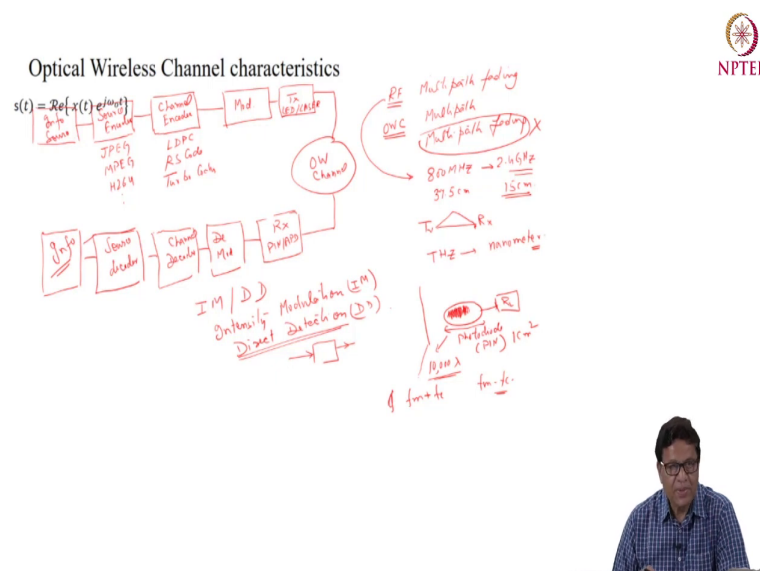


Optical Wireless Communications for Beyond 5G Networks and IoT
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Lecture - 06
Part 2
Indoor OWC channel modelling...contd

So, now, we will start to understand what is a optical wireless channel characteristic.

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And, it is different from RF for example, in RF you have multi path fading or there is a fading happening in RF channel. But, in optical wireless channel or optical wireless communication there is multi path, which we discussed just now. But there is no multi path fading, there is no multi path fading.

So, if you see in optical wireless communication channel, there is multi path; one is line of sight and other could be non-line of sight and they arrive they follow different paths. So, they arrive at different times so, it results into multi path. But what happens for example, in RF if you see the frequencies of RF operation, this basically is from 800 megahertz to say 2.4 gigahertz if I am counting 2G, 3G, 4G spectrum.

And, though if you see the corresponding wavelength is 37.5 centimeter to 15 centimeter of this order which is actually the size of the antenna. So, what happens in RF if there is a phase difference of π , then those two signals will kill each other or there will be a destructive interference which will result into fading.

So, in the RF scenario if you have a T x and R x, suppose this path and this path which gets reflected; there is a phase difference of π then they will cancel out each other. So, you will not get any signal at this point because of wavelength. But whereas, in optical wireless what happens? We are working at optical wavelengths which is terahertz and if you see the wavelength, it is of the order of nanometer.

So, what happens you at the receiver, suppose this is your receiver detector photo diode, photo diode and let us see this is a pin photo diode and it has area of say few centimeter square or 1 centimeter square. Then, at the receiver you will get this you know phasing sorry the destructive interference happening, constructive interference happening at nanometer scale.

Whereas, this dimension if you see here, the dimension of the receiver is about 10,000 λ . So, what it actually what happens, this this is the power the total power gets averaged. So, there is no effect of you know interference as it happens in RF. So, there will not be any fading and instead always you will get some continuous power which is you know integrated over the whole area.

So, that is why we do not have a multi path fading in optical wireless channel systems. Because, the area of the detector is large as compared to the wavelength and each typical

photo diode may have you know dimension equivalent to $10,000 \lambda$. So, you might have these kind of fast variations happening here, but on an average the everything is collected and the intensity is added and you get some output across the resistor load.

So, this is the main difference between RF and OWC that it suffers from multipath, but does not suffer from multipath fading. Now, let us try to understand a typical optical wireless communication system. So, optical wireless communication system will have a info source, information source and then there is source encoder.

So, there are many algorithms for source encoding through name a few JPEG is one for example, or MPEG or H264 so on and so forth. And, then after source encoder you have channel encoder which takes care of the error in the channel. So, you can have either LDPC code or you can have Reed Solomon code or Turbo codes and then you have some sort of modulator, we will discuss about this modulator little later.

And, then this goes to a optical source which is either a LED or a laser and this is your optical wireless channel. And, on the receiver side, you have the R x which is could be a PIN diode or APD diode and then you demodulate. And, then you have the reverse of channel coding, channel decoder and then you have the source decoder, source decoder and then you get back your information.

So, this is a typical a system of optical wireless. In optical wireless because you are driving a either a LED or a laser which requires real and positive signal. So, one of the common technique, which is used for modulating a laser diode or LED diode is called as IMDD which is Intensity Modulation Direct Deduction.

So, you modulate the intensity. So, whenever there is a logical 0, your source or LED is switched off. There is it is not emitting any light and whenever there is a logic 1, it is emitting light. So, you have the variation of the intensity which is going in the optical wireless channel; high intensity, low intensity and so on so forth.

And, in the receiver the light directly falls on to the PIN diode and you are able to recover your signal. So, that is why it is called as Direct Detection, DD. So, this is the IM part and this is the DD part. So, now let us try to understand that optical wireless channel is essentially a baseband channel.

So, unlike you have in RF where you modulate the signal with a carrier and you know you then you have the modulated signal which has components at ω_c for example, ω_c plus ω_m or ω_c minus ω_m and so on and so forth. So, whereas, this optical wireless channel is essentially a baseband channel. So, let us discuss in detail how it is a baseband channel.

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Optical Wireless Channel characteristics

$s(t) = \text{Re}\{x(t) e^{j\omega_c t}\}$
 $y(t) = \text{Re}\{\rho(t) e^{j\omega_c t}\}$
 $\rho = \text{sum of all multipaths}$

$$\rho(t) = \sum_{k=0}^{(N-1)} a_k x(t - t_k) e^{j\theta_k}$$

 Assume constant envelop ($x(t) = 1$)

$$y(t) = \text{Re}\left\{\sum_{k=0}^{(N-1)} a_k e^{j(\omega_c t + \theta_k)}\right\} \dots (A)$$

 $a = \left|\sum_{k=0}^{(N-1)} a_k e^{j\theta_k}\right|$ and $\phi = \arg\left\{\sum_{k=0}^{(N-1)} a_k e^{j\theta_k}\right\}$

Baseband Channel

Real and positive

$a_k \rightarrow \text{amp}$
 $t_k \rightarrow \text{Time of the } k^{\text{th}} \text{ path}$
 $\theta_k \rightarrow \text{Phase}$

$N \rightarrow 1$



So, just now we have seen that there is no multipath fading here. Now, let us understand that optical wireless channel is actually a baseband channel, unlike the RF counterpart. So, let us

prove it mathematically that it is a baseband channel. So, suppose you have a input signal which is a real because anything which is given to a laser diode has to be a real and positive.

So, the signal which is given is say for example, $\cos(\omega_0 t)$ and at the output; this is your for example, inside a room this is your T_x , this is your R_x . So, this signal from T_x which is $\cos(\omega_0 t)$ and at the receiver it gets signal from all the paths. So, this $\rho(t)$ actually is contribution from all the paths.

So, it will be $\rho(t)$, where ρ is sum of all multipath multipaths. So, this is the $y(t)$, the signal at the receiver at the receiver and this $\rho(t)$ can be written as suppose there are N minus 1 such paths N minus 1; so, K is going from N minus N_0 to N minus 1, a K is the amplitude and $x(t) \cos(\omega_0 t + \theta_K)$.

So, a K is the amplitude or the intensity or the power, some components related to the power, t_K is the time of the K th path and θ_K is the phase in the K th path, phase of the K th path and θ_K is phase of the K th path. So, this is the sum of all multipaths. Let us assume that it is a constant envelope that is $x(t)$ is equal to 1, because we are interested in knowing that this is a baseband channel.

So, I can always assume that it has a constant envelope so, that it makes the derivation little simpler. So, my $y(t)$ will be real value putting in this equation, the value of $\rho(t)$ here. So, real K is equal to 0 to N minus 1 paths and then a K is the amplitude and then you have $e^{j(\omega_0 t + \theta_K)}$.

So, if I calculate the amplitude and the phase are given by these two expressions. So, this is the phase part and this is the amplitude part of the receiver signal.

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IM/DD based OW channel

$$-1 < x(t) \leq 1$$

$$I_T(t) = A[1 + \mu x(t)] ; A = \text{DC bias}, 0 < \mu \leq 1$$

$$I_T(t) = |f_T(t)|^2 = A[1 + \mu x(t)]$$

$$f_K(t) = \sqrt{a_K} \sqrt{A[1 + \mu x(t - t_K)]} e^{-j\omega_0(t - t_K)}$$

$$f_R(t) = \sum_{k=0}^{(N-1)} f_K(t)$$

$$I_R(t) = |f_R(t)|^2 = f_R(t) f_R^*(t)$$



So, let us now assume that the $x(t)$ is normalized. I mean my $x(t)$ can be anything like this it has minus 1, some A to sorry; let me write this as some amplitude say I and this is say minus I here. So, I am normalizing my $x(t)$. So, that it is between minus 1 and 1 and I am adding DC to it, some DC is added to the signal so, that I get positive signal.

So, my I_T , the total intensity becomes $A[1 + \mu x(t)]$, where A is the DC bias which I have added. So, that to make it a positive signal which can be given to optical transmitter and this μ is between 0 and 1. So, when μ is 0, I am considering in IM; Intensity Modulation.

So, under this case, under intensity modulation μ will be 0; when you have 0, input and when you have high signal, μ would be 1. So, this is 0 and 0 power say or high power of the

source. So, basically, I am changing the intensity. So, this μ is between 0 and 1. So, my $I(t)$ will total be this is the electromagnetic field of optical radiation.

The intensity will be square of the electromagnetic field of optical radiation. So, this is $f(t)$ a mod square is equal to $A \cos(\omega t + \mu x)$. This is after adding DC bias and intensity modulating rate. So, for the K th path length, the electromagnetic field for the K th path is given by $\sqrt{\alpha_K}$.

This is the you know it will suffer some attenuation, because there are different paths, each path travelling different path lengths. So, this will suffer some attenuation which is say α_K . This is attenuation for the K th path which actually depends on the distance and into $A \cos(\omega t + \mu x)$ and it is arriving at time $t - \tau_K$ into $e^{-j\omega \tau_K}$.

So, this is for the electromagnetic field for the K th path, e m field of optical radiation for the K th path. So, the received power or received electromagnetic field is sum of all these paths and I am assuming there are $N - 1$ paths. So, K is equal to 0 to $N - 1$ $\sum_{K=0}^{N-1} f_K(t)$. And, my received intensity will be square of received electromagnetic field of optical radiation. So, this is whole squared which is $f_R(t)$ into $f_R^*(t)$.

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Contd.

$$= \sum_{K=0}^{(N-1)} \sum_{l=0}^{N-1} \beta_{Kl} e^{j\omega(t_K - t_l)}$$

$$\beta_{Kl} = \sqrt{\alpha_K \alpha_l} \sqrt{A[1 + \mu x(t - t_K)]} \sqrt{A[1 + \mu x(t - t_l)]}$$

$$I_R(t) = \sum_{K=0}^{(N-1)} \alpha_K A[1 + \mu x(t - t_K)] + \sum_{K=0}^{(N-1)} \sum_{l=0}^{(N-1)} \beta_{Kl} e^{j\theta_{Kl}} \quad K \neq l$$

where $\theta_{Kl} = \omega_0(t_K - t_l)$...sensitive to path length changes

Handwritten notes on the slide:

- $\lambda \rightarrow 2\pi$
- $c(t_K - t_l) \rightarrow 5 \times 5 \times 3 \pi$
- $c \rightarrow c_m$
- $\frac{1}{\lambda} \rightarrow \frac{1}{2\pi}$




So, let me find out. So, I will just put the value of; so, I will put the value of $f_R(t)$ here in this expression which is from here, from this equation. And, put this value here, take the complex conjugate and I will get summation K is equal to 0 to N minus 1 summation l is equal to 0 N minus 1.

Let me, I will explain you why these two summations have come. So, actually if you multiply the earlier thing to get the intensity at the receiver, then this will be given by $I_R(t)$ given by this expression, where β_{Kl} is given by this. So, what I have done here? I have broken this whole thing into two parts. One is K is equal to l and when K is not equal to l .

So, the whole received intensity has been broken into two parts. When K is equal to l so, these are two different paths and when the paths K is equal to K is not equal to l . So, after breaking you get actually two components. So, this is a K into a $1 + \mu x(t - t_K)$ minus t_K . This

is K is equal to 1, for all the terms which have K which have which are K is equal to 1 and these are all the terms which are which have K not equal to 1.

So, this is the expression for $I_R(t)$, the received intensity here. This is K not equal to 1. So, if you see this part here ω_0 , this part; this is sensitive to path length. So, every time the path changes by λ , this is phase change of 2π . So, this part is quite sensitive and if you see the difference in path length is actually given by $c \cdot t_K - c \cdot t_L$.

And, for a typical room for a typical room $c \cdot t_K - c \cdot t_L$, c is the velocity of light and for a typical room of say 5 cross 5 cross 3 3 is the height and then then width are 5 meters, this may range into centimeters. And, here this λ , the phase changes changing by 2π .

Whenever there is a change whenever there is a phase change of the path length change is λ equivalent to λ which is you know in terahertz on a nanometer wavelength, the phase change is 2π and the path length is of the order of few centimeters. So, actually if you see there are many this change of phase is happening very very fast, within us typical path length of centimeter, because this is going to be some nanometer. I mean if you see this λ , this nanometer.

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Contd.



$\theta_{kl} \equiv$ r.v with uniform distribution for any k and l

$$(k \neq l) \quad [0, 2\pi]$$

$$I(t) = E[I_R(t)]$$

$$= \sum_{K=0}^{(N-1)} \alpha_K A [1 + \mu x(t - t_K)] + 0$$

Remove DC $\rightarrow a_K$

$$y(t) = \sum_{K=0}^{(N-1)} a_K x(t - t_K)$$

Thus, the entire process is baseband.



So, I can always assume this θ_{kl} , the one which I had this θ_{kl} , this angle or θ_{kl} , θ_{kl} which is basically this. It is a random variable with uniform distribution for any kind of l , for any k and l and it changes from 0 to 2π . So, if I take the intensity which will be expected value of $I_R(t)$ which I am receiving at the receiver from different paths.

Then, if I take the expected value, the second term in the earlier equation where you know I had bifurcated the two into two terms; one was K is equal to l , other was not K is not equal to l , where this had θ_{kl} . So, if you take the average value of the θ_{kl} , because it is a uniform distribution random variable, this value is going to be 0.

So, what you are left with intensity at the receiver which is given by this expression. And, if I this is the DC factor, at the receiver I remove the DC and you know combine this α_K and μ with some constant say a_K . So, what I get at the receiver is $y(t)$ is equal to $\sum_{K=0}^{(N-1)} a_K x(t - t_K)$.

to 0 to N minus 1 into a K x t minus t K. So, this is clearly a baseband signal. So, the whole process, entire process is baseband. So, this is what we wanted to prove.

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Lambertian Radiation

$$I(\phi) = I(0) \cos \phi$$

$$I(\phi_{1/2}) = \frac{I(0)}{2} \Rightarrow \cos \phi_{1/2} = \frac{1}{2}$$

$$\Rightarrow \phi_{1/2} = 60^\circ$$

Typically a source follows

$$I(\phi) = I(0) \cos^m \phi$$

$$\frac{I(\phi_{1/2})}{I(0)} = \frac{1}{2} \cos^m \phi_{1/2}$$

$$m = -\frac{\ln(2)}{\ln(\cos \phi_{1/2})}$$

Commercial LEDs can be represented as

$$I(\phi) = \frac{m+1}{2\pi} I(0) \cos^m(\phi)$$

$$\phi \in [-\pi/2, \pi/2]$$

Handwritten notes and diagrams:

- Diagram of a light source emitting radiation at an angle ϕ from the normal. The half-angle $\phi_{1/2}$ is marked, with a note $2\phi_{1/2} \rightarrow 120^\circ$.
- Diagram showing the relationship between the half-angle $\phi_{1/2}$ and the Lambertian parameter m . A note says $m = 1 \rightarrow \phi_{1/2} = 60^\circ$.
- Diagram of a sphere with a circular cap of radius $m \cdot \phi_{1/2}$ on its surface.
- Handwritten notes: $\Rightarrow$ No fading, $\Rightarrow$ Real of the signal, $\Rightarrow$ Baseband, $\Rightarrow$ 3D radiation angle.

Now, let us discuss about the Lambertian radiation. So, so far, we have discussed that there is no fading in optical wireless channel, no multipath fading and this is you require real and positive signal for modulation of the laser diode or the LED. And, also whatever you are getting, the modulated signal is actually a baseband. The entire process is baseband.

Now, let us try to understand the characteristic of a laser, how in what form the radiations are emitted from a source. So, while doing analysis for channel modeling, it is always safe to assume that the source is a Lambertian source. Let us understand what we mean by a Lambertian source.

In Lambertian source if this is your source, it is emitting power, radiation profile is something like this and at the middle it is intensity say I_0 . And, if you want to see the intensity at this point for example, say at angle ϕ ; this will be given by this simple expression I_ϕ is equal to $I_0 \cos \phi$. So, such a source is called as a Lambertian source.

And, this source actually you define the irradiance angle. So, so suppose this is your source, it is emitting in this angle for example. So, normally the manufacturers of the source or LED manufacturers, they define this half angle which is referred as $\phi_{1/2}$ which is also called as irradiance angle.

So, $I_{\phi_{1/2}}$ is the angle, where the intensity has fallen to I_0 by that is the maximum is at I_0 at this point and it has fallen I_0 by 2 half of the value, that angle is called as $\phi_{1/2}$. It may be emitting in this angle also, but I for classifying the source, I need to find out the half irradiance angle which is $\phi_{1/2}$.

So, $I_{\phi_{1/2}}$ is equal to I_0 divided by 2 which gives me $\cos \phi_{1/2}$ and this is equal to $1/2$ and this is valid when you have $\phi_{1/2}$ is 60 degree. And, if you see the total angle, total angle will be $2 \times 60 = 2 \phi_{1/2}$. I am considering here is this was half angle; the total angle will be 120 degree ok. So, this is the ideal source, but in practice you do not have ideal source.

So, sometimes it is not necessary that you know you require a 60 degree $\phi_{1/2}$ or irradiance angle, you may require you know 90 degree or narrow source 30 degree. For defining this, we introduce another parameter which is called as Lambertian parameter or the mode number which m is here is Lambertian parameter. It basically tells you about the directionality of the radiation.

So, the radiation can be very narrow, the radiation can be in a broad angle. This is a source, can be further broad. So, this parameter defines the directionality of the source. So, for example, this could be m is equal to 1, this will be some very high value of m say 50. So,

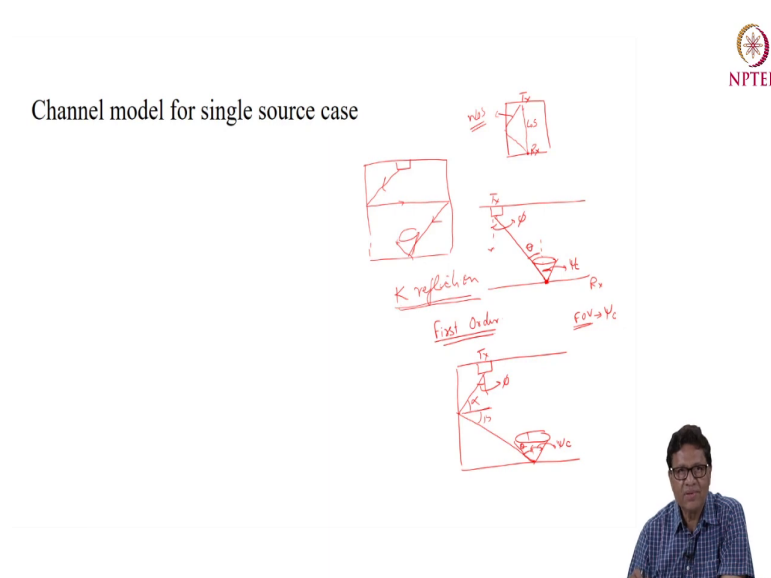
source can be modeled as I_ϕ is equal to $I_0 \cos^m \phi$, where m is the Lambertian parameter.

And, if I want to calculate the value of m , then I_{half} radiance I is divided by I_0 is equal to half. This should be equal to $\cos^m \phi_{\text{half}}$ and from here I can calculate the value of m which is given by $\frac{\log \text{natural of } I_{\text{half}}}{\log \text{natural of } \cos \phi_{\text{half}}}$. So, this is how you can calculate the value of m . I mean this is a relationship between the Lambertian parameter and half irradiance angle.

So, the commercial LEDs which are available which are normally used for elimination, they can be this is a very good approximation for commercial LEDs, where I_ϕ is equal to $\frac{m+1}{2\pi} I_0 \cos^m \phi$, where m is the Lambertian parameter and I_0 is the intensity, this I_0 is the intensity at this point and into $\cos^m \phi$. So, this is how a typical commercial LED can be represented.

And, this ϕ is actually between $-\pi/2$ to $\pi/2$. So, this is your source and this is a π by 2 and this is $-\pi/2$. And, it is emitting light in all the directions and it is governed by this I_ϕ at any angle, this is I_0 and I_ϕ is given by $\frac{m+1}{2\pi} I_0 \cos^m \phi$. So, now we will start how to model the channel if you have single source.

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So, before understanding, let us discuss how the light travels inside a indoor setup. So, suppose this is you know T_x , this is R_x . So, as I mentioned earlier there is one LOS path, direct to the receiver LOS. The other is a reflected path. This is non line of sight and if you see little in detail; suppose, this is your transmitter.

And this is your this is your plane and this is the photo diode and it has FOV given as say ψ_c . So, and so, the line of LOS path will go to will go from here to here and this if I draw a perpendicular here, this makes an angle ϕ and suppose I draw a perpendicular from here, this is the FOV part.

So, this this angle is say the maximum angle FOV ψ_c and it is falling at an angle ψ here and this angle is a θ . So, has a typical when you are considering a line of sight and when

you are considering first order. So, if I draw a similar diagram here, this is the receiver FOV area and this is the transmitter.

So, first order light hits the wall and then it is collected in the receiver. If you draw a perpendicular here, this can make some angle α , β and similarly this angle is ϕ here and this angle is θ and this total angle is ψ . So, this is first order reflection, but you can also have second order reflection.

Let me draw here the second order reflection. So, this is the source, this is the receiver FOV. So, the light is reflected on this wall, is reflected on this wall and then after second reflection, it is collected by the receiver right. So, this is a case of second order reflection.

So, and this way you can go on and you can have you know infinite reflections or we will assume there are K reflections, where K can be infinite. But we will see that you know everything is not contributing the receiver significantly. So, there is some limit on K which is good enough for modeling the channel.