

**Optical Wireless Communications for Beyond 5G Networks and IoT**  
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**Lecture - 05**  
**Part 3**  
**SNR for PIN and APD**

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**SNR**

$$SNR = \frac{R^2 P^2}{2e(I_d + I) \Delta f + \frac{4kBT}{R_L} \Delta f}$$

Shot Noise dominated  $\leftarrow$   $I = R P$

$$SNR = \frac{R^2 P^2}{2e(R P + I_d) \Delta f}$$

Thermal Noise dominated

$$SNR = \frac{R^2 P^2 R_L}{4kBT \Delta f}$$

Noise equivalent power (NEP)

$I_d \gg I$

$$NEP = \frac{1}{R} \left( 2eI_d + \frac{4kBT}{R_L} \right)^{1/2} W/\sqrt{Hz}$$

$R = 0.65 A/W, I_d = 1 nA, R_L = 1 K, T = 300 K$

$$NEP = 6.3 \times 10^{-12} W/\sqrt{Hz}$$

Handwritten notes: PIN { Shot Noise, Thermal Noise; APD { Excess Noise; Diagram showing Tx, 50 km, Rx with Shot/Thermal Noise arrows; Circled equations  $SNR = 1$  and  $I \ll I_d$ .

So, the NEP expression is given in this fashion and if I want to calculate the value of NEP for typical photo diode where I assume responsivity as 0.65 amperes per watt, dark current is 1 nano ampere and RL 1 kilo ohms and temperature of 300 degree kelvin; I get NEP as 6.3 into 10 raise to power the minus 12 watts or 6.3 pica watts per square hertz. So, this is the value of ne NEP.

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### SNR Calculation

PIN  
 $\mathcal{R} = 0.65 \text{ A/W}$ ,  $I_d = 1 \text{ nA}$ ,  $R_L = 1 \text{ K}$ ,  $P = 500 \text{ mw}$ ,  
 $\lambda = 850 \text{ nm}$ ,  $\Delta f = 100 \text{ MHz}$   
 $I = \mathcal{R} P = 0.325 \text{ }\mu\text{A}$

$$\text{r.m.s. shot noise (Signal)} = (2e\mathcal{R}P\Delta f)^{1/2} = 3.2 \text{ nA}$$

$$\text{r.m.s. shot noise (Dark current)} = 2eI_d\Delta f = 0.18 \text{ nA}$$

$$\text{r.m.s. thermal noise} = \left(\frac{4k_B T}{R_L} \Delta f\right)^{1/2} = 40.7 \text{ nA}$$

$$\text{SNR} = 63 \cong 18 \text{ dB}$$



So, now let us now try to calculate the SNR for a PIN, PIN diode first and then we will calculate SNR for APD. So, for PIN again assuming some typical values of responsivity id dark current assuming load resistance as 1 kilo ohm and 500 mill watts of power and suppose I am working at 850 nanometer and the bandwidth is 100 megahertz.


So, I can calculate the or the current flowing which is 0.325 micro amperes and r.m.s shot noise because of the signal is  $2e$  responsivity power into delta f square root. And this is about 3.2 nano amperes and the shot noise because of the dark current is  $2e I_d \Delta f$  which is 0.18 nano amperes. So, you see here the shot noise because of the signal current is much more than the shot noise because of the dark current.

So, in the calculation you may neglect it, because the value because shot noise is due to signal is much higher than the shot noise due to the dark current. And the thermal noise if you see is

40.7 nano amperes which is much higher than the shot noise signal and obviously much higher than the shot noise dark current.

So, it is a thermal noise dominated system. And if you plug in the values of signal power and noise power then you get this ratio 63 and convert into db you gets 18 dB or signal to noise ratio. So, this is what you will get at the receiver for these set of components.

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SNR for APD

$$I = M R P$$

$$i_{Ns}^2 = M^{2+x} 2e(RP + I_d) \Delta f$$

Material	$x$
Si	0.3
InGaAs	0.7
Ge	1.0

$$SNR = \frac{M^2 R^2 P^2}{2e M^{2+x} (RP + I_d) \Delta f + \frac{4k_B T \Delta f}{R_L}}$$




Now, let us calculate the signal to noise ratio for APD, as we discussed last time that APD gives you a gain of M there is an amplification gain because of the avalanche effect. So, the current which is produced is M times than what it is produced in a PIN diode. So, it will be M responsivity into power and the shot noise current will be again there will be increase of shot noise current, because there is an avalanche effect.

So, the shot noise component average value of the square shot noise component is  $M$  raise to power 2 plus  $x$  and  $x$  actually depends on the material into  $t^2 e$  this is  $i$  plus  $I_d \Delta f$ . So, there is a factor of  $M$  raise to power two plus  $x$  and this  $x$  for silicon it is 0.3 for InGaAs it is 0.7 for germanium it is 1.

So, depending upon the material you have to put this value of  $x$  to find out the average square of mean square shot noise current. So, the SNR for an APD will be this is a single power notice a factor of  $M$  square here and then this is the shot noise which has  $M$  raise to power 2 plus  $x$  and usual shot noise because of the signal and because of dark current here and this is the thermal noise component.

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### Example

$M = 50, x = 0$  ✓  
 $\mathcal{R} = 0.65 \text{ A/W}, I_d = 1 \text{ nA}, R_L = 1\text{K}, P = 500 \text{ mW},$   
 $\lambda = 850 \text{ nm}, \Delta f = 100 \text{ MHz}$

Signal current =  $16.25 \mu\text{A}$  ✓  
 rms Shot noise (signal) =  $161 \text{ nA}$   
 rms Shot noise (dark) =  $8.9 \text{ nA}$   
 rms Thermal noise =  $40.7 \text{ nA}$   
 SNR =  $9548 \approx 39.8 \text{ dB}$

*Significant increase in SNR*




Example, so if the gain is 50 and let us assume for simplicity  $x$  is 0. So, and responsivity is 0.65 amperes per watt, 1 nano ampere of dark current, 1 kilo ohm of load resistance 500 milli

watt of power, lambda 850 and suppose bandwidth is 100 megahertz and the signal current which you get is 16.25 micro amperes and rms shot noise because of the signal will be 161.

I mean you have to use the earlier expressions to find out these values and rms shot noise because of the darker in component is 8.9 nano amperes. And the thermal noise rms value is 40.7 nano amperes we have which we have calculated earlier and if you plug in all these values in the expression of SNR for APD is 9548 which gives you 39.8 dB. So, there is a significant increase of SNR significant because of this M factor significant increase in SNR when you use APD.

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SNR for APD

$$SNR = \frac{R^2 P^2}{2eM^2(I + I_d)\Delta f + \frac{4K_B T \Delta f}{R_L M^2}}$$

$M \uparrow, SNR \downarrow$        $M \uparrow, SNR \uparrow$

Optimum  $M$


Differentiation denominator with respect to  $M$  and equate to 0.

$$M_{opt}^{2+x} = \frac{4K_B T}{xeR_L(RP + I_d)}$$

Si (APD)  $x = 0.3$ ,  $R_L = 1K$ ,  $P = 100 \text{ nW}$

$I_d \cong 0$ ,  $R = 0.65 \text{ A/W}$ .

$M_{opt} = 42$



So, now let us try to understand more about this factor M is there any optimum value of M or M can be any high value which will give you gain? So, this is the expression for SNR for APD responsivity square into p square divided by 2 eMx. So, the earlier expression which I

had written actually there was a term of  $M^2$  which I have brought it down and that is why you see a factor of  $M^2$  in the earlier expression  $x$  was actually  $M^2$  plus  $x$ . So,  $M^2$  gets cancelled and this  $M^2$  is also coming here.

So, this is a modified version I am bringing  $M^2$  which was in the numerator to denominator and doing some manipulation, what we get is this expression which is shown here. Now if you see here if I increase  $M$  the first component in the numerator that value will increase it will have an effect of SNR going down.


So, increasing  $M$  here will make SNR going down, because the noise component the shot noise component has increased. Whereas, if I increase  $m$  here the second component in the denominator will help will actually increase the SNR, because this  $M$  is coming in the denominator. So, increasing  $M$  the first component it is decreasing the SNR and increasing  $M$  the second component is increasing the SNR. So that means, there is some optimum value of  $M$  which will maximize the SNR.

So, how do you find out that optimum value of  $M$ ? You have to differentiate denominator with respect to  $M$  and equate to 0 you will get a optimum value of  $M$ . So, if I do this differentiation and equate to 0 I get  $M_{\text{optimum}} = \frac{2}{x} \sqrt{\frac{k_B T}{q I_{\text{photo}} + I_{\text{dark}}}}$  where  $k_B$  is Boltzmann constant,  $T$  is temperature,  $q$  is electronic charge,  $I_{\text{photo}}$  is photo current and  $I_{\text{dark}}$  is the dark current, so this is the optimum value of  $M$ .


And for silicon APD as I mentioned earlier the value is  $0.3 \times R_L$  I am assuming as 1 kilo ohm and  $P$  say 100 nano watt  $I_d$  is say equivalent to 0, there is no dark current and the responsivity is 0.6 amperes per watt. Then I calculate I get  $m_{\text{optimum}}$  as 42 so this will maximize your SNR.

So, it is not always necessary to increase the value of  $M$  to get a good SNR, because we see that one component of the noise is decreasing the SNR other might be increasing the SNR. But the overall effect that is you know getting maximum SNR we require some optimum value of  $M$  which in this case or in this example is 42.

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PIN	Parameter	Symbol	Unit	Si	Ge	InGaAs
	Wavelength	$\lambda$	nm	400-1100	800-1650	1100-1700
	Responsivity	$\mathcal{R}$	A/W	0.4-0.6	0.4-0.5	0.75-0.95
	Dark Current	$I_D$	nA	1-10	50-500	0.5-2.0
	Rise Time	$\tau_r$	ns	0.5-1	0.1-0.5	0.05-0.5
	Bandwidth	$B$	GHz	0.3-0.7	0.5-3	1-2
	Bias Voltage	$V_B$	V	5	5-10	5
APD	Parameter	Symbol	Unit	Si	Ge	InGaAs
	Wavelength	$\lambda$	nm	400-1100	800-1650	1100-1700
	Avalanche Gain	$M$	-	20-400	50-200	10-40
	Dark Current	$I_D$	nA	0.1-1	50-500	10-50 @ $M=10$
	Rise Time	$\tau_r$	ns	0.1-2	0.5-0.8	0.1-0.5
	Gain Bandwidth	$M \cdot B$	GHz	100-400	2-10	20-250
	Bias Voltage	$V_B$	V	150-400	20-40	20-30



So, let me conclude this photo diode by giving you some typical values of the PIN diode and the APD diode. I mean these are the possible solutions which we will use in our optical wireless communication. And so for example, for wavelength for silicon the range where it is can give you good responses from 400 to 1100 nanometres.

This is in nanometer this is nanometer units are written here and the germanium the range is little wider from 800 to 1650 and InGaAs actually gives you good in good you know response in 1100 to 1700. So, that is a typical wavelength range for these different materials. The responsivity if you see for silicon it is 0.4 to 0.6 for germanium about 0.4 and InGaAs has higher responsivity of the order of 0.75 to 0.95.

The dark current in case of silicon is low, but germanium gives you high dark current and it is very low in InGaAs the rise time  $\tau_r$  for silicon is 0.521. Because there is the bandwidth of the

device actually depends on  $t_r$  this is what we have seen when we are doing response time calculations we got  $f$  is equal to approximately  $1/t_r$  or  $t_r$  and for germanium 0.1 to 0.5 InGaAs is 0.05 to 0.5.

So, according to you can calculate the typical bandwidth and the typical bandwidth actually it is given here it is 0.3 to 0.7 gigahertz this is 0.5 to 0.3 gigahertz and this is 1 to 2 gigahertz and the bias voltage in PIN is typically for silicon 5 volts and for InGaAs is also 5 volts you can work in germanium also at 5 volts, but little higher voltage may give you some additional advantage.

So, similar values in APD for silicon the wavelengths are same as you notice, but the avalanche gain which is the; which is the main advantage of APD is about 20 to 400 in silicon, this is what you can achieve for germanium 50 to 200 and InGaAs not a very high number 1 can get but up to 41 can have. The darker current values are shown here and the rise time values are point 1 to 2.

For example, in silicon and 0.5 to 0.8 in germanium and then 0.1 to 0.5. So, related is what is your bandwidth or in this case it is gain bandwidth. So, that is a multiplication of  $M$  and the bandwidth this is how you typically define an APD. So, the gain bandwidth product is in gigahertz for silicon you can go up to 400 gigahertz gain bandwidth product and for germanium you can go up to 10 gigahertz and for InGaAs it is 250 gigahertz.

And the bias voltage required for APD because you have to create avalanche effect you require a very high voltage and for silicon you require about 150 volts for germanium it is quite moderate about 20 to 40 volts and similarly you know InGaAs you require about 20 to 30 volts. So, this is in brief the comparison between PIN and APD and so this with this we conclude the discussion of optical components.

So, we have discussed optical sources that is laser transmitter and led and also we have today discussed about photo detectors the PIN the APD, what is the basic principle of PIN and



APD. How what are the main quantities which are defined in PIN the responsivity the efficiency for PIN and APD.

What are the difference noise component which are produced in the detection process and then we have calculated the signal to noise ratio for PIN and APD. And briefly we saw some typical values or typical components in terms of wavelength responsivity, dark current, rise time bandwidth and bias voltage for different types of PIN and APD.

So, thank you very much and in the next lecture we are going to discuss about indoor optical channel, where we will be using these devices for communication.

Thank you.