Optimal Control

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Lecture – 07
Free End Point Problem (Continued)

Welcome friends, in this class we will continue our previous class discussion which was on the free end point problem.

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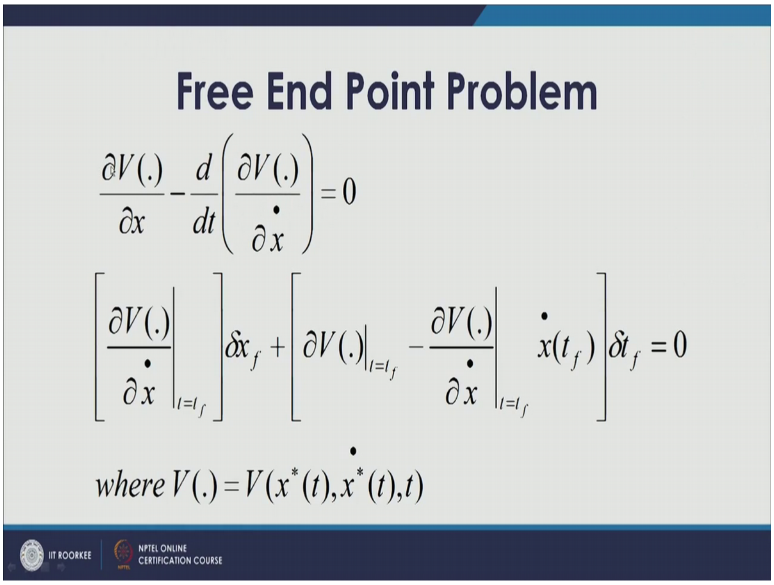

In the previous class we have drive the Euler equation plus the terminal condition, and any free end point problem is we can define as in the different cases bias for the variation of the delta x f and the delta t f the different cases can arise. The two cases we have seen in the previous classes one is the fixed end point problem in which both t f and the x of t f was fixed; initial point is also fixed terminal point is also fixed so, at terminal point delta x f and delta t f will be 0.

The second case we have seen that in which t f is fixed and the x t f is free. So, in that case my t f point 0 so this whole will become 0 and in solving a problem first we will solve the Euler equation, which will lead us to a differential equation whose solution will have the certain constants, and these constants we evaluate by using the initial point, initial condition as well as the terminal condition. In x t f free t f fixed point, my terminal condition was del V by del x dot by del x dot evaluated at t equal to t f point equal to 0.

So, in this class we will start our discussion with another case which will be our third case in which we are making t f to be free and x t f is fixed.

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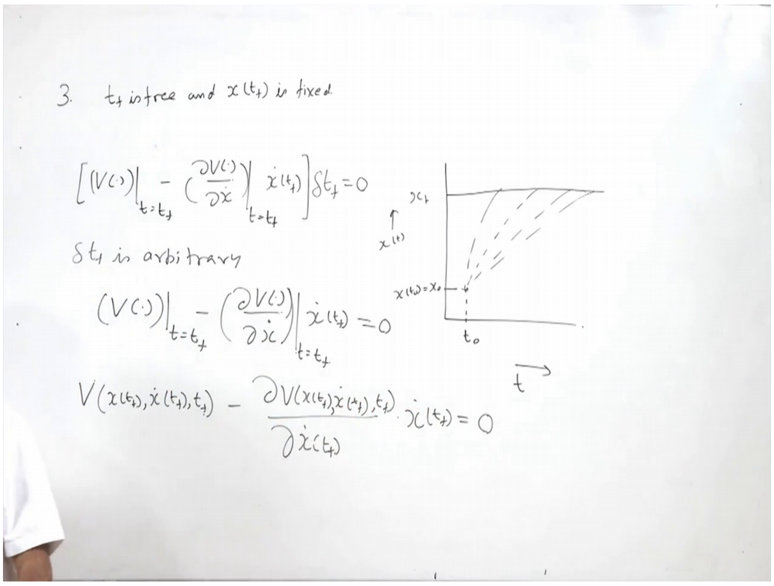

So, this is the case like means if this is my x f, this is the x t, this is t and let say this is my initial point t 0, x t 0 is say x 0. So, my final trajectory can terminate anywhere with this where x t f is fixed. So, this means in this case also my t f is varying.

So, as we have discussed in the previously also once my x t f is fixed. So, delta x f will be 0; if delta x f is 0 so in this case my this first term will become 0, and my terminal condition will be V dot which we are evaluating at t equal to t f point, minus del V by del x dot again evaluated at t equal to t f point, x dot t f delta t f equal to 0. As delta t f is t f is free. So, delta t f is arbitrary. So, we can simply write as V dot at t equal to t f point minus del V dot by del x dot again evaluated at t equal to t f point, x dot of t f that must be equal to 0. In general if we will write V x t f, x dot t f, t f minus del V x of t f, x dot of t f t f by del x dot of t f multiplied with x dot of t f equal to 0. So, we have the 2 conditions now one is if. So, the one is the initial point is given. So, I am given with the t 0 and x t 0 x f is fixed is specified.

So, x f is given t f is free which will lead to my terminal condition V minus del V by del x dot at t f point, multiplied with the x dot of t f equal to 0. So, this we can use as my terminal condition. So, to explain it in explain it further again let us consider example for this case which will give us the more insight how to use these conditions to solve a problem.

So, we take another example.

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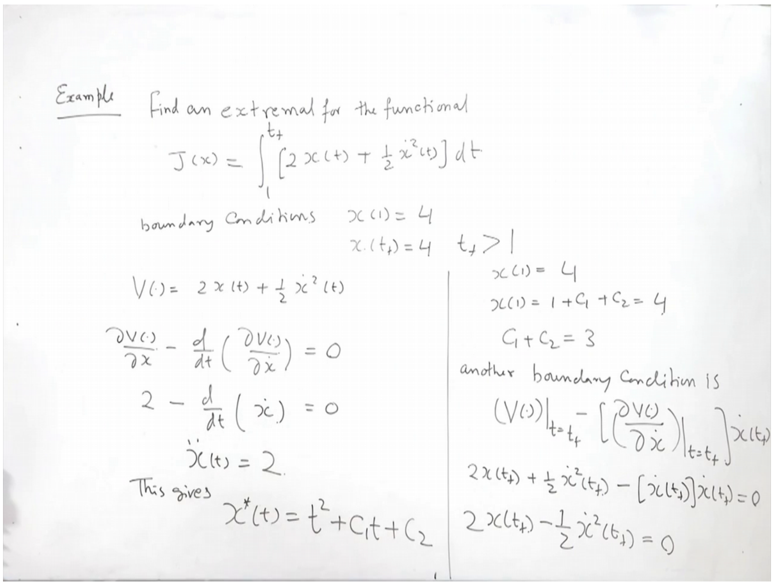

Find an extremal for the functional this is one to t f means the initial time is given as the 1, 2 x t plus 1 by 2 x dot square t, d t. So, our objective is to find the extremal of the functional J x which is subjected to the boundary condition, boundary conditions are the initial point is given as x 1 equal to 4 and x of t f because t f is unspecified this is also 4 and here my t f is greater than 1; 1 is the initial point. So, t f naturally will be greater than 1. So, this is my problem statement, we have to find out the extremal of the given functional J x with boundary condition x one equal to 4, x t f equal to 4.

So, at the first step we have to solve the e l equation given by the first equation here. So, in this case my V is 2 x t plus 1 by 2, x dot square of t. Euler equation is del V dot by del x minus d by d t of del V by del x dot that must be equal to 0. So, with this value of the V del V by del x is from here I can simply write that 2 minus d by d t of d V by del x dot. So, this is nothing but giving you x dot equal to 0. So, my equation is x double dot t equal to 2.

So, e l equation give me the another differential equation which is nothing but x double dot t equal to 2 and the solution of this. So, this gives me x t which is t square plus C 1 t plus C 2. So, if I will solve so, x dot is 2 t further I will integrate this to get the x star t as t square plus C 1 t plus C 2. So, this is the solution of my x double dot t equal to 2 which am getting here. Now I can use my initial conditions I am given with the x 1 equal to 4. So, x 1 in this case is 1 plus C 1 plus C 2 which is equal to 4. So, my equation is C 1 plus C 2 equal to 3. So, by the initial condition we can find out one equation in C 1 and C 2 which is giving me C 1 plus C 2 equal to 3.

And the second condition or say the another equation to find out the C 1 C 2 variable we will get it from my terminal point, in this case t f is free x f is 0. So, this term will be 0 and we are left only with this. So, what was my equation? So, my another boundary condition is V dot minus del V dot by del x dot x dot t f equal to 0.

So if, I will use the V as 2 x t plus half, x dot square t I will get 2 x of t f plus 1 by 2 x dot square of t f minus. So, this is giving me the V at the t f point, 2 x of t f plus half x dot square of t f because my V is given me this value; minus if I will differentiate this with respect to x of t f. So, that is nothing but x dot of t f multiplied with x dot of t f and this is equal to 0. So, am simply getting as to x of t f minus 1 by 2 x dot square of t f that is equal to 0. So, what we can get by this.

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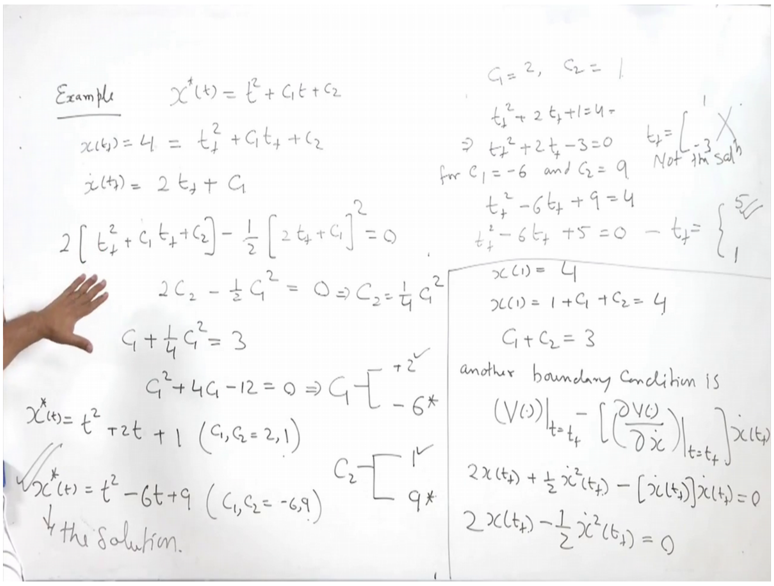

So, I have x t f equal to 4, x t f is given to me as 4 and what was my x t and if you will see we have find the x star t as t square plus C 1 t plus C 2. So, we can write this as this is nothing but my t f square plus C 1 t f plus C 2. So, this is my x t f, and x dot t f if I will write it from this; this is nothing but my 2 t f plus C 1. Now I have my x t f as this, x dot of t f as this I can place this value in this equation, 2 x of t f minus 1 by 2 x dot of t f square equal to 0. So, if I will place it. So, I have twice x of t f is t f square plus C 1 t f plus C 2 minus 1 by 2; x dot of t f is 2 t f plus C 1 square and that must be equal to 0. So, if I will solve this we get 2 C 2 minus 1 by 2 C 1 square equal to 0.

So, all other factors will be cancelled out and this if we will solve we will get this value. So, either 2 equation in the C, one is 2 C 2 minus 1 by 2 C 1 square equal to 0 and another this C 1 plus C 2 equal to 3. So, these 2 equations we have to solve to find the value of the C 1 and C 2. As we can see my this equation is quadratic in nature, so naturally I will get the 2 values for C 1 and 2 values for C 2. So, which value we will select that will be decided by the condition; which is given what is the t f which is greater than one?

So, let us first see what the value is we are getting for C 1 and C 2 in this case. So, this gives me C 2 equal to 1 by 4, C 1 square placing this value of C 2 in this equation C 1 plus C 2. So, C 1 plus C 2 in place of C 2, I write 1 by 4 C 1 square and this is equal to 3. So, this gives me equation C 1 square plus 4 C 1 minus 12 equal to 0, and this gives me the value of the C 1 as the 2 values of the C 1, 1 is plus 2 and another is minus 6. So, I get the 2 value of the c 1 by solving this equation plus 2 and minus 6. Using the value of the C 1 I can find out what is the value of the C 2; so C 2 corresponding to 2 if C 1 equal to 2. So, C 2 will be 1 with minus 6 C 2 will be nothing but 9. So, I have I can take the value as 2 or 1 or 6 or 9.

So, this means my optimal trajectory may be. So, has the 2 solutions, x star t as if I will take the 2 and one this is t square plus 2 t plus 1 with C 1, C 2 as 2 and 1. And another solution is x star t as, t square minus 6 t plus 9 with the value of C 1 C 2 as minus 6 and 9. So, which is my solution which solution I will select for this. So, this we can select by finding the value of the t f. So, we can see from this equation if I know the C 1 C 2 for the different value of the C 1 C 2.

This will satisfy let say C 1 equal to 2, C 2 equal to what I am taking the C 2; C 2 is 1. So, this equations give me t f square plus 2 t plus 1 equal to 4, this implies t f square plus 2 t minus 3 equal to 0. If I will solve this; this gives me t f equal to 1 and minus 3. For second solution for C 2 equal to sorry C 1 equal to minus 6, and C 2 equal to 9 I have t f square plus 2 t f naturally this is the t f, 2 t f sorry you to correct it. So, with C 1 is 6. So, this is minus 6 t f plus 9 equal to 4 or simply t f square minus 6 t f plus 5 equal to 0 and this gives me t f equal to 5 and 1. So, what was our condition my t f should be greater than one. So, in this case my t f is 1 and minus 3. So, this is not the solution in this case my t f equal to 5. So, I can select the second equation. So, this is the solution.

So, in such a problem we have to find out what are my constants and simultaneously we can also find out what is my t f. So, in the solution my optimal trajectory here will be t square minus 6 t plus 9 and the optimal time will be t f equal to 5. So, all the unknown parameter we can find out simply by solving using these 2 equations. So, in today lecture we have taken the three cases in which we have the fixed end point problem, the second case was our t f was fixed and the x t f was free, and the third case was we have the x t f as fixed t f as free.

So, in the next class we will discuss about when both are free then these 2 equations will exist and the different cases arises in that particular case that we will discuss in the next class. So, today we will stop here.

Thank you very much for your attention.