{10} \right) = X$ or $V = 10 \times 10^{\left(\frac{X}{20} \right)}$
- IMD₃ component: $P_{IMD_3} (dB) = 20 \log_{10} \left(\frac{V_{IMD_3}(t)}{10} \right)$
- $P_{IMD_3} (dB) = 20 \log_{10} \left(\frac{100 \times 10^{\frac{3X}{20}}}{10} \right)$
- $P_{IMD_3} (dB) = 20 \log_{10} \left(10^{\frac{3X}{20}} \right)$
- $P_{IMD_3} (dB) = 3X$

If the in-frequency domain, these are your total signals, at omega 1 and omega 2 and, these are your IMD terms, then this c by I ratio is carrier to inter modulation level. So, what we do? We calculate this in dB and, the difference between these power level this is

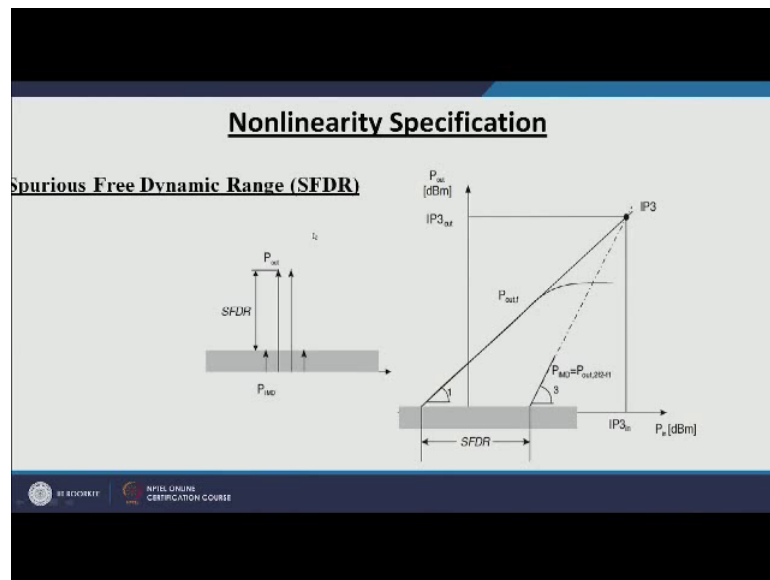
called c by I , difference between the actual carrier signal and your inter modulation product.

Normally it is third order inter mode, as we discussed before. So, because it is more prominent, then we can calculate it for this case particularly, IMD 3 this is the formula. 3 times the actual power minus 2 times of IP_3 , this is what we have calculated earlier right? This is what we have calculated original power minus 2 times of minus IP_3 power. So, this is what we are writing here, this is what we can see, the output power of the signal 3 times minus 2 times IP_3 at the output.

So, after that, we have to calculate the c by I ratio. So, carrier in this case, is actually P_{out} right? So, it becomes c by I and dB c , will be actually P_{out} in dB, minus IMD 3 component in dB, if we do that, dB or dBm, as I told earlier that dBm is nothing but anything x dB plus 30. So, this addition term it can be dBm or it can be dB, because it is a difference between these two part 30 portion will be cancelled right? X carrier minus x I. So, it will be cancelled. So, it does not matter. So, P_{out} dBm minus, IMD 3 in dB m less here.

So, IMD 3 in dB m we have seen 3 times. So, if you solve this further, it becomes 2 times IP_3 , minus 2 times P_{out} dB m or, we can take 2 out and becomes the expression which we are seeing in our ppt right? So, the difference between IP_3 out and the P_{out} and, we take the twice it works very fine for the c by I estimate. So, we can have some estimate by using this thumb rules.

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Now other parameter is spurious free dynamic range SFDR, we have seen this SFDR earlier in when, we were discussing mixers and A to D, D to A converter, this is basically the ratio of output power, with respect to the noise level or any distortion.

So, this distortion which is coming from the IMD if it is higher than the noise level, then this is called SFDR and this SFDR, will be very much equivalent to c by I , in that case if this IMDs prominent. If this IMD is below the noise level then, this SFDR becomes actually peak power to the noise level. So, just the definition, but it appears a lot in the commercial sheets.

So, this is where we are showing the range of SFDR, where our noise is prominent and, our IMDs not there at the time. So, spurious free dynamic range in terms of input power will be, up to this level because, after this level our IMD power it starts increasing and, this component will start going above the noise level. So, in the sense of input power this will be the range, where we will not see any IMD.

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Nonlinearity Specification

PA Noise Factor F (or Noise Figure NF in dB), its bandwidth B and (available) gain G , the $SFDR$ can be computed:

$$N_{out} = kT_0 \cdot B \cdot G \cdot F \rightarrow N_{out,dBm} = B_{dBHz} + G_{dB} + NF - 174dB$$
$$SFDR_{dB} = \frac{2}{3} \cdot (IP3_{out,dBm} - N_{out,dBm})$$
$$SFDR_{dB} = \frac{2}{3} \cdot [IP3_{out,dBm} - NF_{dB} - G_{dB} - B_{dBHz} + 174dB]$$

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Now, this SFDR can be computed in terms of the noise factor, if you remember our earlier lectures we have covered this topic. When the output from the chain of these components in a transmitter or in this case receiver, it is the total gain of the system, noise figure of all the systems, input noise which is calculated as minus 174 dB. So, it was the Boltzmann constant it was our temperature in kelvin. So, with that calculation kTb it was giving this value and, this b is actually detector bandwidth.

So, we already have this calculation, n_{out} output noise power. So, if we know this thing and we calculate this converted from watt to dBm and, we apply $IP3_{out,dBm}$ minus $n_{out,dBm}$, then the twice of this gives you, your c by I ratio, this is what we have calculated here and divided by 3 gives you SFDR in dB. So, this is the ratio they have given, but for the third order basically. So, we can use this calculation, to calculate SFDR with respect to our c by I or with respect to our noise figure, if it is known.



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Multi-carrier or wideband signals:

Peak-to-average power ratio (PAPR)

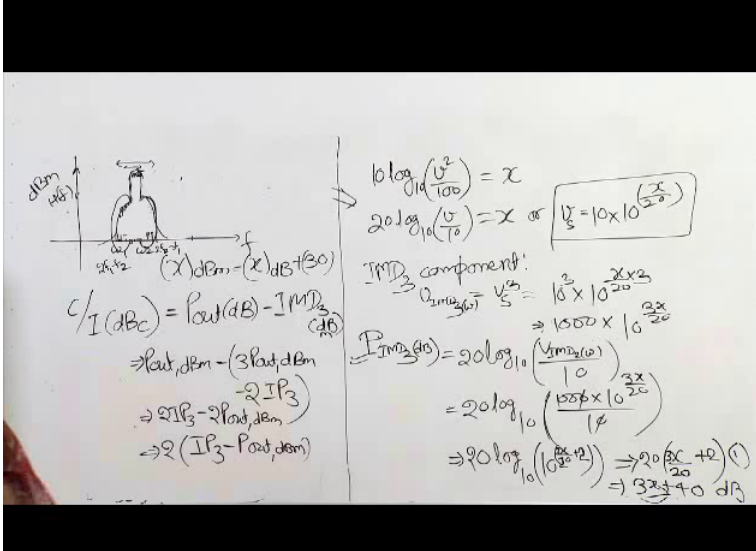
- It is the ratio between the peak power P_{peak} (related to peak amplitude) and the average power P_{avg} (related to mean amplitude) of a signal.
- Also called the crest factor and is given by:

$$PAR(dB) = 10 \log \left(\frac{\max(|x(t)|^2)}{\text{mean}(|x(t)|^2)} \right) = 10 \log \left(\frac{P_{peak}}{P_{avg}} \right)$$

Now, we have discussed till now single carrier signal and two-tone signal. Now, let us move to the multi carrier or wideband signals, when we are having two tones we have seen that, we have some of the IMD components.

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The whiteboard contains the following content:

Graph: A graph showing a signal waveform with a peak labeled $(X)_{dBm} - (X)_{dB} (30)$ and a mean level labeled $(X)_{dBm} - (X)_{dB} (30)$. The y-axis is labeled dBm and the x-axis is labeled f .

Equations:

$$C/I(dBc) = P_{out}(dB) - I_{IM3}(dB)$$

$$\Rightarrow P_{out,dBm} - (3P_{out,dBm} - 2IP_3)$$

$$\Rightarrow 2IP_3 - 2P_{out,dBm}$$

$$\Rightarrow 2(IP_3 - P_{out,dBm})$$

IMD₃ components:

$$P_{IM3}(dB) = 20 \log_{10} \left(\frac{V_{IM3}(\omega)}{V_0} \right)$$

$$= 20 \log_{10} \left(\frac{1.556 \times 10^{\frac{3X}{20}}}{1} \right)$$

$$\Rightarrow 90 \log_{10} \left(10^{\frac{3X}{20}} \right) \Rightarrow 90 \left(\frac{3X}{20} \right) (1)$$

$$\Rightarrow 3 \times 3 \times 10 = 90 \text{ dB}$$

Other equations:

$$10 \log_{10} \left(\frac{V^2}{100} \right) = X$$

$$20 \log_{10} \left(\frac{V}{10} \right) = X \text{ or } V = 10 \times 10^{\left(\frac{X}{20} \right)}$$

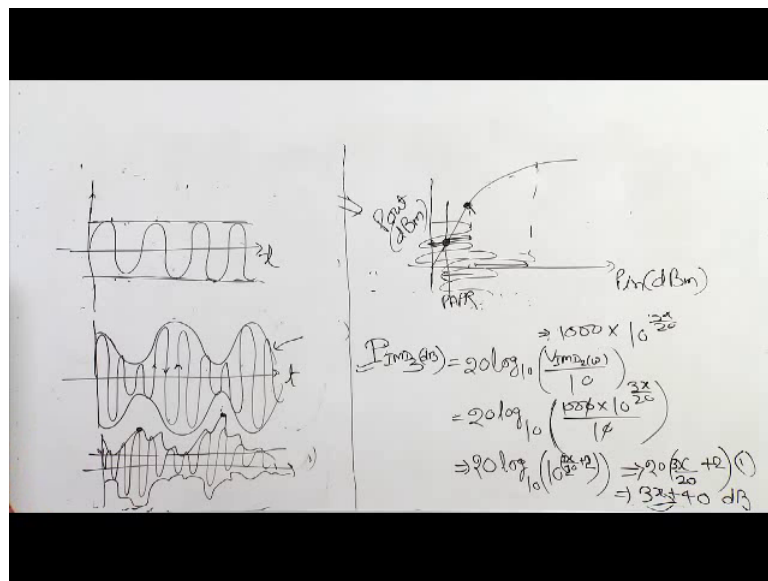
Now, let us assume it is multi tone and, multi tone in a way that, it is quite nearby so many carriers in this small space, multicarrier multi tone system. So, how it is output will be looking like. For two tone system we saw that we had some IMD 3 components here, $2 f_1$ minus f_2 and $2 f_2$ minus f_1 , this were here now because of other components, we

will see something like that. So, this was the original range and, now it become the range and these all are, the distortion component.

Basically, their IMD components, but they are so, near to each other that, we cannot separate them from our eyes. So, basically it will look to us like something like that, because we cannot distinguish all of these components separately. So, this is how it will look, some signal which is our wideband signal and then, some distortion which is moving like that, up to some distance.

So, this is what we called wideband signals and there we have, little bit differently from the single carrier and their total signals. One of the concept which we keep hearing is, peak to average power ratio, this is the ratio between peak power and, the average power because, our signal is multi carrier. So, it keeps fluctuating.

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If we want to have example, in time domain if we see our single carrier signal, it will look like, the signal is wearing, but it is envelope is constant, it is always going through this envelope. In total signal your envelope, become something like that. So, your original signal is like this, signal is this which I am plotting here and the envelope is wearing like this. So, these kinds of signal are called wearing envelope signals and, with two tone it is like this, with the multi tone it becomes even more distorted.

So, you will have some strange kind of envelope, you can have anything because, they are more and this is how the waveform moves. So, basically when you calculate the average, the average can lie somewhere here of all the values, but the peak can reach to some higher values.

So, because of that this PAPR comes into picture, which is the ratio of the peak and the average power and how it matters? Suppose this is your input power in dBm and, it is your output power in dBm and, this is your power amplifier how it looks like right. So, it was linear till here and after that, it becomes starting getting saturated.

So, I want to drive this amplifier here. So, that is not that much distorted, if my average power lies here, then my signal average power is here, but because peak is going to other values here. So, peak the signal when we are applying this, it is not constant and some of the peaks can even go here. So, in those cases some of the IMD components will generate be generated very easily. So, what I have to do, I have to back off it by PAPR of the signal; it means the peak value from the average whatever is the distance, I drive my PA here which is the PAPR of the signal.

So, my signal power is average power is here, but some of the signal amplitudes they reach till here. So, peak is falling here, but average is here and, all the efficiency calculation everything they happen with respect to the average power. So, this is the power which will define all those components; means output power will be coming from here and, all though it sometimes it will go higher.

But mostly it will remain here, your efficiency will be calculated with respect to this point. So, this PAPR is very important factor for this case. So, we are stopping the lecture here and, tomorrow we will continue with this kind of wideband signals and the specifications of nonlinearity, how this kind of signals get impacted by the nonlinearity?

Thank you.