

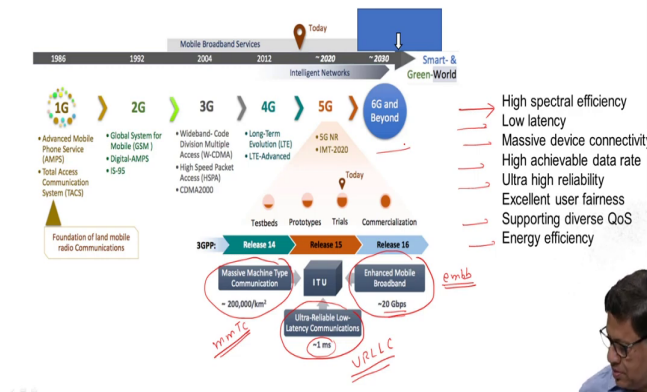
**Optical Wireless Communications for Beyond 5G Networks and IoT**  
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**Lecture - 16**  
**Part - 1**  
**NOMA**

Hello everyone. So, today we are going to discuss about a new technique which is called as NOMA Non- Orthogonal Multiple Access. So, far we have done orthogonal multiple access where we studied in detail about OFDM. Today we will start another technology another multiple access technique which is called as NOMA.

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**5G and IoT Requirements**



So, before I start discussion on NOMA let us understand what are 5G and IoT requirements and how these requirements will be met by NOMA. So, as we know we started with 1G

which was Advanced Mobile Phone Services AMPS and then we have 2G then you have 3G, 2G was GSM, 3G was W-CDMA and high speed packet access, 4G was LTE and currently we have 5G systems in place.

And these 5G systems they give you high spectral efficiency which means you are able to transmit more bits per second in a given bandwidth and it also offers low latency. The latency is of the order of few milliseconds and it supports massive device connectivity of the order of 200,000 per kilometer square and it gives you a high data rate or high throughput or a high achievable data rate.

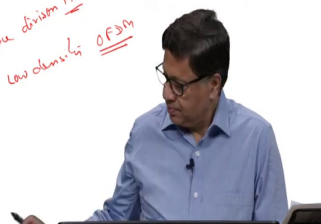
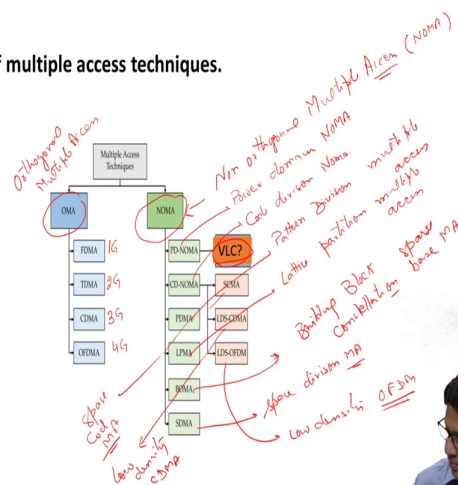
Offers you ultra high reliability and promises excellent user fairness and it supports various quality of service requirements of different users and this 5G technology is energy efficient. And this 5G technology basically is around 3 main pillars. One is which is called as Enhanced Mobile Broadband that is eMBB which promises about 20 gigabits per second at the user interface.

And the second is ultra reliable low latency communication the communication is reliable and low latency of the order of 1 millisecond. And third pillar is Massive Machine Type Communication which is mMTC it can connect as high as 200,000 devices per kilometer square and this is ultra reliable low latency communication.

So, currently these systems are deployed in the network and people have now started working on 6G and beyond. So, these are the some of the requirements of 5G and IoT massive connectivity, ultra reliable low latency communication and enhanced mobile communication system, enhanced mobile broadband communication.

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### Classification of multiple access techniques.



Now, there are various multiple access techniques both in orthogonal space as well as non-orthogonal space. So, this is orthogonal multiple access OMA, OMA Orthogonal Multiple Access. Under this you have FDMA Frequency Division Multiple Access basically 1G system was around this, then TDMA the 2G system was around this and then CDMA 3G system was around this technology and then 4G uses OFDM or optical sorry, Orthogonal Frequency Division Multiple Access.

CDMA is Code Division Multiple Access; TDMA is Time Division Multiple Access. And similarly in NOMA Non Orthogonal, this is Non Orthogonal Multiple Access. Again, there are various variants of NOMA one is PD-NOMA which is called Power Domain NOMA and then you have code division NOMA.

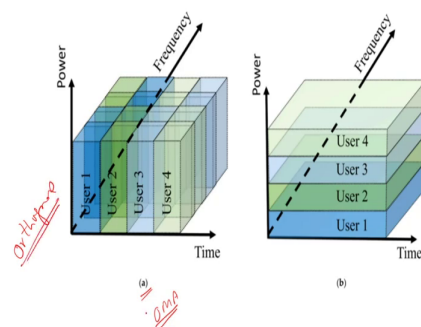
and then you have pattern division multiple access pattern division multiple access and then you have lattice partition multiple access; lattice partition multiple access, then you have a BOMA, which is a Building Block Sparse Constellation Based Multiple Access.

And then you have SCMA oh sorry, SDMA which is Space Division Multiple Access this is Space Division Multiple Access. And under CD NOMA that is Code Division NOMA there are different variants which is SCMA, SCMA is Sparse Code this is Sparse Code Multiple Access and this is low density CDMA and this is low density OFDM.

So, in this lecture we are going to discuss about LOMA and NOMA scheme and in particular we will use or we will take an example of PD-NOMA that is Power Domain NOMA. And we will see how PD-NOMA power domain NOMA can be applied to VLC environment or visible light communication environment.

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Spectrum sharing for (a) OMA; (b) NOMA.



So, this is the basic difference between OMA and NOMA. In OMA you have resources or rb's resource blocks allocated to the users. And this is how this is shown here in this diagram a where frequency resource block is elevated to different users. And in NOMA all the resource block are given to the user and the users have different power levels, but they have all the resource blocks.

So, whether it is a weak user or a strong user they make use of all the resources available. So, that is the basic difference between orthonormal multiple access and non-orthonormal multiple access multiple access. So, this is orthogonal and all these resource blocks they are orthogonal to each other.

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### NOMA over OMA



- Spectral efficiency
- Throughput
- User fairness
- Low latency
- Massive connectivity



So, these are some of the advantages of NOMA over OMA as we know spectral efficiency and throughput, they are better in NOMA. Because in NOMA users have all the resources irrespective of whether it has a poor channel condition or a strong channel condition each user actually has all the resources available to them.

So, which basically increases the spectral efficiency and the throughput and it also in terms of user fairness and low latency and massive connectivity NOMA performs better as compared to OMA. Because for example, in OMA there are weak users and strong users and channel priority is given to strong users and thereby delaying the access to the weak user which actually results in unfairness or fairness is low.

And also, the latency in the system increases and it is not able to connect you know large number of devices whereas; some of these issues are taken care of in non-orthonormal

multiple access. So, it is fair it has low latency and it can cover large number of users. So, it has better features in terms of fairness latency and connectivity as compared to OMA.

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### Successive Interference Cancellation (SIC)



Two users

$$y = x_1 + x_2 + n$$

↓

$P_1$   $P_2$  → noise

max when  $P_2 = 0$

max when  $P_1 = 0$

$$R_1 < \log_2 \left( 1 + \frac{P_1}{N_0} \right)$$

$$R_2 < \log_2 \left( 1 + \frac{P_2}{N_0} \right)$$

$$R_1 + R_2 < \log_2 \left( 1 + \frac{P_1 + P_2}{N_0} \right)$$

Is it possible to get higher sum?

$$R_2 = \log_2 \left( 1 + \frac{P_2}{P_1 + N_0} \right) \equiv A$$

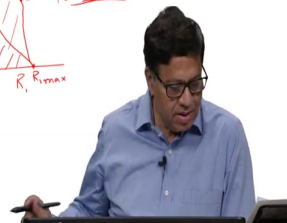
$$R_1 = \log_2 \left( 1 + \frac{P_1}{N_0} \right) \equiv C$$

$$R_1 = \log_2 \left( 1 + \frac{P_1 + P_2}{N_0} \right) \leftarrow \equiv D$$

$$R_2 = \log_2 \left( 1 + \frac{P_2}{N_0} \right) \leftarrow$$

$x_1, x_2$  — i.i.d. independent and identically distributed

from rate



So, before we start discussion on NOMA, it is actually based on fundamental technique which is Successive Interference Calculation which is also called as SIC. So, let me spend next 5-10 minutes on explaining what is this successive interference cancellation and then how it is used in NOMA.

So, let us take a simple example if you have two users, they have data  $x_1$  and  $x_2$ . So, the received data will be  $y$  is equal to  $x_1$  plus  $x_2$  plus noise. So, this is the noise term. So, this is the received data and  $x_1$  is the from user 1 and  $x_2$  from user 2. And suppose  $x_1$  is transmitting at power  $P_1$  and  $x_2$  is transmitting at power  $P_2$ .

Then if you follow the Shannon capacity theorem the rate will be less than  $\log_2(1 + \frac{P_1}{N_0})$  over  $N$  naught. And  $R_1$  will be a max when actually  $P_2$  is 0. So, this will be max when  $P_2$  is equal to 0. Similarly,  $R_2$  the achievable rate is less than  $\log_2(1 + \frac{P_2}{N_0})$  and it will be max when  $P_1$  is 0.

Now, let us see the sum rate  $R_1 + R_2$ . It will be less than  $\log_2(1 + \frac{P_1 + P_2}{N_0})$  where these  $x_1$  and  $x_2$  are iid's. So, we will assume that  $x_1$  this data which is user 1 user 2 data are iid's that is Independent and Identically Distributed Independent and Identically Distributed.

So, now if I plot these rates  $R_1$ , this is say  $R_2$  it will follow this kind of this thing and here you have this is  $R_1$  max when  $P_2$  is equal to 0  $R_1$  max and this is  $R_2$  max this point. So, this is for a normal scheme. Now, let us see for now you can get actually sum rate which is as per the area defined under this curve.

So, now my next question is whether is it possible to get higher sum rate? Is it possible to get higher sum rate? So, now let us apply NOMA technique and see whether we are able to get sum rate higher than what we have got using OMA technique. So, in NOMA what happens the higher power is detected first. So,  $R_2$  will be  $\log_2(1 + \frac{P_2}{P_1 + N_0})$  because the contribution from the user 1 is stated as noise.

So, the signal having higher power is detected first and rest of the signals they give they contribute to the noise, it is noise for them. So, that is why I have written this  $1 + \frac{P_2}{P_1 + N_0}$  power in the user 2 divided by  $P_1$  which is the noise part and the  $N_0$  is the normal noise. So, if I see this point this will be  $R_2$  will be somewhere here.

So, this point will be somewhere here because this is  $P_2$  divided by  $P_1 + N_0$  noise component is higher. So, it will be less than  $R_2$  max. And similarly, if I see  $R_1$  now once I have detected the  $P_2$  then I subtract the user from that signal and I am left with only signal of user 1.



So,  $R_1$  will be  $\log_2(1 + \frac{P_1}{N})$  because  $P_2$  has already been subtracted that is the basic principle of successive interference calculation, you detect the highest power and then after you are detected you subtract from the rest of the signal and then again, this cycle goes on.

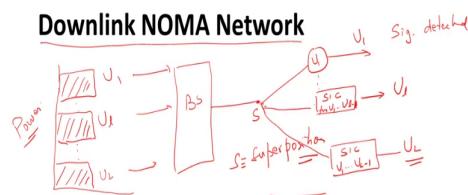
So,  $R_1$  is equal to  $\log_2(1 + \frac{P_1}{P_2})$ . And similarly on the other side I can have  $R_2 = \log_2(1 + \frac{P_2}{N})$  and  $R_2$  will be  $\log_2(1 + \frac{P_2}{N})$ . So, in this case the power  $P_1$  has higher power which has been detected, the  $P_2$  is the noise in the system and once you have subtracted  $P_1$  you are left with this  $R_2$ .

So, if I plot all these values here then I get. So, this is  $R_1$  max it is somewhere here and this is somewhere here. So, I so this is say C and D, this point is actually  $R_2$  is A; this will be A here and this point is C and similarly you have the D point. So, this is  $R_1$  log 2. So, D will be somewhere here. So, this is D and then you have  $R_2$  max will be somewhere here.

So, basically now you are able to get you know you can have combination of  $R_1$ ,  $R_2$  which lies in a curve which is given by a larger area as compared to orthonormal multiple access. So, which basically tells that if I see the sum rate for case of NOMA there is a possibility of getting higher sum rate provided you have chosen the power of different users in some appropriate fashion.

So, the sum rate in case of NOMA is higher and this is basically use of successive interference cancellation technique to get data from to get data demodulated or get extracted from the composite signal.

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Now, having understood SIC Successive Interference Cancellation, let us try to discuss a downlink NOMA scenario. And we will try to calculate the achievable rate in case of downlink as well as in the case of uplink. So, let us start with downlink. So, downlink is you have users having different powers.

So, suppose this is user  $U_L$  capital  $L$  this is the power of capital  $L$   $L$ th user and then you have somewhere in the middle say  $U_{L-1}$  user and then you have this is say  $U_1$  user and there are many users in between. So, this is the power allocated to different users this is power. And these are transmitted and towards the base station and these signals are superimposed. So, this is  $S$  is the point this is super as a superposition and then user with the highest power is detected first.

So, in this case you detect the user once we get data of the user 1 this is signal detected and then so on and so forth.  $U_2$ ,  $U_3$  and for  $U_L$ th user you will perform a successive interference cancellation technique for  $U_1$  to  $U_{L-1}$  this is  $L-1$ . So, this will give you  $U_L$ th signal. And similarly, to get data from that  $U_L$ th you have to do SIC from  $U_1$  to  $U_{L-1}$ .

So, this is how you extract the data using successive interference cancellation technique. So, this is the basic diagram of a downlink NOMA network. So, where you have users having different powers  $U_1$  is the highest power you detect that user first then subtract from rest of the users detect the second one and so on and so forth till you extract the data from the VK signal which is  $U_L$  in this case.

So, normally the bad user or the which has a poor channel condition is given higher power and a good user which has a good channel condition is given low power. So, this is how the power allocation is done for different users.

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$$s = \sum_{i=1}^L \sqrt{a_i P_s} x_i \rightarrow \begin{array}{l} L = \text{No. of Users} \\ a_i = \text{Power Coefficient} \\ P_s = \text{Power} \\ x_i = \text{data} \end{array} \quad \underline{\underline{\text{SNR}}}$$

$$\sum_{i=1}^L a_i = 1 \quad \checkmark$$

$$a_1 \geq a_2 \dots \dots \geq a_L \quad \checkmark$$

$$|h_1|^2 \leq |h_2|^2 \leq \dots \leq |h_L|^2 \quad \checkmark$$

$$y_l = h_l s + n_l \quad \checkmark$$

$$= h_l \sum_{i=1}^L \sqrt{a_i P_s} x_i + n_l \quad \checkmark$$



So, now let us try to understand signal to signal SNR analysis for NOMA signal. Calculation of signal to noise ratio for NOMA signals. So, now at the super imposition super position point the S where all the powers are combined is given by i is equal to L to L where L is number of users and each user is given some power which is defined by the power coefficient a i, a i is the power coefficient and P s is the power and x i is the data of the ith user.

So, the S is actually summation of root of a i into P i P s into x i a i is power coefficient P s is the power and x i is the data on the x x i use ith user. And this power coefficient when you sum over all the users is equal to 1. And we assume that a 1 is greater than a 2 is greater than a L.

Now, depending on the channel condition you allocate power to different users. So, in this case the power coefficient is highest at a 1. And if you see the channel condition of a 1 is very

weak whereas, the  $h_1$  is the  $h_1$  square the channel coefficient is smaller as compared to  $h_2$  or  $h_3$ ,  $h_4$  and  $h_L$  is the highest channel coefficient and whereas, it is given the lowest power  $a_L$ .

So, again I am repeating the users which are having bad channel conditions are given more power as compared to users which have good channel conditions. So, this is basically depicted from these two equations. So, the data on the  $L$ th user will be given by  $h_L$ ,  $h_L$  is the coefficient of the  $L$ th user into  $s$  is the superposition signal plus noise in the  $L$ th user. So, this is the data in the  $L$ th user. And if I put the value of  $s$  here in this equation. So, this becomes  $h_L$  summation  $i$  is equal to 1 to  $L$   $a_i P_s x_i$  square root plus  $n$ .

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**SINR analysis**

$$SINR_{j \rightarrow l} = \frac{a_j P_s |h_l|^2}{P_s |h_l|^2 \sum_{i=j+1}^L a_i + \sigma^2}$$


*The instantaneous SINR of the jth user to detect the lth user.  $j \leq l$   $j \neq L$*


$$= \frac{a_j \gamma |h_l|^2}{\gamma |h_l|^2 \sum_{i=j+1}^L a_i + 1}$$

*where,  $\gamma = \frac{P_s}{\sigma^2} = \text{SNR}$*

$$(SINR)_l = \frac{a_l \gamma |h_l|^2}{\gamma |h_l|^2 \sum_{i=l+1}^L a_i + 1}$$

$$(SINR)_L = a_L \gamma |h_L|^2$$





Now, let us do the SINR analysis. So, SINR  $j$  to  $l$  actually this is the instantaneous SINR of the  $l$ th user to detect the  $j$ th user and  $j$  is less than  $l$  and  $j$  is not equal to  $L$ . This can be written


as a  $j$   $P_S$  coefficient of the  $j$ th user  $P_{s, h_l}$  is the  $L$ th user given by this is the noise term which is coming from rest of the components where  $i$  is from  $j + 1$  to  $L$  remaining users into a  $i$  plus the noise which is  $\sigma^2$ .

So, this is the interference term this is the noise, this is the interference term. And this power is the instantaneous  $s_i$ , instantaneous power of the  $L$ th user to detect the  $j$ th user. So, I can write this  $a_j$  this  $\gamma$  can be written as  $P_s$  over  $\sigma^2$ . So, this can be written as  $a_j \gamma_{h_l}^2$  divided by  $\gamma_{h_l}^2$  summation this  $a_i + 1$ .

So, I have used the identity  $\gamma$  is equal to  $P_s$  by  $\sigma^2$  which is actually the SNR signal to noise ratio. So, SINR for the  $L$ th user will be  $a_l \gamma_{h_l}^2$  divided by  $\gamma_{h_l}^2$  summation  $i$  is equal to  $l + 1$  to  $L$  this is again the interference term. And here it is one because I have replaced  $P_s$  by  $\gamma$  as  $P_s$  by  $\sigma^2$  as  $\gamma$ .

So, SNR for the  $L$ th user is given as follows. And if I see the SNR for the last user the  $L$ th capital  $L$ th user where there is no interference then this term will be 0 for that particular  $L$  capital  $L$ th user then I am left with a capital  $L$  this was the capital  $L$ th user  $\gamma_{h_L}^2$  divided by  $\gamma_{h_L}^2$ .

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$$\begin{aligned}
 \text{Sum rate} & \rightarrow \text{downlink} \\
 R_l^{\text{NOMA-d}} &= \log_2(1 + \text{SINR}_l) \\
 &= \log_2 \left( 1 + \frac{a_l \gamma |h_l|^2}{\gamma |h_l|^2 \sum_{i=l+1}^L a_i + 1} \right) \\
 R_{\text{sum}}^{\text{NOMA-d}} &= \sum_{l=1}^L \log_2(1 + \text{SINR}_l) \\
 &= \sum_{l=1}^{L-1} \log_2 \left( 1 + \frac{a_l \gamma |h_l|^2}{\gamma |h_l|^2 \sum_{i=l+1}^L a_i + 1} \right) + \log_2(1 + a_L \gamma |h_L|^2)
 \end{aligned}$$


*Handwritten notes in red:*  
 - A bracket on the first sum from  $l=1$  to  $L-1$  is labeled  $(L-1)$ .  
 - The second term is labeled  $L^{\text{th}}$  user.

Now, let us try to calculate the sum rate. The sum rate of the downlink for this is for  $R$   $l$ th user rate NOMA this is  $d$  stands for downlink is given by  $\log_2 1 + \text{SINR}_l$ . Now, if I take the sum rate before I take the sum rate. So, this is expanded here  $\log_2 1 +$  this we have calculated from our previous discussion  $a_l \gamma |h_l|^2$  divided by interference term and plus 1.

The sum rate for the downlink will be sum over all the users that is  $\log_2 1 + \text{SINR}_l$  and this can be written into two terms one is up to  $L$  minus 1 users and this is for the  $L$ th user. So,  $i$  is equal to 1 is equal to 1 to  $L$  minus 1 and you have the SNR value for this I have replaced here and this is for  $L$ th user where there is no interference. So, this particular term will be 0. So, this is for the  $L$ th capital  $L$ th user. So, the sum rate of NOMA  $d$  is given by this expression.

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$$\begin{aligned}
 &= \sum_{l=1}^{L-1} \log_2 \left( 1 + \frac{a_l}{\sum_{i=l+1}^L a_i + \frac{1}{\gamma |h_l|^2}} \right) + \log_2(1 + a_l \gamma |h_l|^2) \quad \leftarrow R_{sum}^{NOMA-d} \\
 &\gamma \rightarrow \infty \quad \text{high SNR} \\
 &R_{sum}^{NOMA-d} \approx \sum_{l=1}^{L-1} \log_2 \left( 1 + \frac{a_l}{\sum_{i=l+1}^L a_i} \right) + \log_2(\gamma |h_l|^2) \quad \leftarrow \gamma \rightarrow \infty \\
 &\approx \log_2(\gamma |h_L|^2) \quad \leftarrow
 \end{aligned}$$


So, again writing this again here this is the total sum rate this is actually R NOMA sum and I am doing for downlink. So, this is this value. So, now let us try to see what happens at high SNR that is just to get some better insight what happens at high SNR when gamma is tending to infinity.

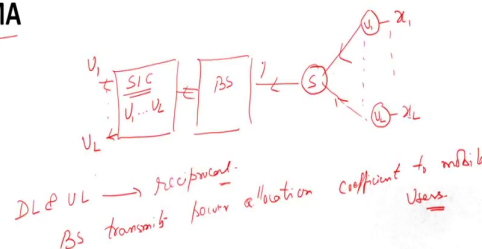
So, when gamma is tending to infinity R NOMA d sum will be equal to because gamma tending to infinity this term will be high. So, 1 by you know gamma tending to infinity will give you 0. So, I am left with approximately  $\log_2 1 + a_l$  divided by i is equal to 1 plus 1 to capital L a i because this term will be 0 plus the second term will be because 1 is small as compared to a l gamma h l square and when you take log 2 a b the log 2 a that is a l will be very very small as compared to other term which has gamma.



So, I will be left with  $\log_2 \gamma \text{ mod } h_1 \text{ square}$ . So, this is for when  $\gamma$  tending to infinity. So, this expression above expression can be approximated in this fashion. And this basically because again  $\gamma$  is tending to high so, basically this term will dominate as compared to the first term. So, we get a very approximate and clean result for R NOMA sum for the downlink which is  $\log_2 \gamma \text{ mod } h_1 \text{ square}$ . So, this is the rate for the downlink sum rate.

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### Uplink NOMA



So, now this is this we have got expression for sum rate for the downlink. Now, let us try to understand the uplink and also calculate the sum rate for the uplink. Everything is, ok. Now, we will discuss about the uplink NOMA. So, we have calculated the sum rate for the downlink NOMA. Now, we will calculate the sum rate for the uplink NOMA.

Now, let us first understand the how uplink NOMA works. So, you have data  $x_1$  of the user 1. This is the user 1 so on so forth you have  $x_L$ th user,  $x_L$  capital L is the data on the Lth, capital Lth user. This is  $U_L$ . So, these data are superimposed in this block and this receive data goes to base station. And this base station performs the SIC function.

So, this basically part of the base station, it does sub-sub carrier successive interference cancellation for  $U_1$  to  $U_L$  and  $U$  detect  $U_1$  to  $U_L$ , the data from all the users. So, this is how it works in case of uplink NOMA. So, the downlink and the uplink are actually reciprocal. In one case, the SIC is happening at the user interface; in this case, uplink the SIC is happening at the base station level.

And base station for example, in uplink NOMA base station transmits, power allocation, how different powers are allocated because we are discussing power domain NOMA here allocation, coefficient to mobile users. So, this is in brief the uplink NOMA. Now, let us try to calculate the sum rate for the uplink NOMA. So, this is the point r here.

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## Uplink NOMA



$$r = \sum_{i=1}^L h_i \sqrt{a_i P} x_i + n$$

noise

SINR analysis

The BS decodes the signals of users in an order according to power coefficients of users

$$\text{SINR}_l = \frac{a_l \gamma |h_l|^2}{\gamma \sum_{i=1}^{l-1} a_i |h_i|^2 + 1}, \quad l \neq 1$$
$$\text{SINR}_1 = a_1 \gamma |h_1|^2$$



If you see the  $r$ , the received signal is summation coming from different users,  $h_i \sqrt{a_i P} x_i$  plus noise.  $P$  is the power,  $a_i$  is the power coefficient,  $P$  is the total power which is scaled down scaled by power coefficient factor  $a_i$ . So, doing SINR analysis, the base station decodes the signals of users in an order according to the power coefficient of the users.

So, a power coefficient is high, that user that data is detected first, then it is subtracted from rest of the signal and this process continues till you get the data from the last user. So, SINR for the  $l$ th user is given by  $a_l \gamma |h_l|^2$ , this is the interference term plus 1 and  $l$  is equal to not equal to 1 because in that case, there will not be any interference. So, this will be 0. So, SINR for the one 1th user will be  $a_1 \gamma |h_1|^2$ .

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### Sum rate analysis

$$R_{sum}^{NOMA-U} = \sum_{l=1}^L \log_2(1 + SINR_l)$$

$$= \log_2(1 + a_1 \gamma |h_1|^2) +$$

$$\sum_{l=2}^L \log_2 \left( 1 + \frac{a_l \gamma |h_l|^2}{\gamma \sum_{i=1}^{l-1} a_i |h_i|^2 + 1} \right)$$

$$= \log_2 \left( 1 + \gamma \sum_{l=1}^L a_l |h_l|^2 \right)$$

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$L=1$   $\frac{T_{00}}{R_{sum}^{NOMA-U}} = \log_2(1 + a_1 \gamma |h_1|^2)$

$L=2$   $\log_2(1 + a_1 \gamma |h_1|^2) + \log_2 \left( 1 + \frac{a_2 \gamma |h_2|^2}{\gamma a_1 |h_1|^2 + 1} \right)$

$\log_2(1 + a_1 \gamma |h_1|^2) + \log_2 \left( \frac{\gamma a_1 |h_1|^2 + 1 + a_2 \gamma |h_2|^2}{\gamma a_1 |h_1|^2 + 1} \right)$

$\log_2(\gamma a_1 |h_1|^2 + a_2 \gamma |h_2|^2 + 1)$

$L=3$  ...



And if you see the sum rate analysis, do the sum rate for the uplink, this is uplink. A summation over all the users  $\log_2(1 + SINR_l)$ , the similar way as we had done for the downlink. So, this will have two terms, this is for the one  $l$ th user and this is for lesser user from  $l$  is equal to 2 to capital  $L$ . So, this is  $\log_2(1 + \gamma a_l |h_l|^2)$  plus  $\log_2 \left( 1 + \frac{a_l \gamma |h_l|^2}{\gamma \sum_{i=1}^{l-1} a_i |h_i|^2 + 1} \right)$ .

Now, let us try to understand how we have got this expression. So, this expression from here, how we have got this? So, let us do what two users to understand this. So, two users then  $R_{sum}^{NOMA-U}$  will be  $\log_2(1 + \gamma a_1 |h_1|^2)$  plus this is for  $L$  is equal to 1. So, so there is only one user and this will be  $1 + \gamma a_1 |h_1|^2$  mod square.

So, if there are two users, then this will have two terms, that is  $\log_2(1 + \gamma a_1 |h_1|^2)$  plus  $\log_2 \left( 1 + \frac{a_2 \gamma |h_2|^2}{\gamma a_1 |h_1|^2 + 1} \right)$ . So, I am taking from this equation A actually for two users. So,

this will be  $1 + a_2 \gamma h_2^2$  mod square divided by  $\gamma a_1 h_1^2 + 1$ . So, this can be written as, we can rewrite this as  $\log_2 1 + a_1 \gamma h_1^2$  whole square plus  $\log_2$ .

I am taking  $h_1^2$  mod square plus 1 and this will be  $\gamma a_1 h_1^2 + 1$  plus  $a_2 \gamma$ . Where I have simplified this equation and this is  $\log_2 a$  plus  $\log_2 b$  which is nothing but  $\log_2 ab$ . So, this will give me  $\log_2$  and then this will get cancelled here and then you are left with  $\gamma a_1 h_1^2$  whole square plus  $a_2$  plus  $a_2 \gamma h_2^2$  whole square, this is  $a_2$  and this is  $\gamma$ .

So, if I extend the logic for 1 equal to 3 users, you will have one more term which is  $a_3 \gamma h_3^2$  mod square so on and so forth. So, basically if you want to solve this, then you will get summation of  $\gamma$  will come out, summation of  $a_1, h_1^2$  mod square. So, this is how this has been solved. And now let us try to evaluate this expression for at high SNR which is  $\gamma$  tending to infinity. So,  $\gamma$  tending to infinity will give me because this will get neglected here because  $\gamma$  is very high.

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$$\gamma \rightarrow \infty$$



$$R_{sum}^{NOMA-U} \cong \log_2 \left( \gamma \sum_{l=1}^L |h_l|^2 \right) \leftarrow$$



So, I am left with approximately  $\log_2 \gamma \sum_{l=1}^L |h_l|^2$ . So, this is the expression for R NOMA sum rate for uplink.

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### Comparing NOMA and OMA

$$R_l^{OMA} = \alpha_l \log_2 \left( 1 + \frac{\beta_l \gamma |h_l|^2}{\alpha_l} \right)$$

$\beta_l$  and  $\alpha_l \rightarrow$  power coefficient and the parameter related to specific resource of  $U_l$



$$R_{SUM}^{OMA} = \sum_{l=1}^L \alpha_l \log_2 \left( 1 + \frac{\beta_l \gamma |h_l|^2}{\alpha_l} \right)$$

$$\alpha_l = \beta_l = \frac{1}{L}$$

$$R_{SUM}^{OMA} = \sum_{l=1}^L \frac{1}{L} \log_2 (1 + \gamma |h_l|^2)$$



Now, let us start comparing NOMA and OMA. So, for OMA there is Orthogonal Multiple Access. The rate is given by  $\alpha_l \log_2 \left( 1 + \frac{\beta_l \gamma |h_l|^2}{\alpha_l} \right)$ , where  $\beta_l$  and  $\alpha_l$  are power coefficient,  $\beta_l$  is actually power coefficient. And the  $\alpha_l$  is the parameter related to specific resource of  $U_l$  lth user.

So,  $\beta_l$  is the power coefficient and  $\alpha_l$  is the parameter that is related to specific resource of  $U_l$ . It could be frequency for example. So,  $R_l^{OMA}$  is given by  $\alpha_l \log_2 \left( 1 + \frac{\beta_l \gamma |h_l|^2}{\alpha_l} \right)$ . So,  $R_{SUM}^{OMA}$  rate will be sum over all the users and the same expression here.

And in OMA it is this  $\beta_l$  and  $\alpha_l$  are actually equally divided by all the users which is  $1/L$ . So, putting  $\alpha_l = \beta_l = 1/L$ . Then we are left with  $R_{SUM}^{OMA}$

rate OMA which is summation  $\frac{1}{L} \log_2(1 + \gamma |h_l|^2)$  because  $\beta_l$  and  $\alpha_l$  will get cancelled. So, I am left with  $R$  this is sum rate for OMA.

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$$\gamma \rightarrow \infty$$

$$R_{sum}^{OMA} = \sum_{l=1}^L \frac{1}{L} \log_2(1 + \gamma |h_l|^2)$$

$$\text{Using } |h_1|^2 \leq |h_2|^2 \leq \dots \leq |h_L|^2$$

$$R_{sum}^{OMA} \cong \sum_{l=1}^L \frac{1}{L} \log_2(\gamma |h_l|^2) \leq \sum_{l=1}^L \frac{1}{L} \log_2(\gamma |h_L|^2)$$



So, evaluating this expression when  $\gamma$  is tending to infinity that is at high SNR condition, this OMA sum rate will be given by  $\frac{1}{L} \log_2(1 + \gamma |h_l|^2)$ . Now, the child coefficient is in this order  $|h_1|^2$  is less than  $|h_2|^2$  and so on and so forth.  $|h_L|^2$  is the highest child coefficient.

So, using this we can write  $R_{sum}^{OMA}$  is equal to  $\frac{1}{L} \log_2(1 + \gamma |h_l|^2)$  will be less than because I have replaced this  $|h_l|^2$  by this  $|h_L|^2$  which is actually the highest. So, this inequality is valid. So, I can write this  $R_{sum}^{OMA}$  as less than or equal to this quantity.



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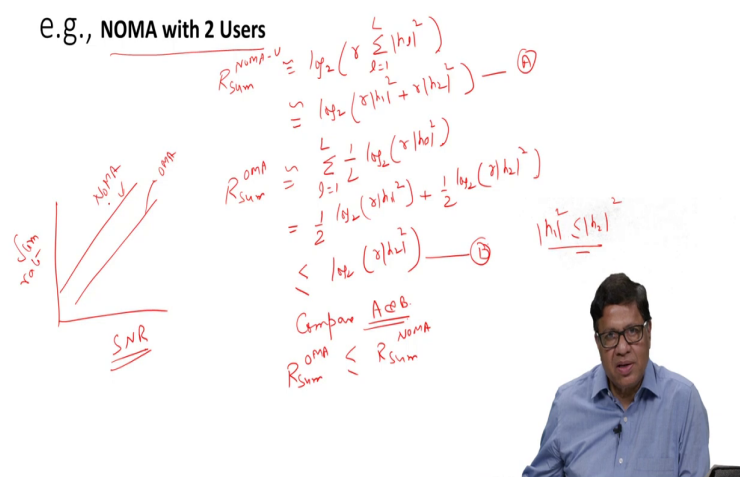
$$\begin{aligned}
 & \leq \sum_{l=1}^L \frac{1}{L} \log_2(\gamma |h_L|^2) \\
 & = L \frac{1}{L} \log_2 \gamma |h_L|^2 \\
 & = \log_2 \gamma |h_L|^2 \quad \text{OMA } R_{sum} \\
 & = R_{sum}^{NOMA-d} \\
 & \therefore R_{sum}^{OMA-d} \leq R_{sum}^{NOMA-d}
 \end{aligned}$$



And this will be less than equal to 1 by L from the previous equation. And if I do summation so, it will be L times the same thing plus this plus this L times. So, L into 1 by L log 2 gamma h L square. So, L will get cancelled and this is the expression I get which is R sum OMA log 2 gamma h L square. And if you see here this expression is nothing but same expression as R NOMA sum downlink.

So, we can always say because this less than is coming here this less than or equal to. So, what we therefore, we can say that R that is the rate sum rate of OMA for downlink is less than or equal to R NOMA downlink sum rate for NOMA downlink. So, the sum rate is higher in case of NOMA.

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Now, let us see this example small example with two users. So, let us see the understand best example with two users. So,  $R_{sum}^{NOMA}$ , let us take for example, the uplink is given by  $\log_2$ . This we have derived in the earlier you know slides. So, this will be  $\gamma$  is equal to  $\frac{1}{L} \sum_{l=1}^L |h_l|^2$ . And so, this will be equal to for two users  $\log_2 \gamma$   $\frac{1}{2} \log_2 (\gamma |h_1|^2)$  plus  $\frac{1}{2} \log_2 (\gamma |h_2|^2)$  this is for two users.

Whereas, if you see the  $R_{sum}$  rate OMA for two users for under  $\gamma$  tending to infinity high SNR condition is  $\frac{1}{2} \log_2 (\gamma |h_1|^2)$  plus  $\frac{1}{2} \log_2 (\gamma |h_2|^2)$ . And this  $L$  is actually for OMA is half because there are two users. So,  $L$  will be  $\frac{1}{2}$ . So, this will give me you know  $\frac{1}{2} \log_2 (\gamma |h_1|^2)$  plus  $\frac{1}{2} \log_2 (\gamma |h_2|^2)$ . And we know this  $|h_1|^2$  is less than  $|h_2|^2$ .

So, if this is valid then I can write this as it will be definitely less than  $\log_2 \gamma h^2$  whole square. Now, seeing this equation and this equation A and B that is compare A and B we can easily see that R sum rate OMA this is an example of a two users is less than R sum rate NOMA.

So, if you plot if you plot SNR versus sum rate this will be some sort of you know this curve will be for NOMA and this will be for OMA. So, sum rate in case of NOMA power domain NOMA we have seen for both uplink and downlink it is higher as compared to OMA.

So, this basically gives a basic feel about the NOMA and then we will also see how this NOMA can be using VLC environment. And later on, also we will see because the NOMA has been used along with MIMO. So, we will also see how NOMA and MIMO combination can be used in visible light communication systems.

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NOMA	OMA
$R_1 = \log \left( 1 + \frac{P_1  h_1 ^2}{P_2  h_1 ^2 + N_0} \right)$ bits/sec/Hz	$R_1 = \alpha \log \left( 1 + \frac{P_1  h_1 ^2}{\alpha N_0} \right)$ bits/sec/Hz
$R_2 = \log \left( 1 + \frac{P_2  h_2 ^2}{N_0} \right)$ bits/sec/Hz	$R_2 = (1 - \alpha) \log \left( 1 + \frac{P_2  h_2 ^2}{(1 - \alpha) N_0} \right)$ bits/sec/Hz



So, just to briefly compare this OMA scheme and NOMA scheme in OMA scheme. So, this is NOMA the  $R_1$  rate user 1 this is for two users. Is this is log to the base 2 everywhere is given by 1 plus  $P_1 |h_1|^2$  square  $P_2 |h_1|^2$  mod square plus  $N_0$  because this is the interference and this is the signal of the user 1.

So, this is bits per second per hertz and for  $R_2$  because you have subtracted the user 1 signal you are left only with user 2 signals. So, the  $R_2$  is given by  $\log_2 1$  plus  $P_2 |h_2|^2$  mod square divided by  $N_0$  bits per second. So, this is for NOMA whereas, for OMA alpha is the alpha or here the power coefficient the parameter the resource block and which is given as  $R_1$  into alpha  $\log_2 1$  plus  $P_1 |h_1|^2$  mod square divided by alpha  $N_0$ .

And for  $R_2$  the resource block will be will be 1 minus alpha. So, there is 1 minus alpha will come here. So, this is the  $R_1$  and  $R_2$  for OMA scheme. So, this is in brief for two users the

rates of different users. And as we have seen in earlier discussion that the sum rate if there if you have chosen the power of the signals in such a way you may get sum rate higher as compared to the sum rate of NOMA higher as compared to sum rate of OMA.

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### Main issues with NOMA



- Hardware complexity....due to SIC implementation → delay
- Error propagation in SIC implementation...estimation of high power signals *CFO, TO, hardware implementation*
- Optimal pilot allocation...perfect or near perfect CSI is required
- Carrier frequency offset and timing offset estimation...overlapped signals *CFO, TO*



There are some issues with the NOMA let us discuss those issues as well. So, it is a actually the hardware complexity is very high. Because you have to do the SIC implementation either at the base station level or at the user level. So, the hardware complexity becomes little higher when you have more number of users. So, basically you have to do this excessive information you know 1 times because if there are 1 users.

So, this sometimes results into a delay. So, there are techniques for doing to get rid off this complexity. In such cases you do you make some clusters of users. So, that you are doing SIC

only for limited number of users. So, there are techniques to improve the delay part and you know reduce the hardware complexity part.

Error propagation in SIC implementation. So, basically what we are doing, we are first estimating the channel. So, the channel estimation becomes very crucial. So, sometimes there is you know center frequency offset or timing offset or some hardware impairments you may not be able to estimate the channel coefficient highly accurately. So, it might result into wrong decoding of the signal which you are actually subtracting so that you know error continues.

So, one has to be really careful in estimating the channel so that the error does not propagate it propagate in SIC implementation. And the third thing is you have to have optimal pilot allocation. For this to get some perfect or near perfect channel state information you have to allocate some pilot. So, the design of pilot how many symbols are needed, how frequently they are needed. So, those are some of the areas which one has to work on. And then carrier frequency offset and timing offset estimation.

In case of orthogonal multiple access for example, in RF carriers frequency offset that is CFO and timing offset they are estimated fairly accurately because you can distinguish the signals at the receiver end. So, this correction can be easily done in case of OMA signals whereas, since the NOMA signals are overlapped signals. So, this CFO estimation and timing offset estimation is becomes a really challenging task.

So, these are some of the issues and the research community is working to handle such issues. And we will find a good use of NOMA as we have seen in RF in VLC as well. So, my next discussion will be use of NOMA in VLC environment. So, we will try to understand why NOMA is useful or advantageous in NOMA and what kind of benefit we can get using NOMA.

Thank you very much.

