Course Name: Optimization Theory and Algorithms

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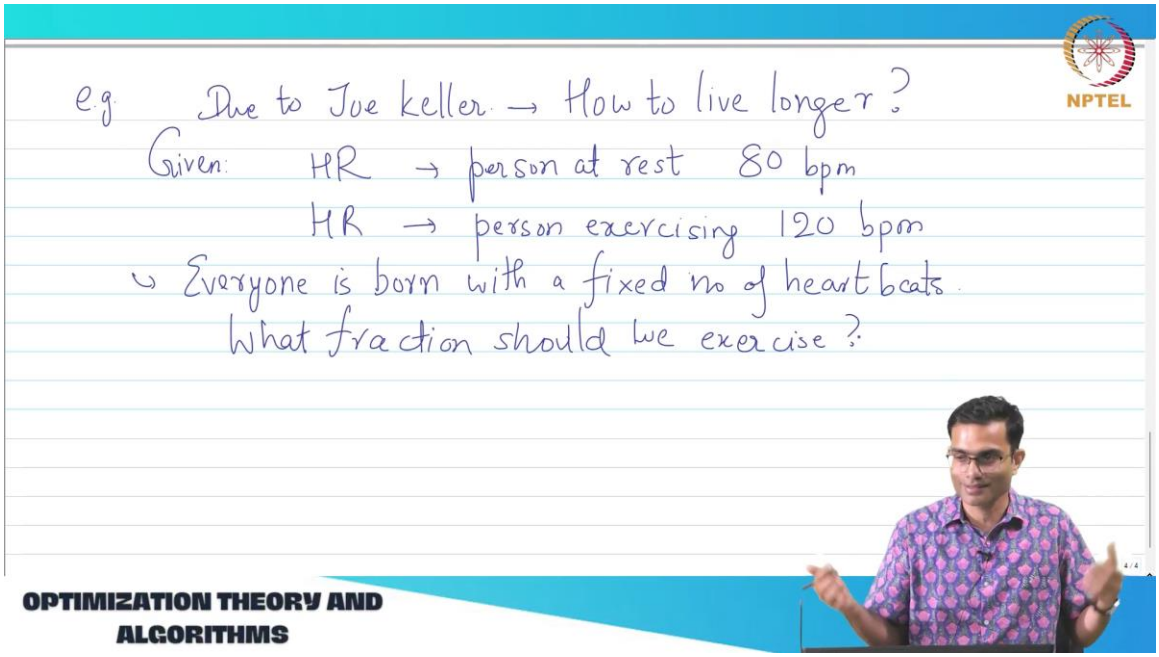
Week – 01

Lecture - 03

Introduction to the course - 3 - an optimization example to live longer

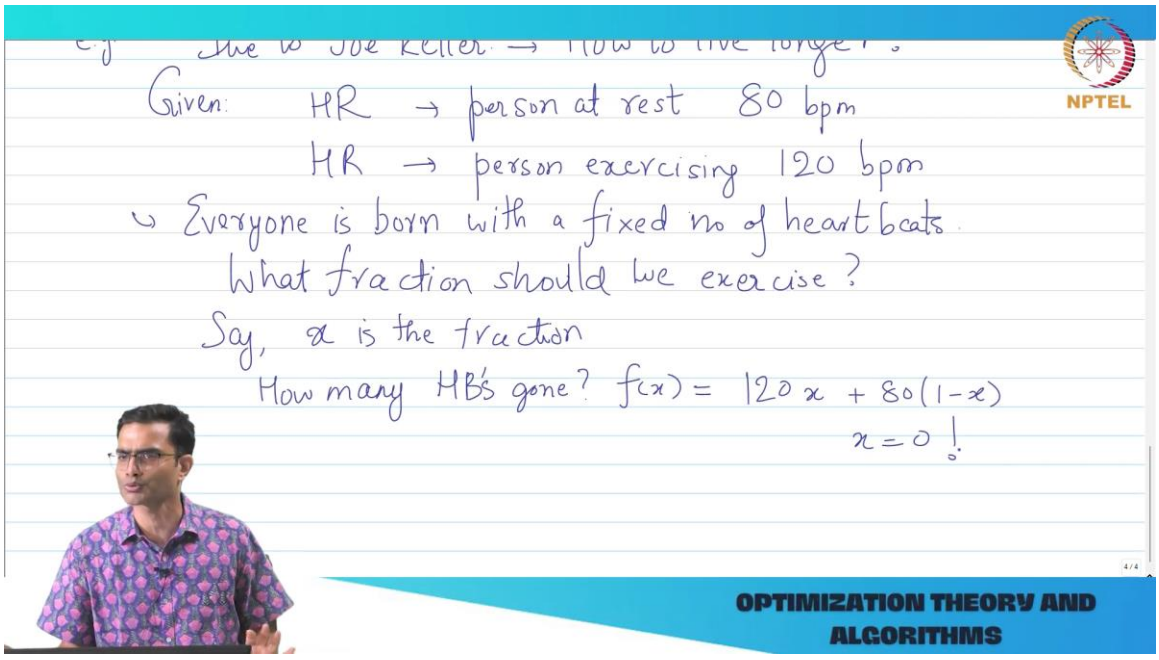
So, now let us take an example. We will take one fun example. This example is due to a person called Joe Keller. So, it is a fun way for us to kind of absorb the concepts that we have spoken about in this lecture and the premise of this problem is how to live longer ok. So, let us look at this fun thing. So, how to live, okay.



So, what is the information that I am going to give you? I am going to give you some information. So, given. So, HR is heart rate, okay. So, heart rate of person at rest, 80 beats per minute, ok.

The heart rate of person exercising, any guesses? Ok, not bad, let us take 120. So, now let us say that everyone is born with a fixed number of heartbeats, let us say. You just do not know how many they are, but everyone is born with a fixed number of heartbeats, born with, please do not get depressed. So, the question is can we figure out what is the what fraction of the day we should spend exercising? We will come to that. What fraction should we? So, immediately I saw the answer as zero which is the correct answer.

Given this information in the problem right. Can anyone see why? Can someone else tell me why should the answer be zero for this problem? Because exercise increases the heart rate. If I have a fixed number of heartbeats, I'll exhaust them sooner, I'll die sooner. Right? But, so, the person who never went out for exercise, who only stared at their Instagram reel, will answer zero. Because, you know, what else is there, right? So, but intuitively, what was the problem that was being solved? I mean, you didn't do it this way.



You just automatically said it. But can we form an objective function for this problem? So, let us say, x is the fraction. So, in a day or in a minute, does not matter, how many heartbeats are gone if I am exercising a fraction x of the time, how many heartbeats let us call that

$$f(x).$$

So, I am exercising a fraction x of the time when I am exercising what is my heart rate? 120 right. So,

$$f(x) = 120x,$$

these are the number of heartbeats during my exercise time and when I am not exercising how much am I losing?

$$f(x) = 120x + 80(1 - x),$$

Ok.

Now, what kind of a we can even draw this right as a function of x what will this be? is a straight line right and you can without even doing the calculation you can see that there are two end points x equal to zero, x equal to one. Again, I am doing this intuitively where did I get x equal to 0 and x equal to 1 from? We will call them common sense constraints because heart rate cannot be negative, cannot be greater than 1. Then I look at the solution at 0 and I look at it at 1. And at x equal to 0, it is going to be 80. At x equal to 1, it is going to be 120.

Obviously, 80 is less than 120. So, I will choose x equal to 0. Now, I say that, hey wait a minute, this does not make any sense because, exercising seems to do something people who exercise seem to live longer and ah I mean by and large who can tell, but yeah. So, in the language of optimization where did we mess up? The objective function, yes of course in the objective function, but what in the objective function did we not get right? Parameters means variables. So, x is there what? So, again so I am getting the correct answers this is we have made a modeling error.

We have modeled the objective function incorrectly right. So, there is a modeling error because we know that a person who exercises and then comes to rest versus a person who only watches TV and comes to rest, there's going to be a difference in the heart rate. We have seen that. I mean, if you haven't, then you can observe. Those athlete friends of yours, you can see if you take their heart rate, their resting heart rate is actually much lower.

Then it can be, I mean 50 for example is common. In fact, that's on a side note I used to have some athlete friends who were like extreme athletes and their problem was because the resting heart rate was so low they would sit in class and just go to sleep because this shuts down it's not that they were bored or anything just not having activity they would

just go to sleep the heart rate would go so low so don't exercise that much so modeling error over here So, I am going to model the resting heart rate a little bit like this. So, let us write it like this. Let us start with the intuition that more I exercise, resting heart rate should be lower, right? Ok. So, more exercise corresponds to our variable x itself right.

So, this is x ok. So, when x is small, I am going to call this resting heart rate instead of 80, I am going to call it by some other word. So,

$$g(x).$$

If I do not exercise enough, my resting heart rate let us say is 80.

Couch potato. Potato. Spellings are terrible. Is it correct? no e, ok. x is large if I exercise a large fraction of the time, let us say 50, ok. You notice I am making some kind of assumptions over here actually what should I be correctly doing to get this get these numbers where should I be looking.

Survey is one thing, experiment is one thing, but if I am a little smarter than that, what will I do? Because if I survey people who are living on the top floor of the hostel, I will just get 80 everywhere. If I survey the inter-IIT team, I will get 50 everywhere. So, survey is not really a good idea. Interpolation, but I am interpolating between bad examples then. If I interpolate between 80, 80, 80, 81 then.

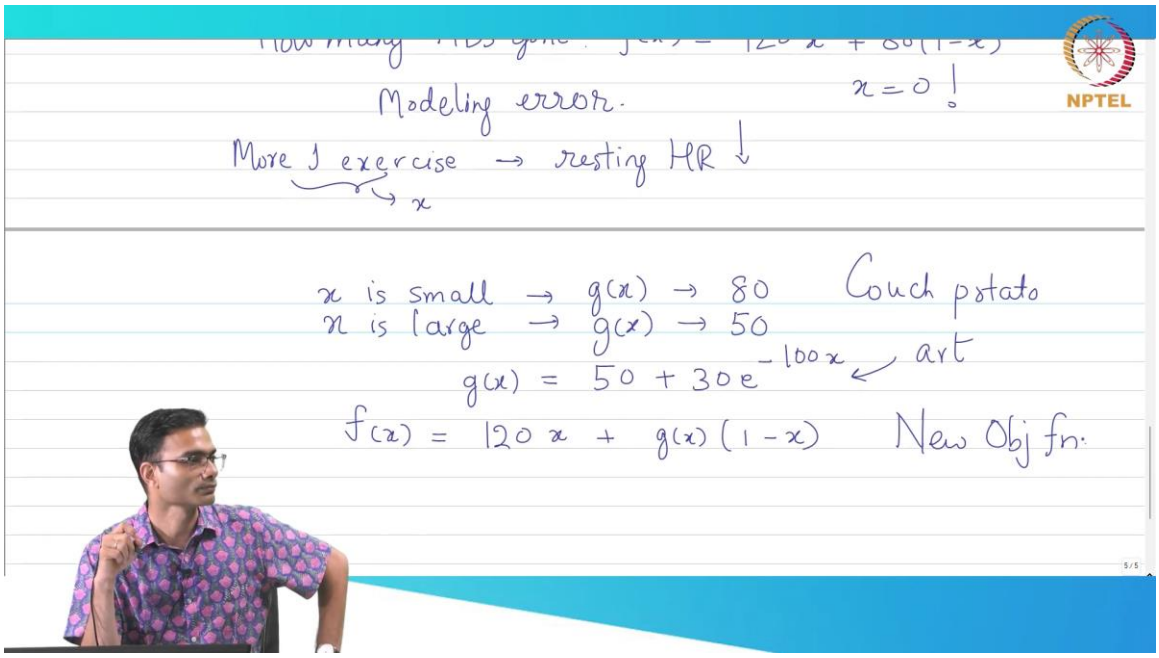
What is the smarter intelligent way of solving this problem? Come on guys. Do it on ourselves. Do it on ourselves. Okay. I do not know if that is better or worse.

Everyone has heard of Google? How about I go into Google and type journal of sports medicine? Guess what? There are scientists who do this for a living. I can just get their data, no? Correct. Right. So, I can get, I can look up a place where experts have done this work.

Right. So, this is another thing that as engineers we have to learn. Do not reinvent the wheel. Do not do it on yourself to start with. Tempting as it may be.

Right. So, go to the right places because what are you trying to fix? Remember, what was your starting point? You wanted to fix modeling error. Right. You don't want to replace modeling error by yet another modeling error. So, that is an example. So, let us say that I do you know I did this exercise and turns out that $g(x)$ is something like this:

$$g(x) = 50 + 30e^{-100x}$$



It is possible when x is small what is the value of this function? 80, right? When an x is large, 50 right? So, this is kind of and it is smoothly we expect it to be smooth we do not expect some crazy stuff to happen right we expect it to be smooth. Now, this is again this is you can almost call this as art right I mean this is coming from this is not coming from you this is coming from some other engineering or sports discipline right. So, you have to take that with a grain of salt this is what you got right. And now with this what is my objective function $f(x)$ is going to be? When I am exercising, I am still taking it as 120 hertz, beats per minute and would you agree that this is the correct new objective function? Right when I am exercising it is 120 when I am not exercising, I am I have my heart rate and my heart rate is given by this function $g(x)$, ok. So, now what, I have done what have I done I have done my problem model.

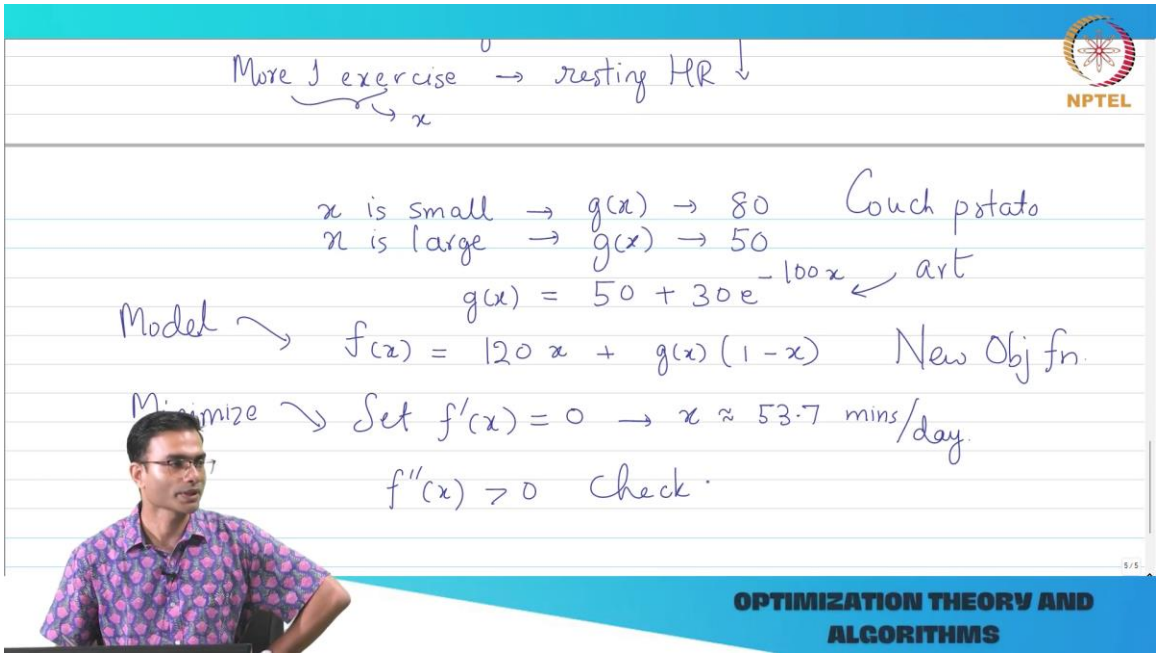
What is the next thing I do solve it, in this simple case solving means what? I want to minimize or maximize? this is number of number of beats in a day, right, I want to

minimize it. So, I want to, so this is model, now I want to minimize it. You could solve this problem in class 12, what would you do?

$$f'(x) = 0$$

Does this look like a nice problem that I can run take the derivative and solve? I am happy to hear yes, right. What is it is a linear function is an exponential function I can differentiate it in my sleep right I will get the answer.

Now, I will give you the answer since I worked it out. It is a believable number, right? 53 minutes a day. So, you can work out, I mean, I am converting the fraction to minutes per day. I have got some number, multiplied it by whatever to give you a number which is 53.



7 minutes a day. So, you know, when you hear from doctor's advice to old people, they say, walk for one hour in the morning, evening. I mean, it is based on stuff like this. It's not just come out. I mean it may have come from some WhatsApp forward that I'm not discounting that. But scientifically how you want to arrive at this stuff is like this.

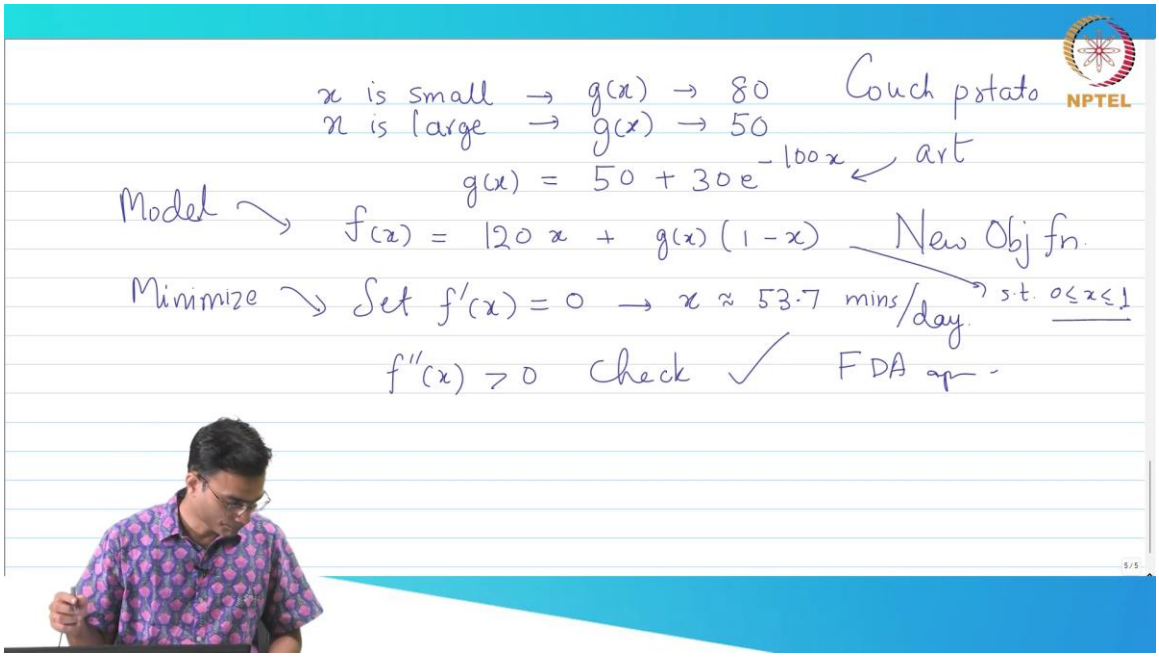
It's not super complicated. Model the problem intelligently, solve it and make a prediction. And you can kind of do this for any problem really, right? The trick is to model the problem with appropriate complexity. If I modeled it with not enough complexity, my recommendation would be a couch potato, because that way you

optimize your heartbeats, but this is a little bit more right. So, the more effort you put into modeling the better solution you get, the price you may have to pay is more complicated optimization algorithm. Everyone is happy with this solution, but we did not do one thing.

What is that one thing? Check for optimality, right. How do you know that this $f'(x) = 0$ happens both at maxima and minima? How do you know that this is not the solution to die soonest? It could be. So, what should we do next? So,

$$f''(x) \ ?$$

should be. You can check it and if it turns out to be greater than 0 then then next what will come FDA approved or something right.



So, that is how this happens, ok. Now, was you can see clearly that this was a constrained optimization problem although when you solved it you did not write over here such that, did you write this? You did not write it right. So, this is your you are still coming out of class 12 kind of solution strategy do something set derivative equal to 0 and what is your you are relying on luck, hope, whatever right. So, in this case it worked out that you got a solution which was which was fine, ok. It may not always happen this way and we will see this in the second half of the course that it may not always happen and you have to there is a proper framework, kind of a generalization within calculus which allows you to work with constraint constraints in the optimization So, I think that is as far as I have

with regards to introduction. So, any questions on what we have been speaking about so far? So, you have got the basic idea over here.

Model, modeling the problem correctly is one of the most important parts of an optimization problem and unfortunately this is the place where most in engineers get tripped up because they are in a hurry to solve the problem, you do not model it correctly you get a ridiculous answer like 80 right and various such things can happen, ok, right.