**Optical Wireless Communications for Beyond 5G Networks and IoT**
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**Lecture - 16**
**Part - 3**
**MIMO**

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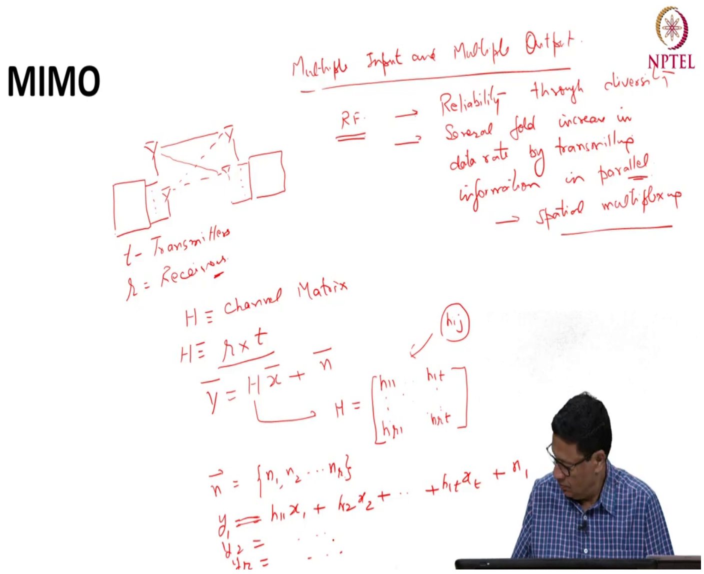


Hello everyone; so, today we are going to understand basics of MIMO; MIMO stands for Multiple Input and Multiple Output. In the early classes we have discussed about NOMA and also discussed how NOMA can be used in VLC. Now, we want to use MIMO and NOMA in VLC environment; so, to understand MIMO, NOMA in VLC environment we need to understand some basics of MIMO.

So, today we will discuss about multiple input and multiple output MIMO. So, in RF MIMO is used for increasing the reliability of the system and this is achieved through diversity, because there are multiple input and multiple output. So, even if one link between one transmitter and another receiver is broken, there are other links available from where you can decode the information.

So, basically there is a diversity there in the system and you are able to increase the reliability of the communication system. The second is the several fold increase in data rate by transmitting information in parallel; so, this is actually also called as spatial multiplexer. So, these are the main uses of MIMO in RF systems, you increase the reliability through diversity. And also, you get increase in data rate, because you are able to transmit parallel information across different channels.

So, a typical MIMO system consists of multiple transmitters; so, this is these are say there are T transmitters and this is the channel here wireless channel and this is receiver and these are receiving antennas. Assume there are receivers r receivers, the signal goes to this antenna for example, it also goes to all the antennas. Similarly signal from here goes to this antenna and this antenna and there are many antennas.

So, we have assumed T transmitters T transmitting antennas and are receiving antennas. So, the channel matrix which is formed the channel matrix is actually r cross t and so, H is r cross r the dimensions here and the noise is also getting added. So, the equation the system equation becomes, why? Now why becomes a vector, because there are multiple outputs is equal to channel matrix into x, x is again a vector. Because, there are multiple inputs plus noise which is also a vector in each channel in each channel

So, this channel matrix can be written as would be r cross t matrix start with h 11, h 1t this will be h r1 and h rt. So, basically h 11 means, receiver 1 and transmitter 1; so, this is basically h ij, i stands for the receiver and j stands for transmitter. So, the channel where channel coefficient between jth transmitter and ith receiver this is what h ij means. And these are the components of the channel matrix coefficients of the channel matrix.

Similarly, the noise there are r receivers; so, it will be a vector and 1 and 2 so on and so forth n r. So, if you see the output which is a vector; so, y 1 for example, there will be R outputs y 12, y 22, y r. So, y 1 will be h 11 x 1 plus h 12 x 2 plus h 1 t x t plus n 1 and similarly you have y 2 and you can have y r, in a similar fashion you can write y r.

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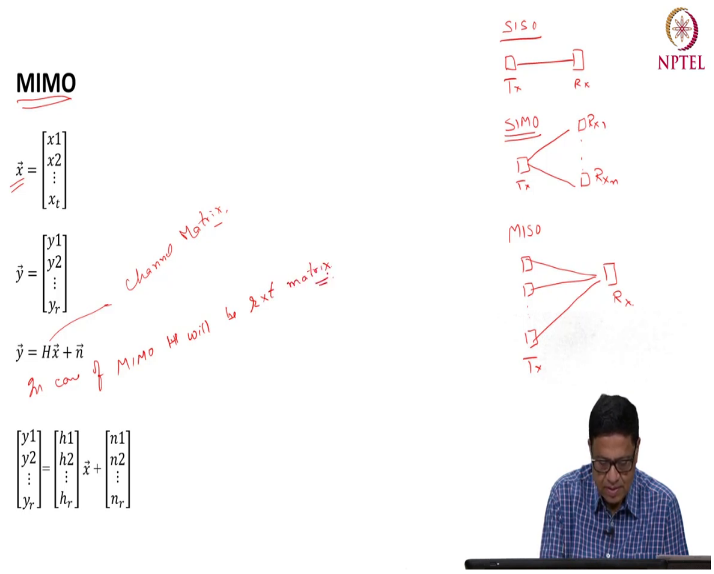


So, there are actually various form of MIMO systems, one is very simple which is single input single output; so, there is one transmitter and one receiver. The other is; the other is SIMO that is single input and multiple outputs; so, you have one transmitter and you have multiple receivers, this is R x form this is say R x n, this is the example of a SIMO. And similarly, you have MISO system which is multiple input single output; so, you have multiple transmitters and single receiver.

These are T x and this is R x and MIMO of course, has multiple inputs and multiple outputs. So, now let us understand a MIMO channel; so, suppose there is a input vector x which has components x 1 to x t there are T transmitters and y there are r receivers; so, y vector will be y 1 y 2 into y r. And y is equal to H which is a channel matrix will be given by Hx plus n. So, this will be y 1 to y r h 1 to h r into x; so, this is an example of a MISO system.

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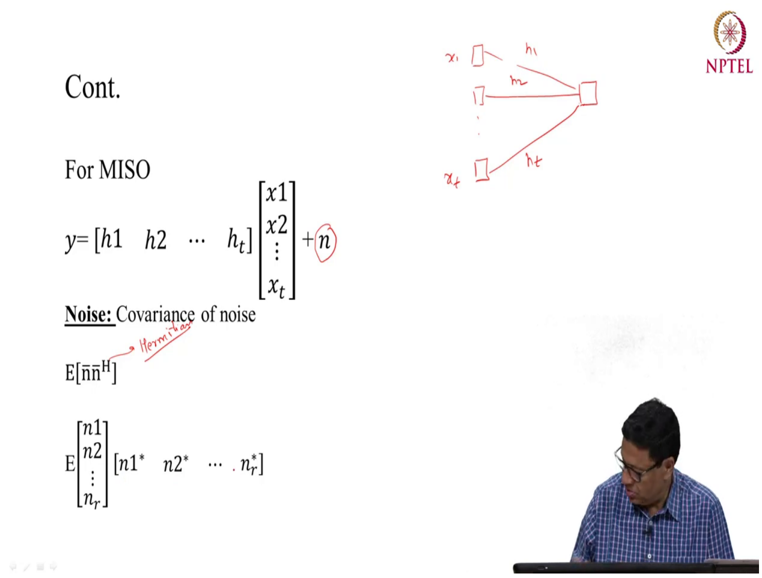


So, this will be actually a matrix here in case of MIMO; so, in case of; in case of MIMO H will be a H will be r cross r matrix. So, for MISO system that is multiple input and single output, this can be written as y this is h 1 to h t. So, let me again draw the MISO system multiple inputs and this is single receiver and this say h 1 this is h 2 this is h t, and this is data here is x 1 this is x t here. So, y is equal to h 1 to h t into x into x 1 x 2 x t plus n.

Now, let us also understand the behavior of noise, what kind of noise is present in the MIMO system? So, let us find out the covariance of noise. So, for finding out the covariance of the noise let us find out because noise is a complex random process. So, expected value of n into n Hermitian this is Hermitian operator is given by expanded value of this.

So, this is n n 1 to n r, because there are r receivers; so, n the components are n 1 and 2 for each branch in 2 then we have to do the transpose and then the complex that is what the Hermitian operator means. So, I have taken the complex and the transpose gives me n 1 complex and n 1 conjugate n 2 conjugate and n r conjugate.

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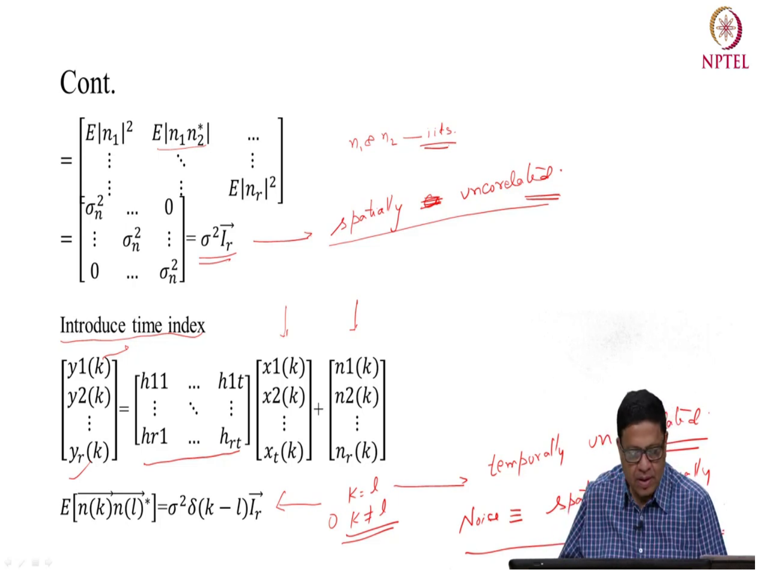


So, if you multiply this you get expected value of n 1 mod square, expected value of n 1 into n 2 conjugate and so on and so forth. So, expected value of n 1 square is nothing but the noise that is sigma n square and other components are 0 because n 1 and n 2 are iids n 1 and n 2 are iids. And expected value of this is a 0 mean process the expected value of n 1 is 0, similar expected value of n 2.

So, this can be broken down to expected value of n 1 into expected value of n 2 conjugate which actually is 0. So, what we get is 0 and what at the output we get sigma n square I r, I r is the identity matrix of rank r or dimension r. So, this actually tells me that the noise component are spatially coherent, because we are only talking currently and in the space direction. In the space dimension spatially coherent is and specially uncorrelated.

So, the value is actually because it has only the diagonal components; so, the noise across the receiver antennas is uncorrelated; so, so, noise is spatially uncorrelated. Now, let us introduce the time index and let us see what happens in the temporal domain or the time domain. So, this is the kth instant y 1 at the kth instant; so, we have to introduce the timing index here now, y 2 at the kth instant and y R at the kth instant is given by the this channel matrix for MIMO.

This is x 1 at the kth instant x 2 at the kth instant and then this is a noise and 1 at the kth instant and so on and so forth. So, this equation introduces the timing part in the system model and if I do the similar calculations as I had done earlier when I was considering the only in the space dimension. And then if I do this and try to calculate expected value of n at the kth instant and expected value and the noise at the lth instant, this gives me sigma n square delta k minus r l I r.

So, this has meaning or this has value when k is equal to l and it is 0 when k not equal to l. So, we also conclude here that it is temporally uncorrelated also, the noise is temporally uncorrelated. So, it is spatially uncorrelated as well as temporally uncorrelated; so, we can finally, conclude noise is spatio temporally uncorrelated. So, this is what we conclude about the noise that it is spatio temporally uncorrelated.

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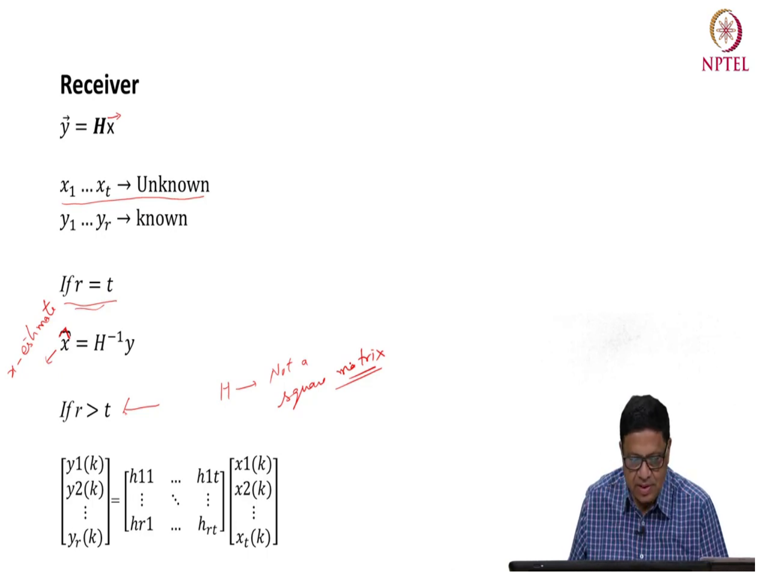


Now, let us discuss about the receiver part; so, as we know y is H x this is again a vector here and H is matrix here. x 1 to x t they are unknown nodes and y 1 to y r, they are known because these are the output. So, we know about the output and we want to decode about the input which are unknown quantities; so, we will use a receiver. So, if R is equal to t that is the number of receivers antennas and transmitter antennas same.

Then we can always estimate the value of x this is x estimate by simply taking the inverse of the channel matrix H inverse y. Because if r is equal to t say square matrix and inverse of a square matrix is possible. So, we can estimate the value of x by taking the inverse and multiplying by the output. But if r is equal to if r is greater than t, then we have something like this, this is y 1 k to y r k is a channel matrix x 1 k x 2 these are the data symbols.

If r greater than t then h is not a square matrix not a square matrix and taking inverse of a square matrix is difficult sometimes it is not possible. So, how do we handle such a situation when number of receivers are greater than the transmitters? r is equal to t we can always solve its a square matrix, we can always find out the inverse, but it is difficult when r is greater than t.

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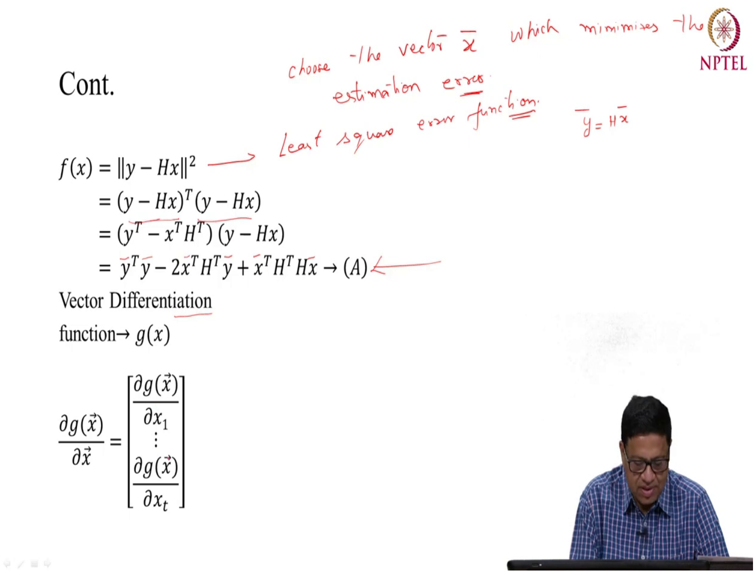


So, for that let us we use a technique where we choose the vector x which minimizes the estimation error which is f x is equal to mod of y minus H x square should be minimum and this is also called as least square error function. So, we want to minimize f x which is y minus H x, as you as you know y is equal to H x matrix; so, we want to minimize this function f x y minus H x.

So, we need to choose a vector x which will minimize this estimation error. So, let us calculate the value of this f x; so, y minus H x mod square can be written as y minus H x transpose into y minus H x. Right now, I am assuming that H is not complex, if it is complex then this transpose operator will be replaced by Hermitian operator.

So, in that case I will take the transpose and the conjugate; so, right now I am assuming this to be real. So, now let us do multiplication of these two terms; so, transpose operator is inside y transpose minus x transpose into H transpose this order gets reversed when you take the transpose into y minus H x. Multiplying all the quantities what we get is this equation which is A here and these are all vectors.

So, I have not put the sign here, but these are all vectors; for example, y is a vector, x is a vector, H is a matrix anyway; so, y is a vector x; so, these are all vectors. Now, vector difference, because I want to differentiate this f x with respect to x vector. So, differentiation in vector is slightly different than what we are normally used to; so, let me just briefly give example of a vector differentiation.

So, vector differentiation suppose there is a function g x, then the differentiation of g x with respect to x vector is given by partial derivative of g x with respect to the components of the components of the x vector which is x 1, x 2 and so on. So, delta partial differentiation g x with respect to x 1, partial differentiation of x vector with respect to x t and there are many terms in between.

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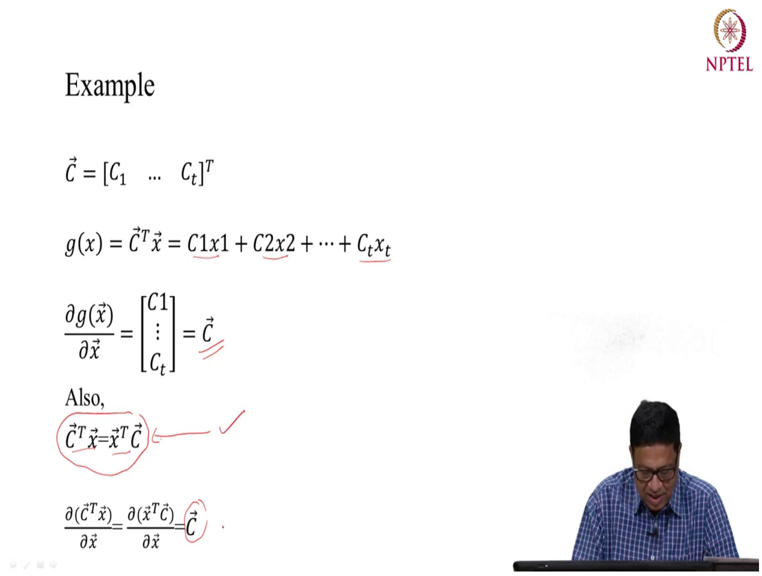


So, let us take an example, suppose there is a vector C which has component C 1 to C t that g x be C T into x. So, if I multiply with x vector I get C 1 x 1 plus C 2 x 2 plus C t x t. Now, if I take the vector differentiation vector with the differentiation with respect to x vector, what I get is C 1 to C t, C t which is nothing but C vector.

Also I can prove this that C T into x vector is equal to transpose x transpose into C. So, this can also be proved and if I take differentiation with respect to x vector for these two quantities it always gives me a C vector. So, basically, we will use this and this to find out the our earlier my example the estimate of x. So, if I go to my earlier equation of this f x and I differentiate this equation A, differentiate this with respect to vector x.

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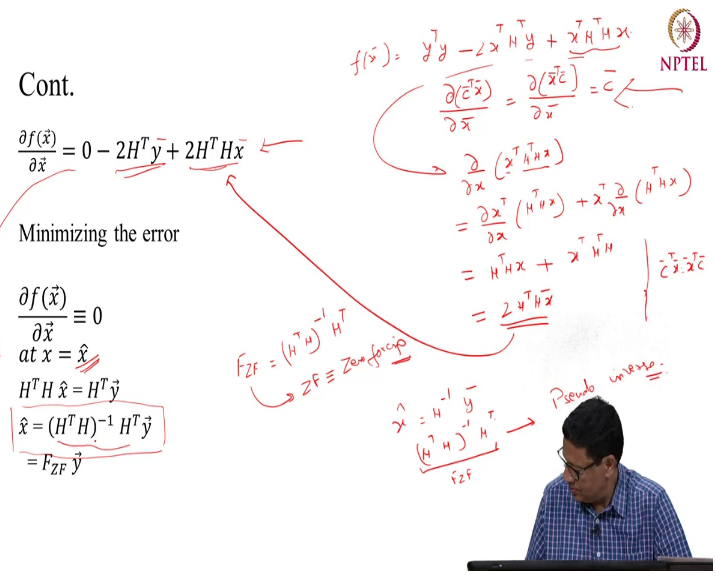


Then I get 0 minus 2 H T y these are again vectors here plus 2 H T H into x vector. So, let me just explain this again; so, my f x was y T y minus 2 x T H T y plus x T H T H x. So, basically, I have used this C T x these are a vector is equal to x T C which is nothing but C.

So, this will give me this will give me d by dx of because this first term is 0 and the second term is 2 H T y 2 H T y this is with respect to x; so, this is minus 2 H T y. I am explaining the third term, the third term will be d by dx of this is x T H x and let us do it by parts; so, this will be x T.

So, this is one part this is another part d x t by dx H T H x plus x t into d by dx of H T H x and this gives me H T H x plus this will give me x T H T H. Where I have used one of those earlier equations which I had discussed which is actually this one I used here. So, this gives me twice H T H x; so, where I have used the other equation that is C T x is equal to x T C and these are all vectors; so, this is how I get this term.

So, minimizing the error I need to differentiate with respect to x which can equate to 0; so, this will give me x estimate. So, if I do this from here, I get H T H into x estimate is equal to H T y and x estimate is given by this H T h whole inverse into H T you know y vector. Let me call this quantity as F ZF where ZF stands for 0 forcing; so, F ZF ZF is 0 forcing.

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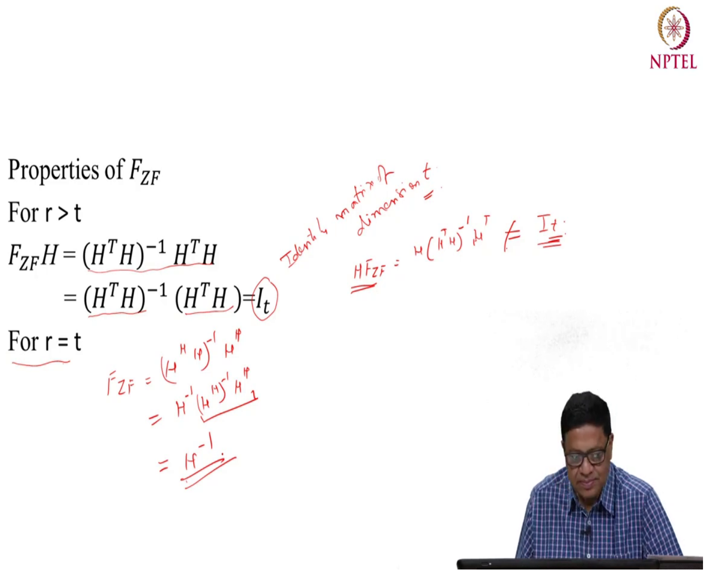


So, here you notice this x is not H is I mean I cannot get x estimate by simply taking inverse; so, it is a different type of inverse here. So, you see this function is H T H minus 1H T. So, this is whole thing is the F ZF and this is called as not the exact inverse the conversion inverse in the conversions is, but it is CDA inverse.

So, for estimating x x we need to have this operation on the channel matrix that is H T into H inverse into H T not simply H inverse y because r and t dimensions of different and it is a rectangular matrix which may not have direct inverse. So, we will have to calculate F Z F for getting the x estimate which is also known as CDA inverse.

Now, let us see the properties of CDA inverse for r greater than t, if I multiply this F Z F by H in this way. Then I see that this is nothing, but identity matrix for dimension T, dimension T. And if I take the it is not inverse, because if I take H F ZF this will be H H T H minus 1H T and this is not equivalent to I t. So, it is not actually inverse matrix in that sense, it is cda inverse because if I multiply H by this F ZF matrix I do not get I identity matrix.

So, its take a special case for r is equal to t when it is a square matrix it will be for r is equal to t for example, F ZF will be H H H minus 1H H. So, if I because this r is equal to t; that means, the inverse of the matrix exist; so, this will be H minus H inverse H H inverse H H and this is 1; so, this actually gives you h inverse. So, with this you with this cda matrix is applicable when r is greater than t that is number of receivers are more than the number of transmitting antennas.

And when they are equal this is simply H inverse this is what we are seeing here. So, let us now take an example how to calculate the how to estimate rather a signal where r and t are of different dimensions.

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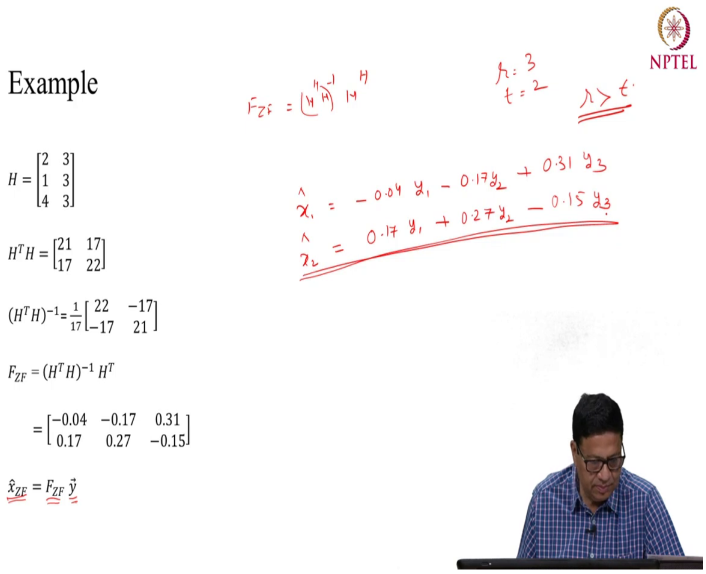


So, suppose there is a channel matrix which has number of experiments have been done and we have we have a we have got this channel matrix. So, first for calculating the for calculating the F ZF, I need to have H H H minus 1 this is whole is 1 into H H. So, first let us calculate because these are all real numbers; so, it will be transport it will not be Hermitian; so, H will be replaced by t. So, H H H t H will be given by this and if I take inverse of this is 1 by 17 and these are the limits.

So, the F F ZF 0 forcing matrix will be given by this formula whatever we had derived and if you plug in all the values this is what you get. So, my estimate x Z F the x estimate will be for a 0 forcing receiver this is 0 forcing receiver will be F ZF into y. And I can write the different for example, the x 1 estimate will be minus 0.04, y 1 minus 0.17 y 2 plus 0.31 y 3 and similarly x 2 will be 0.17 y 1 plus 0.27 y 2 minus 0.15 y 3.

So, there are two inputs and three outputs; so, r is equal to 3 and t is equal to 2 which is a case of r greater than t. So, these are the estimate values of x 1 and x 2 for a case where number of transmitting antennas and receiving antennas are same.

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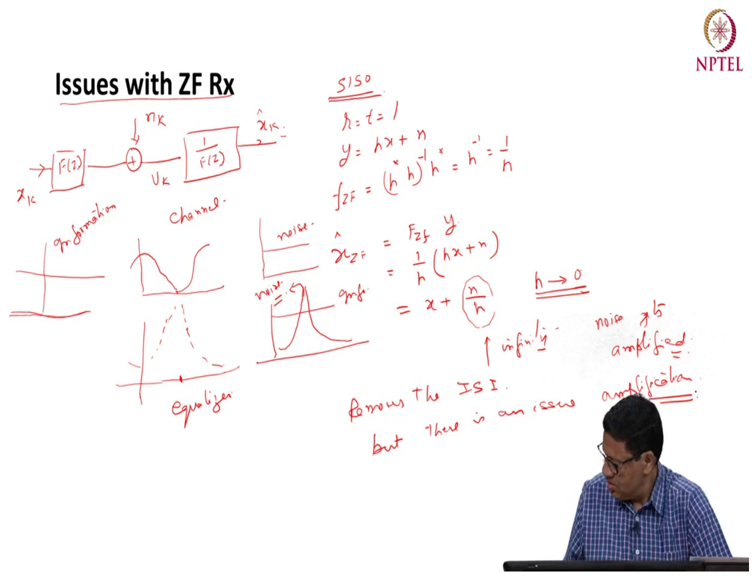


So, let us understand what are the issues with the 0 forcing receiver. Now, 0 forcing receiver let us understand let us see example of SISO system, Single Input Single Output; where, r is equal to t is equal to 1, there is only one transporter one receiver. Now, y is given by h x plus n; where, y is the output H is the channel coefficient x is the input data plus noise.

So, if I take 0 forcing this will give me which is nothing but h inverse is equal to 1 by h. So, x estimate for 0 forcing is actually F ZF this is capital F into y and F ZF we have calculated as 1 by h into y y is h x plus n; so, if I multiply, I get x plus n by h. So, if the channel coefficient if channel coefficient is tending to 0, this quantity actually goes to infinity; in other words, the noise gets amplified which is not a desirable solution.

So, how do we handle this part noise getting amplified? So, let us also understand little more about this noise getting going to infinity. So, for example, as I had mentioned that my system this is my x K say input and then this is the channel matrix is say F Z and the noise is getting added here and what I get is u K. And then on the other hand I have the inverse of the channel matrix F Z and this is I get x estimate k; so, this is our system diagram using 0 forcing receiver.

So, and suppose your noise suppose your information is this, this is say information. And channel happens to be say like this channel has I am deliberately showing its going to 0 at some place, channel characteristics are given by this; so, this is channel and this is say noise. Then the equalizer or the inverse of F Z channel matrix will be, because this is going to 0; so, this will be going to infinity at this point; so, this will going to infinity. So, this is the characters of equalizer which I need to have at the input.

And if I see my data the recovered data which is the information is this and the noise component because of this channel characteristic you know it will go to 0 at this point at this point and then I will get something like this, this is the noise part. So, this is this blow's to infinity; so, this is the noise part goes to infinity. So, it will there is a noise amplification happening; so, this is the drawback of a 0 forcing receiver; whenever h is tending to 0 or close to 0 at that point the noise gets amplified.

So, it is able to remove the ISI completely, but there is a issue of noise amplification. So, it removes the ISI, removes the ISI, but there is an issue of noise amplification. Whenever the channel coefficient is tending to 0 or close to 0; so, how do you solve such a problem? So, that the noise does not get amplified.

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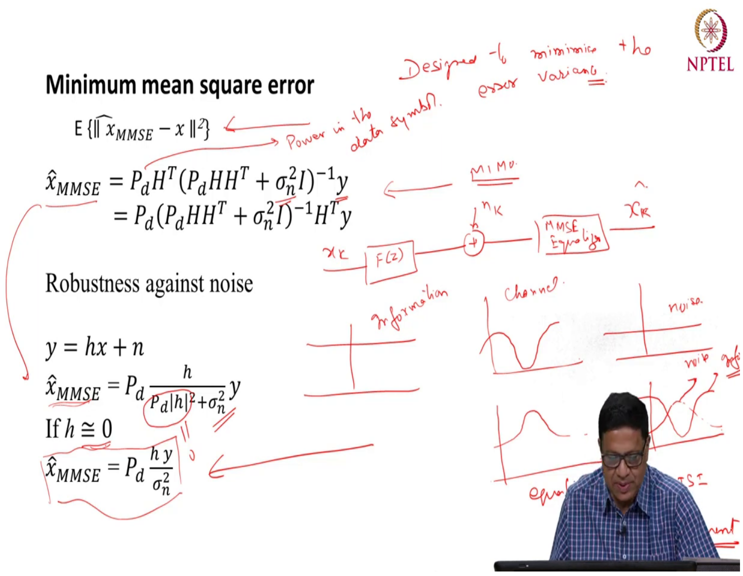


So, for that we use another receiver which is called as minimum mean square error. It is designed to minimize the error variance to minimize the error variance. So, basically x estimate using MMSE minus x mod square this expected value should be minimum; so, this is the MMSE function. And we will not go to the details of calculation of estimate of x MMSE, but this can be done the way we have done for 0 forcing receiver.

So, if you do all those calculations, then x estimate MMSE using minimum mean square error is given by this formula where P d is the power in the data symbol, power in the data symbol. And H is the matrix channel matrix and I have used transpose here, because I have been assuming that H is a real and this not complex, if it is complex then it has to be replaced by Hermitian into Pd HH T plus this is a noise I, identity matrix total thing inverse y.

And the same thing can be manipulated and can be written in a different form which is P d into P d HH T plus sigma and square I whole thing inverse H T y; so, both the expressions are equivalent. So, how let us understand how it is robust against noise; so, y is equal to hx plus n this is simple case of a SISO system. So, my x estimate will be given by P d h P d h square plus sigma n square.

So, this is a case of MIMO system, but I am trying to explain this robustness against noise. Using a SISO system where I assume there is one transfer one receiver which and it is defined by y is equal to hx plus n. So, if I use a siso system, then my this expression x estimate MMSE reduces to P d h P d mod of h square plus sigma n square y.

Now, if h is tending to 0 or equal to 0; so, this quantity will be 0, but the whole thing does not go to infinity, because there is a summation term here sigma n square. So, it does not go to infinity even if h is tending to 0; so, x estimate MMSE will be P d h into y this is 0; so, this becomes divided by sigma n square. So, does not go to the noise is does not get amplified here, this can also be explained using earlier description whatever we had used.

So, this is suppose your channel and this is a noise, this is a xk this is nk and you have the inverse of this or MSE equalizer now here in this case, MSE minimum MSE equalizer and what you get is xk estimate. So, suppose this is your info here, this is information and data information and channel let us see it has a characteristic something like this and this is a noise here.

So, the equalizer characteristic will be something like this, it will not go to infinity now, this is equalizer, this is the channel. It does not go to the infinity which is clear from this equation, because you know still there is a term sigma n square. So, it does not go to blow up to infinity and if you see the output corresponding output will be; so, this is the info here, there is some this is noise.

So, there is some ISI now introduced, but there is no noise enhancement, no noise enhancement; so, MMSE is actually optimal detector, but it is quite complicated. So, there is no noise enhancement and improvement occurs at the cost of not totally removing the ISI; so, it has some ISI, but you do not have any noise enhancement.

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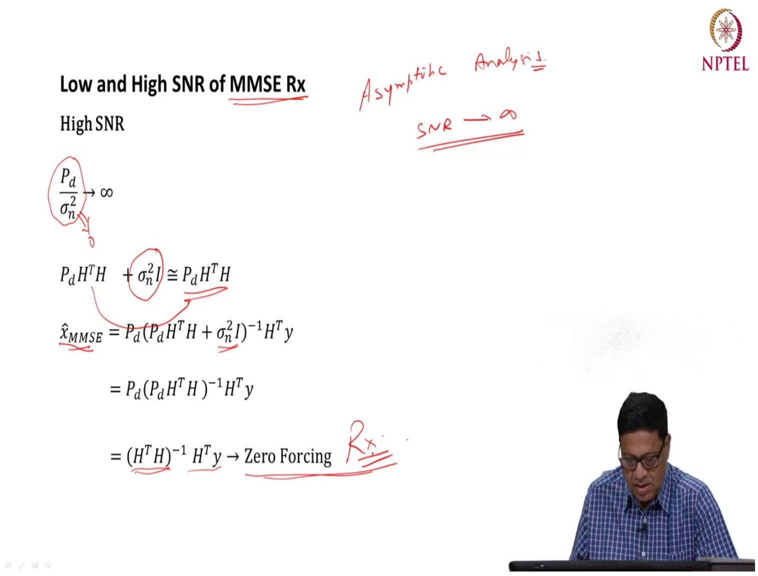


So, now let us see what happens at low and high SNR of MMSE receiver, Minimum Mean Square Error receiver. So, basically, we are doing some asymptotic total analysis, when SNR is tending to infinity or low. So, let us try to see this, let us see at high SNR. So, when high SNR; so, this term which is power square signal power divided by noise will tend to infinity, because the sigma n square is close to 0 in a high SNR condition.

So, if I put this value in the earlier equation which I had written for the when we were discussing the MMSE receiver P d H T H plus sigma n square P is equal to P d H T H, because this is tending to 0; so, we are left with only P d H T H. Now, if you see the estimate which was actually equation earlier a, x estimate MMSE is given by this and if I put this approximation, I get P d P d H T plus H, this is tending to 0 plus H T into H T y. So, this gives me this P d will get cancelled what I get is H T H T y which is nothing but 0 forcing receiver. So, at high SNR where P d by sigma n square tending to infinity; the estimate of MMSE receiver is nothing but 0 forcing receiver.

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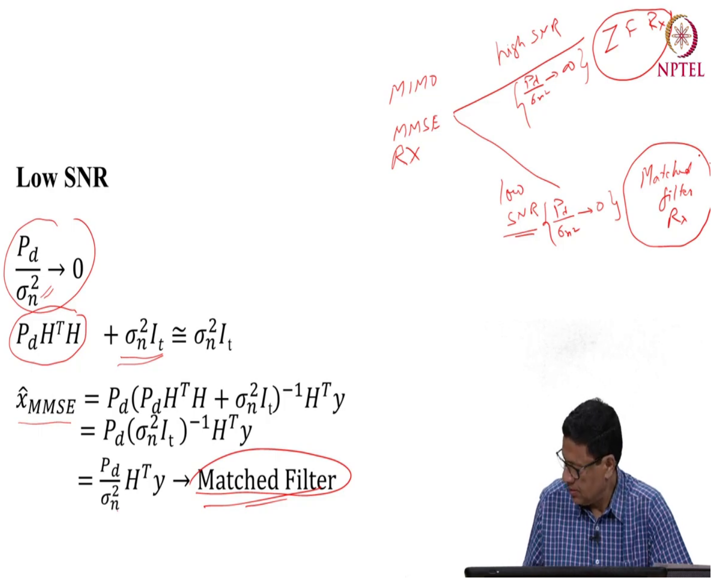


Now, let us see what happens at what happens at low SNR, low SNR means P d by sigma n square is tending to 0, because the noise component is very high; so, this is tending to 0. Again, taking that equation; so, this P d is very low because the low SNR. So, this quantity will be 0 in this case and we are left with sigma n square I t.

And if I put plug in these values, this is approximation in x estimate MMSE, what I get is P d sigma n square I t inverse minus H T y, we can write this as pd divided by sigma n square H T Y. And this is nothing but a matched filter where the impulse responsible is matched to the input; so, this is nothing but matched filter. So, I can write this a diagram form; so, this is for example, MIMO MMSE receiver.

So, there are two conditions which we have evaluated, one is high SNR and low SNR. So, in this case P d over sigma n square is tending to infinity and in this case P d over sigma n square is tending to 0. And we saw in high SNR by putting this approximation, this MIMO MSE MMSE receiver reduces to 0 forcing receiver. And in case of low SNR which is given in this slide, this reduces to a matched filter, matched filter receiver. matched filter receiver.

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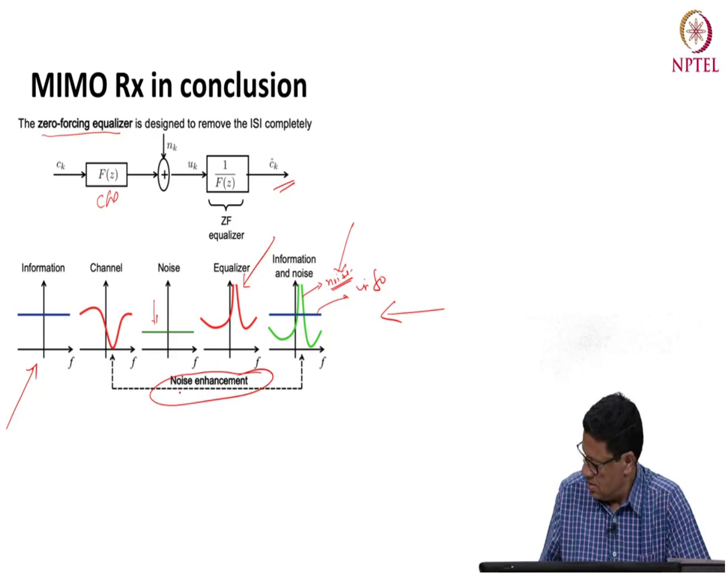


So, in conclusion, MIMO receiver for 0 forcing equalizer; so, this is your channel here, this is the equalizer and this is the estimated output, this is given the information part and this is the channel which is tending going to 0. And if you see the equalizer this thing, this will blow up to infinity because this is 0 forcing receiver.

So, you can see here this going to infinite and this is the noise component and at the output information plus noise. This is your information and the green part which is amplified which is which is getting amplified at when the channel response is channel, the channel coefficient is 0, this is the noise part. So, this there is a noise enhancement which is happening in this case.

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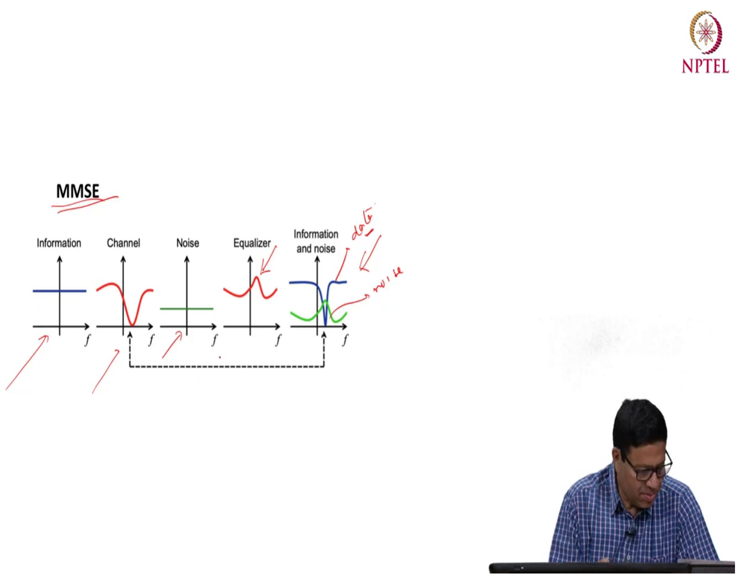


And similarly, for MMSE, this is your information, this is channel, channel in this case is also going to 0 at some frequency and this is the noise part. But because of that sigma n squared term coming in the denominator; it does not go to the infinity; so, there is a you know limit here. So, and if you see both information and noise is given by this; so, this part is the, this part is the green one is the noise and this is the data.

So, there is some sort of ISI which is which gets accumulated in a MMSE receiver, but there is no enhancement, no noise enhancement. So, that is the advantage you get using MMSE receiver. So, this we will stop at this point; now, we have understood the MIMO concept and we also seeing the performance of MIMO using 0 forcing receiver and MMSE receiver. So, we will use some of these receivers in our analysis when we discuss MIMO along with NOMA technique in VSC environment.

Thank you.