Optimal Control
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Lecture - 28
Inverse Matrix Riccati Equation

So, welcome friends; we are discussing about the some alternate forms of the Riccati equations the previous lecture, we have discuss about the specified degree of stability in which by modification into the arithmetic Riccati equation we can ensure the a stability of a closed loop system for a specified degree. Today we will discuss about the inverse matrix Riccati equation which normally appears if we will have a fixed endpoint problem.

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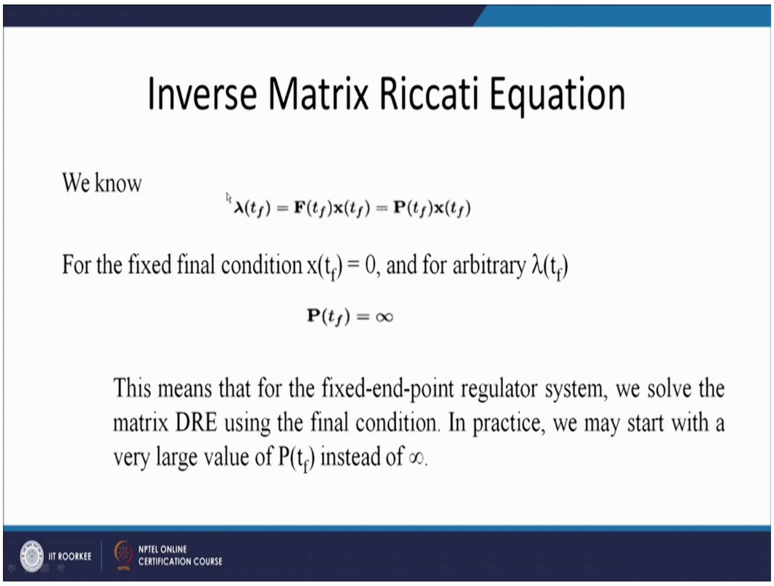

Say as we know we have consider the transformation or say our final condition will appear in a system as lambda t f equal to F of t f x of t f P t f x of t f. So, previously we already have discussed this condition. So, if my x of t f is 0 this means my endpoint is specified and my system is returning to the origin.

So, my x t f will be 0 if x t f is 0. So, what will be my P t f from this equation? So, my P t f will be infinite. So, P t f infinite this means I have to solve my Riccati equation given as the P t f equal to f of t f if my P t f is going to be infinite. So, in my backward solution of the Riccati equation; I have to consider this P t f value to be very very large. So, this is one of the way to solve this Riccati equation. So, we start with a very large value of the P and then backward solve my Riccati equation to get the solution. On the other hand we can have the concept of inverse matrix Riccati equation how this appears that we will see today.

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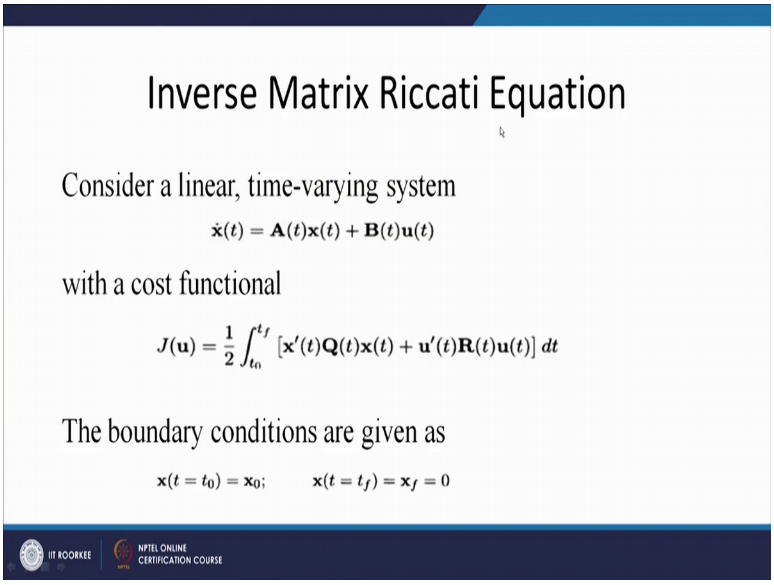

We consider it time varying system define as x dot t A t x t plus B t u t with a cost function in which the f t f we have considered to be the 0. So, it does not has the terminal cost and given as the x prime Q x plus u prime R u as time varying system we have. So, my A B Q and R all are the time varying we consider the boundary condition as at t equal to t 0 it is x 0 and t equal to t f this is x f which in this particular case we are considering to be the 0. So, this equation in a similar manner I can solve using the Hamiltonian approach this already we have done I can define my Hamiltonian as my v plus lambda prime f.

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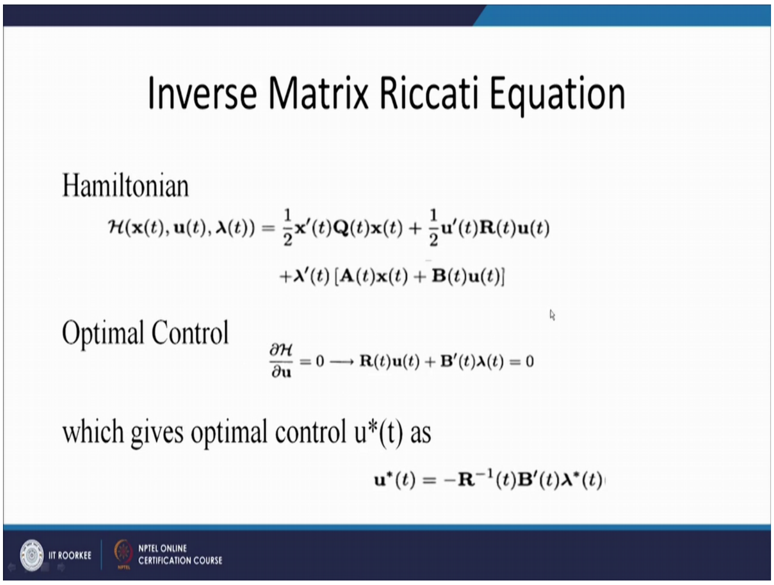

So, v is x prime Q x plus u prime R u with half. So, half of x prime Q x u prime R u plus lambda prime f is nothing, but my A t x t plus B t u t. So, in this solution our first step is to get the optimal control which is del H by del u equal to 0 and this give me nothing, but R t u t plus B prime t lambda t equal to 0. So, by this if I will find out the u t this is nothing, but R inverse B prime lambda and so, lambda we have to express in terms of a closed loop feedback.

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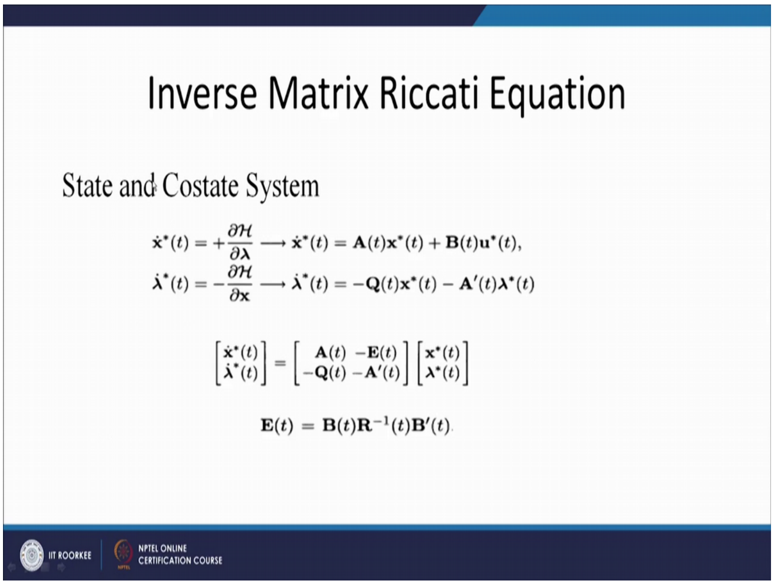

So, this means I have to express lambda in terms of the x, is state and the co state equation if you will drive this is nothing, but by A x plus B u is my state equation and minus Q x minus A x is my co state equation. So, I can write my Hamiltonian system as a minus e minus q minus A prime x lambda this already we have got up to this point in our previous derivation of a normal Riccati equation.

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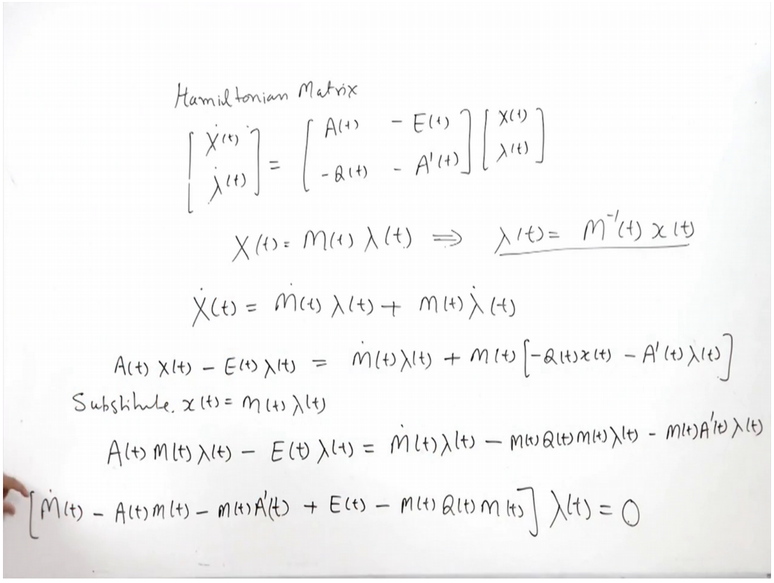

So, with this Hamiltonian matrix; so, my Hamiltonian matrix is X dot t lambda dot t and this is equal to nothing, but my A t minus E t minus Q t minus A transpose t and what the A t; we are considering A t, we are taking as b r inversed B prime and multiplied with x of t lambda of t and if we will write the final condition and we know we will have lambda t f has f of t f x of t f with this consideration we consider a transformation in place of the lambda equal to p x just reverse to this.

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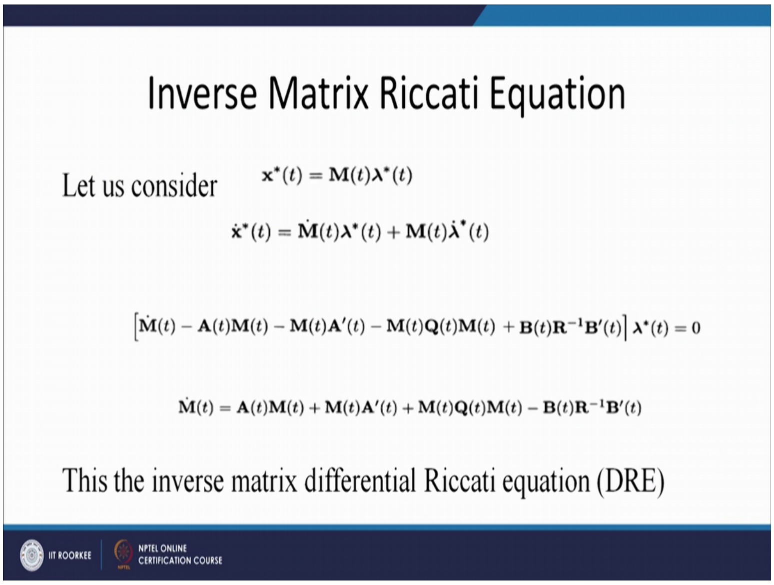

We consider a transformation here is X t as M times of lambda t while the previously we have considered the lambda in terms of the x, we have taken as a lambda equal to p x just we are taking the inverse of this; this means my lambda t M considering simply as M inverse t x t. So, my optimal control which we have considered as we have taken this as r universe B prime lambda t. So, lambda I can consider simply as M inverse t x t. So, this inverse of M is appearing while in the previous case it was appearing as a p t x t. So, this is a basic difference here.

So, this we will replace later on lambda t first we will see how to get the value of my m. So, I will differentiate this equation. So, this is x dot t as M is also a function of time M dot t lambda t plus M t lambda dot t x t and lambda dot t I can place it from the Hamiltonian system in this x dot t is A t x t minus e t lambda t, this I am replacing x dot t from this equation A t x t minus E t lambda t and this is equal to I keep it as such M dot t lambda t plus M t and lambda to t from here minus Q t, Q t x t minus A prime t lambda t that is by lambda dot.

So, in this equation, I am placing the value of x dot and lambda dot I also have x t as M t lambda t. So, substitute x t is sorry this is my M t lambda t. So, if I will place it here. So, this is nothing, but my A t x t in place of x t I will write M t lambda t minus E t I keep it same lambda t again remain here this is M dot t lambda t I multiplying to this. So, this negative minus M t q t and in place of x t again I am placing my M t lambda t and this is say minus M t A prime t lambda t.

So, if I can write this equation substituting this on one side where I will have M dot t minus A t M t minus M t A transpose of t that I am writing from this plus a t and minus M t Q t M t into lambda t equal to 0. So, this means by substituting x dot and lambda dot from Hamiltonian system we got this equation in this we will substitute x t is M t lambda t. So, the whole equation I can write in terms of the lambda t which will be M dot t minus a t M t M t A transpose t e t minus M t Q t M t into lambda t equal to 0.

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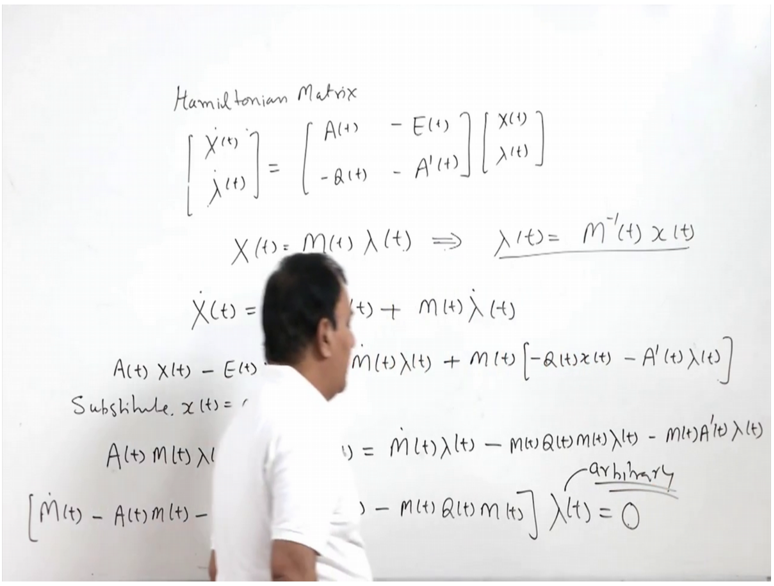

Lambda t is my arbitrary variation. So, the coefficient of lambda t will be 0.

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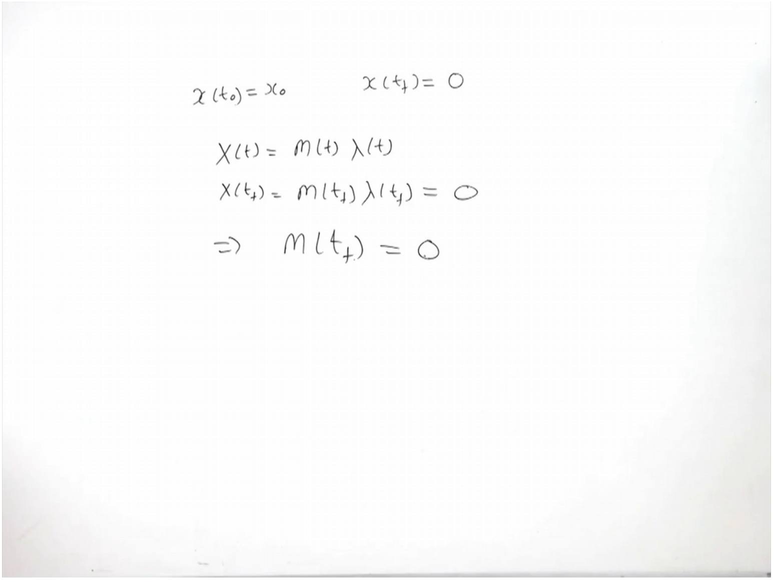

So, I can write if the coefficients equal to 0 M dot t equal to A t M t plus M t A transpose of t minus e t in place of e t we can write B R inverse B transpose plus M t Q t M t. So, this M dot I can write this. So, I am writing M dot equal to A t M t plus M t A transpose t M t Q t M t minus B R inverse p prime and this we call has the inverse matrix differential Riccati equation. So, how to solve this as we have considered the initial condition as x t 0 has x 0 and x of t f as 0. So, in this case because and my transformation is x t equal to M t lambda t.

So, x t 0 is a non zero quantity. So, if I will take this as x t 0. So, lambda t 0 with no con conclusion, but if I will write this as x t f as M of t f lambda t f this will be 0. So, this implies I can take my final condition M of t f equal to 0. So, I can solve this inverse matrix differential equation in a backward form and can find the solution of this equation in terms of the M t if I am solving this equation in backward that will see through an example.

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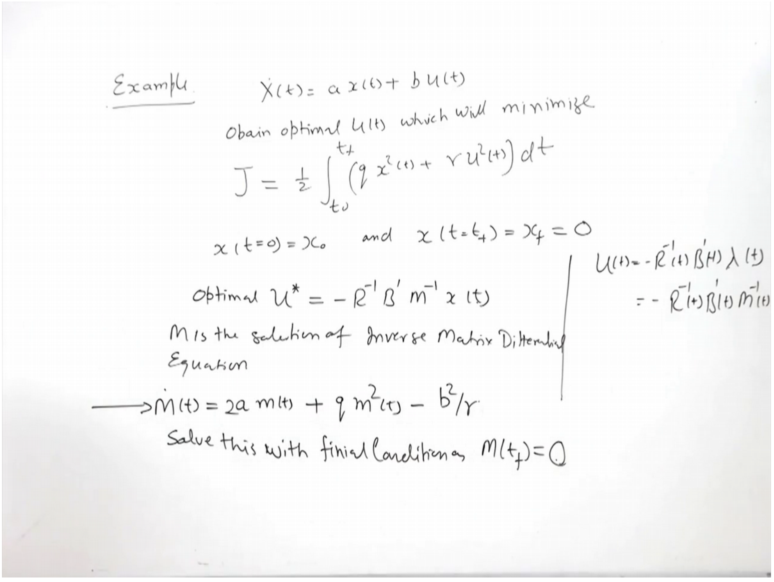

So, we take an example we consider a first order system a of x t plus b of u t, a b we are considering to b the time invariant for example, just for the simplification otherwise they may be time varying also.

Obtain optimal u which will minimize J which is given as 1 by 2 t 0 to t f q x square t plus r u square of t d t and the terminal condition given as x t at 0 point is x 0 and x at t equal to t f point is x f and this we are considering to be 0. So, this is my problem for my given system I have to obtain the optimal value of the u which will minimize my performance index with given initial condition has this I know what is the solution of this my optimal u is R inverse B prime. Now we say M inverse x t because if we will recall what we have taken my u was R inverse B prime lambda t and lambda t we are taking as or we are writing simply as R inverse B prime M inverse t, x t.

So, my solution is sorry my optimal control is u star is minus R inverse B prime M inverse x t where M is the solution of my inverse matrix differential equation and what is this and my this equation is M dot equal to a t M t.

So, M dot t is a M t plus M again a. So, this; a of M t is giving me the A of M t M t a transpose this again giving me the A of M t. So, I can simply write this as first 2 term is 2 A of M t plus M q M where q is my small q. So, this is q M is square this is q M square of t and minus b R inverse b this is give the B square by R and this is minus b square by r.

So, this is my inverse matrix differential equation and this equation I have to solve has my x t f is 0. So, solve this with final condition as M of t f equal to 0. Now this is the simple first order differential equation which can be solved the solution of this equation I will directly take.

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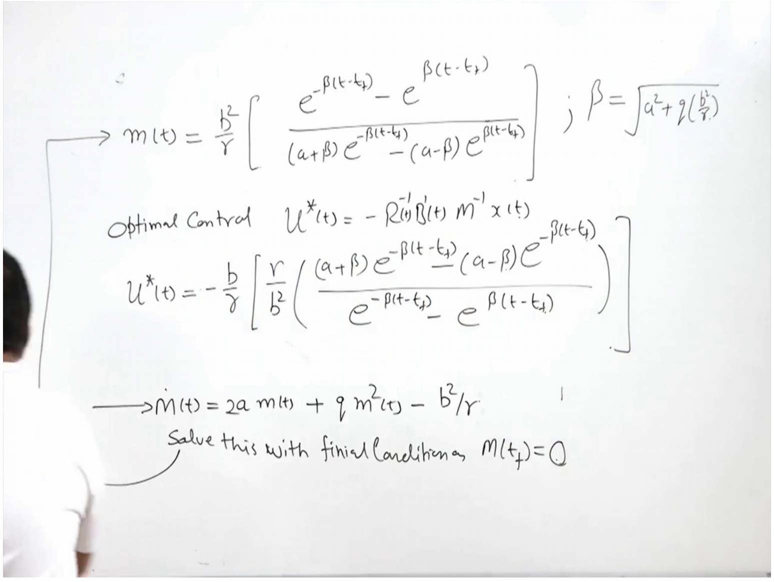

So, the solution of this is M t equal to so, a plus beta e to the power minus beta t minus t f minus a minus beta and in this my beta is a square plus q b square by r and this is the value of my beta.

So, solution of this equation with t f equal to 0 is has M t. Once M t is known then my optimal control u star t is minus R inverse p prime M inverse of x t. So, I can write my optimal control as R inverses 1 by R B prime is b. So, this is minus b by r, M inverse this is my m. So, I have to take the inverse of this. So, this means r upon b square inverse of this quantity a plus beta minus a minus beta. Now numerator will be the denominator in this case minus beta t minus t f.

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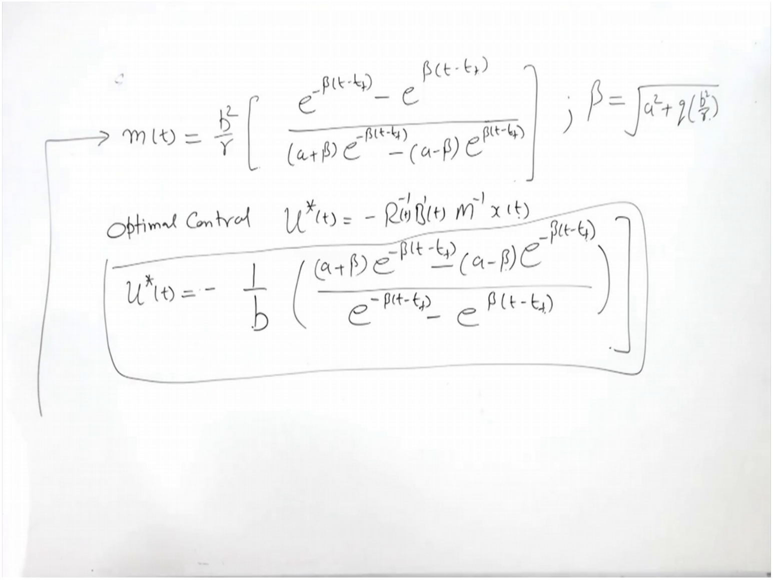

So, the overall u t we can write. So, directly we can write it here. So, if I will cancel this r r will cancel. So, this is nothing, but minus 1 by b. So, this will be my final optimal control, which I can find out utilizing the inverse matrix Riccati equation. So, in this wave the Riccati equation can be used in different forms also we have seen the matrix differential Riccati equation, algebraic Riccati equation, Riccati equation with this specify degree of a stability and then inverse matrix Riccati equation.

So, this class we stop here and in the next class we will start about discussion on the tracking problem. So, till now we have discussed about the regulator problem in which we have taken the reference r as 0. So, my also my is states are returning to the origin so, but if my reference point or my set point will continuously change then this will be a tracking problem and this tracking problem we will start in the; from the next class.

Thank you very much.