Optimal Control

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Lecture – 01
Introduction and Performance Index

Welcome friends to optimal control course. So, we are starting this course from this lecture and the complete course will have the 40 lectures in this. Optimal control is basically optimization approach has been used to develop the control and how we develop this optimal control for a given system that we will study in this course, how we use the optimization process to get the control that we will study in this course.

So, in today's lecture we will discuss what actually optimal control is and how we can formulate a optimal control problem. As we know this optimal control is basically based on the optimization technique and different optimization technique we will study in this course.

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The content of this course are taken from these four references, all four are the books - the first one is the D S Naidu, Optimal Control System published by the CRC press. Our second reference is the D E Kirk Optimal Control Theory: An Introduction which is published by the Prentice Hall. Third reference is M Gopal, Modern Control System

Theory which is published by the New Age International and the last book is the F L Lewis, Optimal Control.

So, reference one to four are normally contain over all the content of optimal control theory. The third reference M Gopal book has discuss the complete modern control systems in which in detail the study space sorry by state space analysis has been analyzed and how these steady state sorry state space analysis is applied to the optimal control that always discussed in modern control system by M Gopal books.

As we said our optimal control is basically based on the optimization. Now question here is what actually the optimization is, optimization means we have to determine the optimal value of a cross function means the system variables we have to determine which is giving me the minimum cost, minimum of say the in our control if we will talk about this we will be the giving me the minimum error in my required output and input where input is my desired input, output is the actual output, the difference is the error and we are trying to minimize this error and once we are minimizing this error what will be my states and what will be my control.

So, if we want the optimal control which will minimize my error. So, error we can represent as by performance index. So, this means, optimization means to determine the system variables if my performance index or we can see the cost function is minimum.

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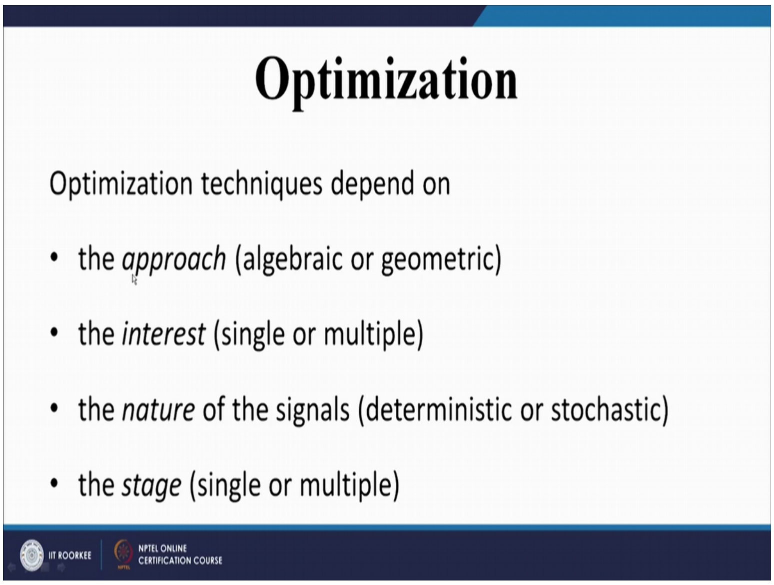

In general optimization techniques depends upon what the approach we are taking my approach maybe algebraic or geometric what is my interest it is a single or multiple interest means as we said our objective is to minimize my cost the cost maybe my interest. It may be I want to minimize the error in my system. So, error will be my interest, it maybe I want to minimize the fuel consumption in a system in an automobile or in any vehicle. So, in that case that will be my interest. So, what I want to minimize that is my interest and interest maybe the single interest or the multiple interest sometimes we want to minimize the multiple thing. So, that we call the multiple interest.

As we are dealing with the control, so means we have to take the signals. So, there my third point on which optimization is depends that is what is the nature of my signal. That may be deterministic or stochastic means I can have the signals in a pure form or I can have the signals affected by the noise which we call the stochastic signal. And the stage a stage maybe single or multiple, a stage means whether in one shot we are getting the optimal values or we are using the multiple stage to get the optimal value.

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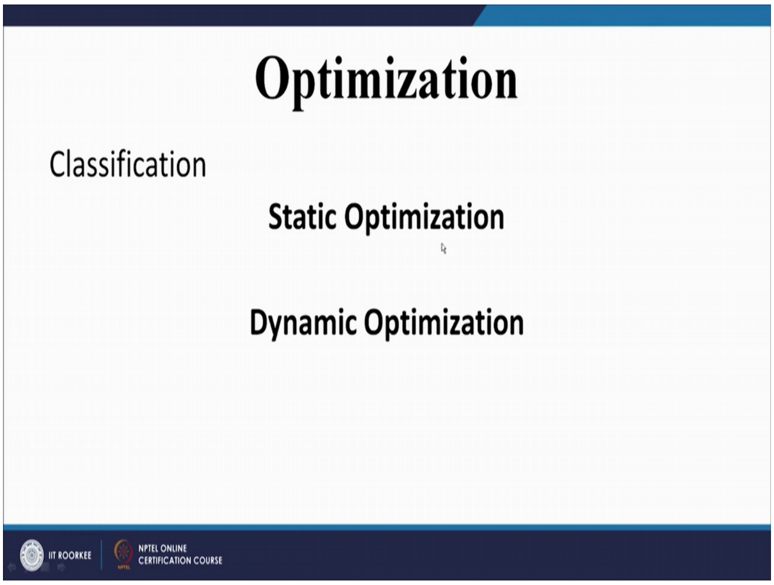

Optimization we can see in the two forms. One we call the static optimization and other we call the dynamic optimization. Static optimization means if we will talk in terms of the control means I want to control the plant under steady state conditions.

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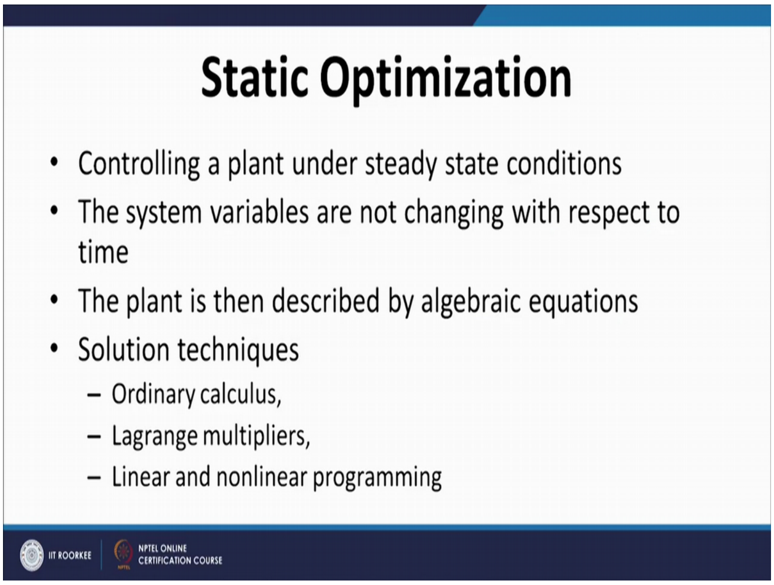

So, I am not considering the transients in the system for example, suppose we want to control a DC machine which is operating at a given speed. So, if I will take the DC machine model it may be dynamic in nature or it may be static in nature, static in the sense we are writing the equation by which we are describing the machine simply by the algebraic equation, like v equal to e plus i r. So, in that case if my speed change from say one level of their speed say 1000 RPM to 1200 RPM then at steady state what are the parameters.

Another example which we normally use in our power system as a electrical engineer if we will see then we want to allocate the power generation of the different generating units. So, in that case we will optimize the system to minimize the cost of our generation. So, to meet out the demand in the system what should be the generation of the different generator, so the cost of generation remain minimum. So, this optimization approach is called the static optimization. From control point of view we can say controlling a plant under steady state condition, my system variables are not changing with respect to time and the plant is simply described by the algebraic equations.

So, such optimization problem can be solved using the ordinary calculus, if we have the constrained optimization then we can use the Lagrangian multiplier approach and the complete formulation may be linear or non-linear. So, the linear or the non-linear programming approach can be adopted to solve such a optimization problem. In control normally we deal with the dynamical system.

Dynamical system is if we are changing any of the input - input maybe the disturbance input or maybe the set point if this is changing then how my output is following to these signals that we have to normally study. And we describe our system in terms of our differential equations and the solution of the differential equation means the system variables we normally observe with respect to time. Like we have taken the example of a DC motor we can take if we are changing the speed of a motor let say the set point we change from 1000 RPM to 1200 RPM. So, how the speed is reaching to 1200 RPM that is important and that period we normally call the transient period.

So, this transient analysis normally is required in case of our control system and once we will such a system we will apply the optimization technique then that optimization is normally called the dynamic optimization.

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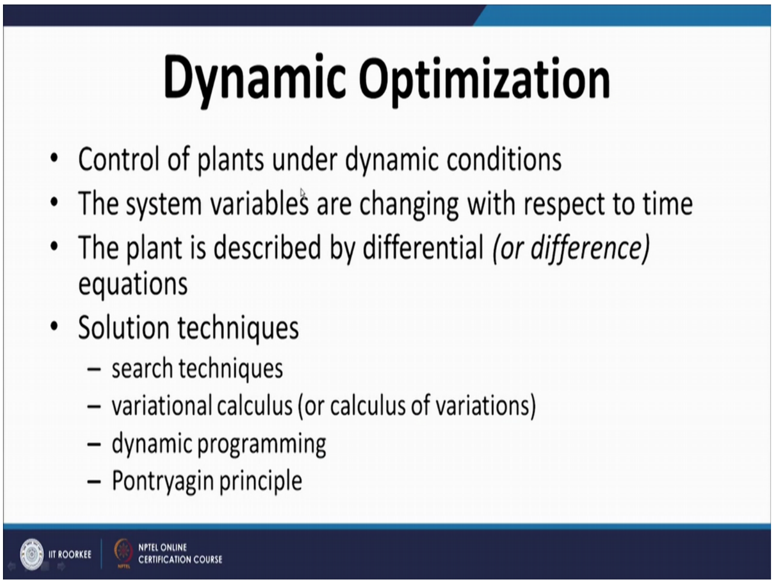

In dynamic optimization we control the plant under the dynamic conditions whether my system is suffered by the disturbance at a given set point. So, my system has to return to the same set point and during this change how dynamically my system is behaving that is our study.

So, we are controlling the plant under dynamical condition, system variables are changing with respect to time as we discussed and the plant is described by the differential equations. If we are the continuous time system we have the differential equations and if we have the discrete system then we have the difference equations. So, that is our dynamic optimization and the solution technique for the dynamic optimization are - the search technique, nowadays the heuristic approaches are very popular to determine the optimal values of a dynamical system this heuristic approach something like your genetic algorithms, pesos and a many number of the heuristic approach are available in the literature we can use to find the dynamic optimization.

Another approach which are the our classical approaches that are the variational calculus or we also called the calculus of variation, dynamic programming, pontryagin minimum principle which normally we used and the objective of this course is to study particularly this three approaches which is variational calculus, dynamical programming and the pontryagin principal.

So, in this course particularly we are concentrating on the dynamic optimization problem and how a dynamic optimization problem can be solved using the variational calculus or calculus of variation or using the dynamic programming and if my system is constraint system then how we can use the pontryagin minimum principal to solve a optimal control problem. So, dynamic optimization techniques in this course we are using to find out the solution of a optimal control problem. What is a optimal control? The main objective of the optimal control is to determine the control signals that will cause a process to satisfy some physical constraint and at the same time extremize a chosen performance criteria which we also called as a performance index or cost function.

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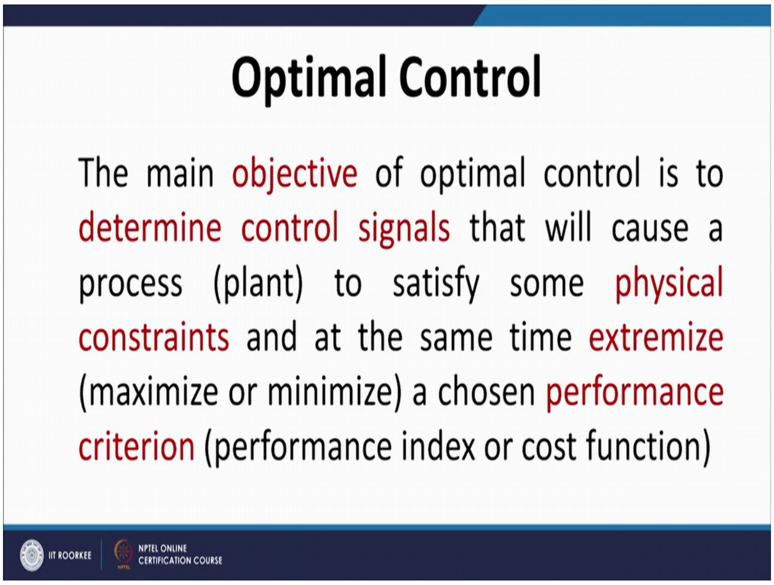

So, how we are setting our objective? Main objective is we have to determine that control signals means what the input going into the plant that should be controlled input and how we are controlling this input that is our main objective. We normally obtained this optimal control signal in such a way that it will satisfy the physical constraint which is nothing, but my plant and at the same time extremizing, extremizing means either minimizing or maximizing a performance criteria. Like we have discuss about the see the minimization of the error, means we select the control in such a way that my error should minimize and optimally how we will get this that we all we will study in this course. So, that is my optimal control. How to solve this problem that all we will study in this course.

So, the first step to a optimal control problem is first of all we have to formulate a problem. How to formulate a problem that we will see, but my in formulation of the optimal control problem, first of all we must know the mathematical model or the mathematical description of the process which we want to control and this normally this modelling we will have any state space form.

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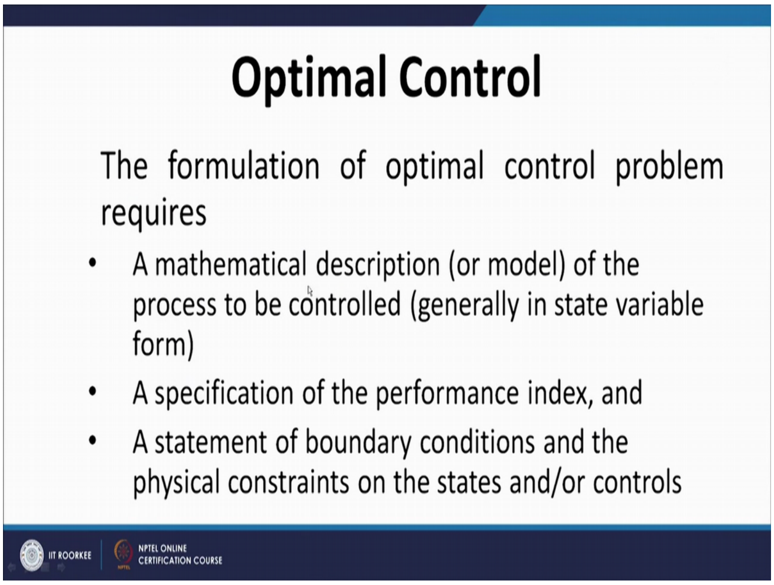

As we already know any physical system can be modeled if it is single input single output. So, we can also modality in the transfer function form and in case of the multi input multi output we normally model in the state space form.

Single input single output system can also be modeled in state space form. So, in this course particularly our prerequisites was you the student must have the knowledge of the classical control as well as the modern control in modern control normally we will use the state space modelling of a system.

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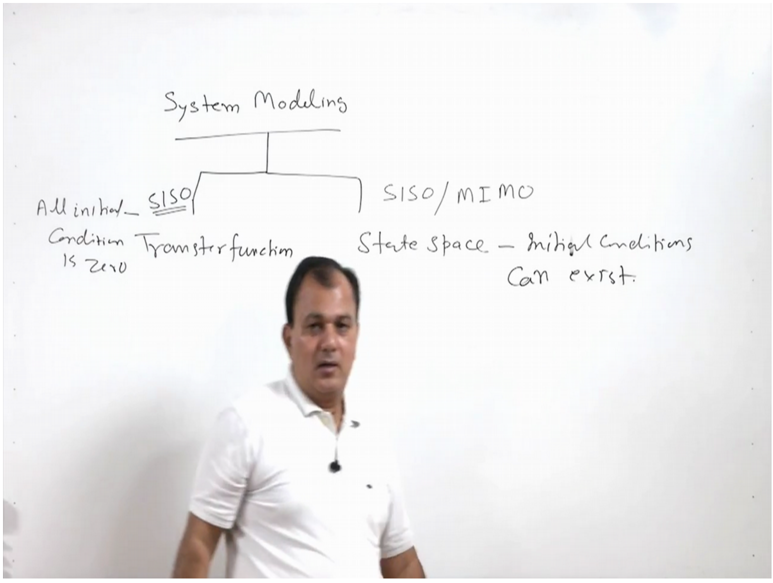

So, in general my system modelling is in two forms the first one is the transfer function and second is in the state space. Transfer function model is normally of single input single output with initial conditions, all initial condition is 0. So, with this we can model a system in transfer function. State space may be single input single output or multiple input multiple output and initial condition can exist. So, state space representation is more general in nature, so therefore, we utilize state space modelling to represent a system or to formulate a problem for optimal control.

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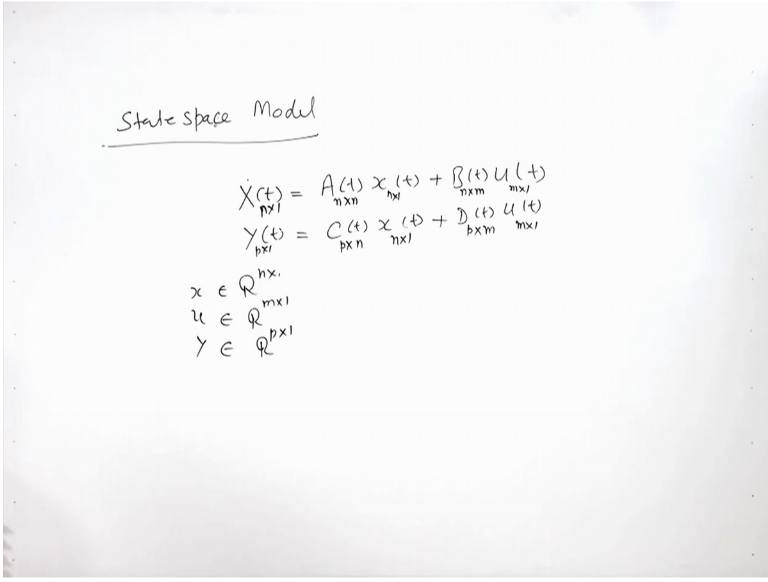

So, what is the state space model of a system? That normally we represent the system is X dot t is A t x t plus B t u t and y t is C t x t plus d, D t u t where x is a vector of n cross 1, u is also a vector let us say m cross 1 and y is also a vector let us say p cross 1. So, if we have a and n order system having the m number of the inputs and p number of the outputs we can write x dot equal to A x plus B u and y equal to C x plus D u. So, if this is the case. So, this is n cross 1, n cross 1 and this is m cross 1. So, my a vector will be, sorry a is a matrix of size n cross n and b will be a matrix of size n cross m, y is a vector of p cross 1 and x is n cross 1 and u is m cross 1. So, naturally C will be a matrix of p cross n and d will be a matrix of p cross m.

So, this is the mathematical model in state space in which we can represent any physical system. Whether a single input single output or multiple input multiple output or whether my system initial conditions are zero for there exist the initial condition. So, this is the general description of a plant in state space which we have to consider.

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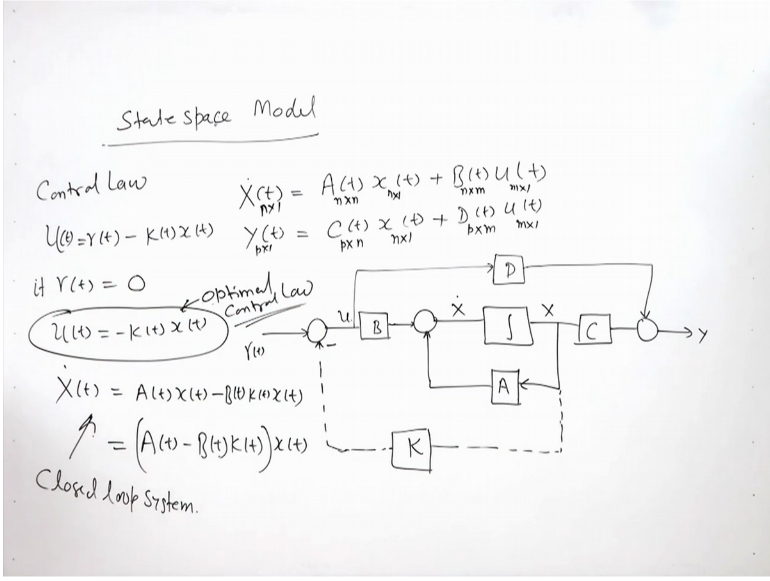

Block diagram form of the system is we can have this is x dot. So, this will be x this is my matrix A x dot equal to A x plus B. So, this will be my u and u is this maybe my r t. So, r and u is same in this case x dot equal to A x plus B u. So, I am not writing that t in this case here this will be my y equal to C x plus D u. So, this is the open loop system description or block diagram of my plant. In optimal control our objective here is we can have the feedback system in which either we are feeding back the state, if we are feeding back by state.

So, this will be my let us say k and if I will take this negative. So, my control law is u equal to. So, I can write it general t is dropped in the figure u t equal to r t minus k t x t. In most of the cases if r t maybe 0 which we call the regulator problem discussing detail later on then my u t simply minus k t x t. So, if I will apply the u t to my state equation then x dot t will be A t x t minus B t k t x t or simply I can write as A t minus B t k t x t and this is nothing, but my closed loop system. So, if my system is controllable I can design a u in such a way whatever be the condition of matrix A in this case I can develop this closed loop matrix as I desired. So, I can get this matrix simply by getting the value of k t. So, this is called the optimal state feedback control. Similarly as we are taking the feedback from the state we can also take the feedback from the output y.

So, if I will take the feedback from the output than this is called the output feedback system and if I am taking the feedback from the state than this is called the state feedback system. In this course particularly we will discuss state feedback system in detail and how to determine the optimal control law. So, I can say how I can get this optimal control law. So, what I can have? I can have is set point r t to be given then this is simply my tracking control problem if r t is 0 then the system is subjected to the disturbance then this is simply my regulator problem.

So, we will discuss in detail what is my regulator problem and what is my tracking problem. So, this is the way how we will model a plant. So, that is my first point first of all we have to get the mathematical description or model of the process to be control and generally we have this modelling in state space form. To formulate optimal control problem once we will have a mathematical model then we have to specify a performance index means what is our objective is to minimize or maximize a particular performance criteria.

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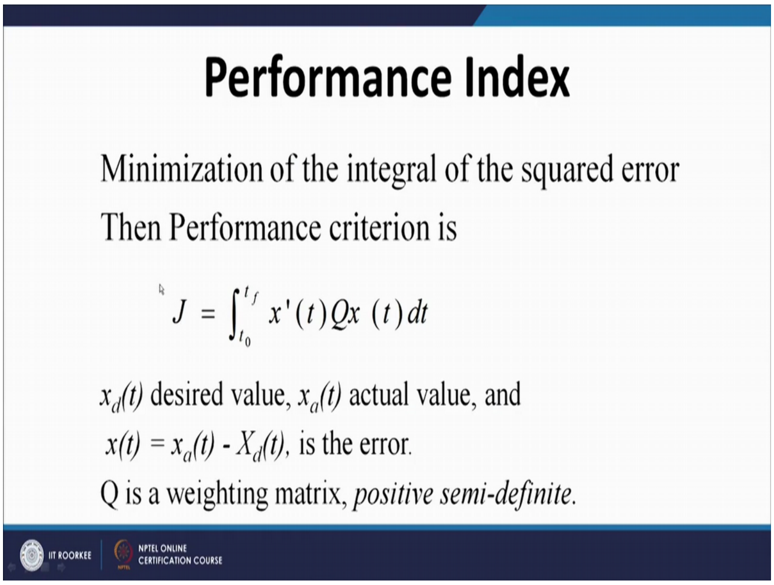

So, what is a performance index? We define a performance index as given here let J equal to t 0 to t f x prime t Q x t. So, this is the minimization of the integral of the squared error, if you will take then my performance index will be J. So, x t we are defining as x a minus x d where x d is the desired value and x a is my actual value or error can also be defined as x d minus x a, in any way we can define the error this is basically the difference between the actual value and the desired value.

So, our next point of discussion here is a specification of a performance index, means we have to decide a performance criteria. A performance criterion is based on the what actually we want from our system like we will take let we want to minimize the integral of the squared error in a given time. So, if my actual input is x a and the desired input is sorry; actual output is x a and the desired output is x d then the error I can define as simply x a minus x t.

So, if I will set my desired to be 0 like we said taken example before which is my set point. So, x d will be, in this case x d is nothing, but my set point. So, if x d will be 0 then x nothing, but we will represent the actual state, so in case of if r is 0 then this nothing, but represent my state, but if I will take a set point then this x nothing, but represent my error. So, we are taking for a linear system we are taking this as x prime Q x. So, x in this case representing nothing, but my error; Q is the weighting matrix. So, x prime x giving me the square. So, this is square of the error and the each state like we have taken the n number of the state. So, in each state there are the n number of the error. So, for each error we are giving a weight which is defined by the Q matrix.

So, my performance criteria is J equal to x prime Q x which is nothing, but my integral of the square error. So, within a given time I want to extremize this either minimize or maximize the value of the J. My next desire may be I will minimize my control efforts. So, control efforts means I will minimize my control energy. So, energy in any form I can represent at the square of my input like I say the i square r, i square r is my power loss I want to minimize the error to reduce this. So, any of the input square can be represented as the energy.

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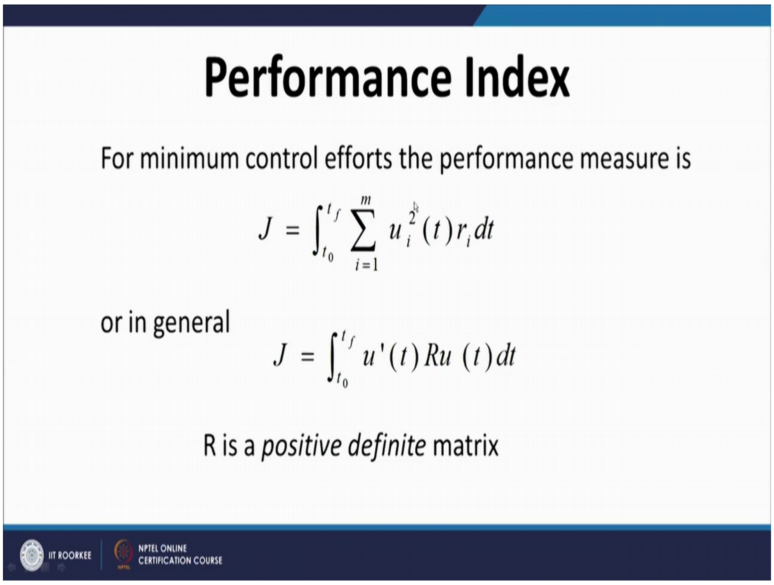

So, u i square r i may be any positive number because if r is 0 negative then energy cannot be negative, and energy cannot be 0. So, r is a positive number. So, if there are the number of inputs I can take J as sigma of u i square means u 1, u 2, u n or in general I can write this if u is a vector of say m cross 1 then u prime r u I can write as my performance criteria which I want to minimize means I want to minimize the control efforts. So, in this r will be a positive definite matrix.

So, ultimately my performance criteria will be u 1 square r 1 plus u 2 square r 2 plus u 3 square r 3 up to the number of inputs I have. And in each case, for the each input I want to the control efforts by the different inputs I want to minimize. So, I can write J as t 0 to t f means in a time period of t f minus t 0, my objective is to minimize the control effort this maybe one of the choice. Like the first choice was J equal to x prime Q x my next choice is the minimization of the control effort u prime r u.

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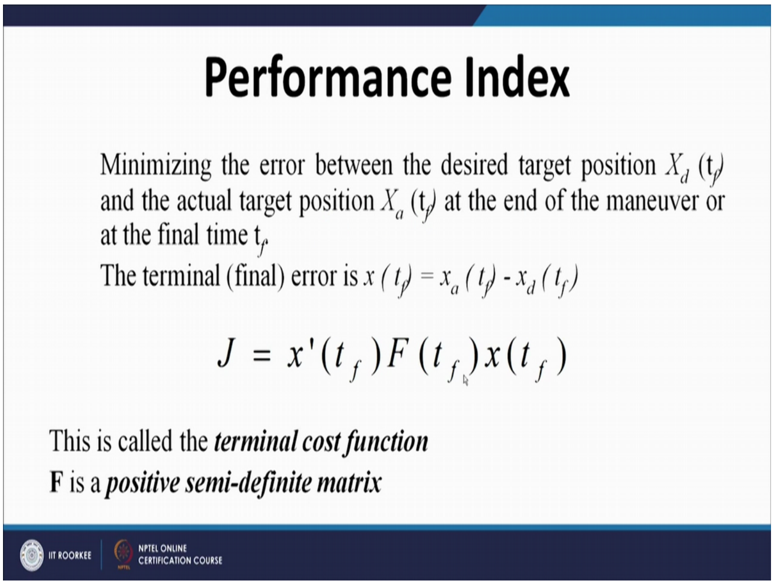

Another form it may take let J equal to x prime t f, F of t f, x of t f, this means I am trying to minimize the terminal cost. That t f is my final point, so when I am reaching to the terminal then I am trying to minimize the error because x we have defined as the error. So, it means after completing of the interval t 0 to t f I am reaching to the t f and after reaching my objective is to minimize this terminal error. So, that will be my terminal cost I can define. So, j I can take as x prime F x of t f.

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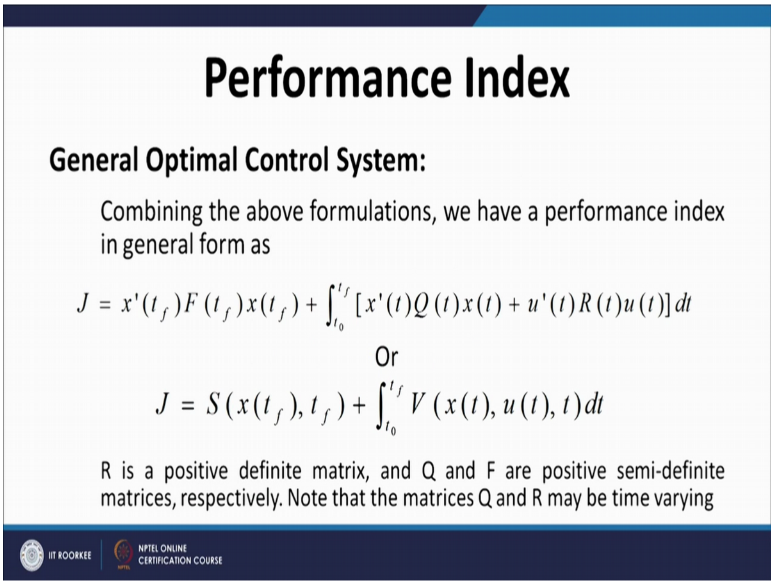

So, in our optimal control problem normally we will take it general performance index which is defined as j equal to x prime t f, F of t f, x of t f plus t 0 to t f x prime Q x plus u prime r u, what this represent here means in one performance index we are trying to minimize the terminal cost, we are trying to minimize the error and we are trying to minimize the control effort. And this all we have discussed for a linear time system. The system which we have described or the mathematical model we have seen that was the linear time control system.

As we have taken as the (Refer Time: 34:45) a b c d matrix are the function of time this means I am considering the time varying system. Even here we are considering Q and R to be the function of time. So, this means this is a linear time varying system we have considered for which we have seen the representation of my system model and we have seen the performance index we can choose for such a system to minimize the performance index. So, complete formulation we will see in the next class this class I stop it here.

Thank you very much.