**Optical Wireless Communications for Beyond 5G Networks and IoT**
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**Lecture - 15**
**Part - 2**
**Color Shift Keying (CSK)**

Hello everyone. So, today we are going to discuss another modulation technique which is called as Color Shift Keying or also called as CSK.

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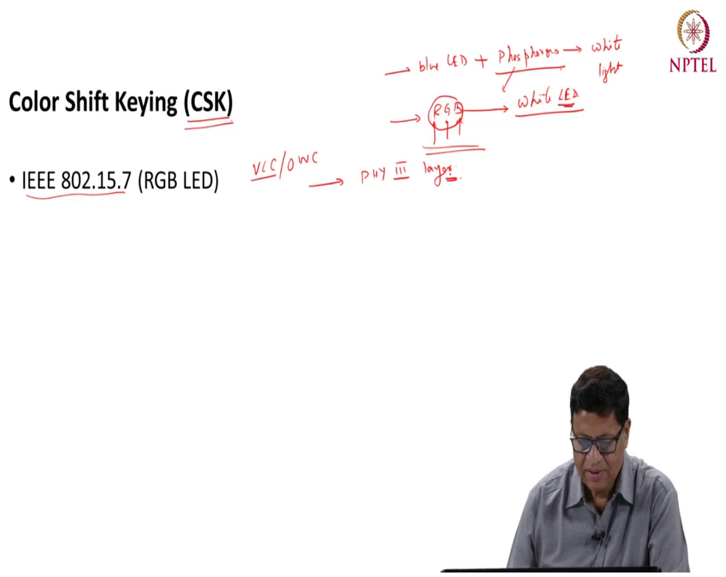


Actually, if you see the LEDs which are used in Optical Wireless Communication for indoor communication, they are basically two types. One of them is you have basically a blue LED and then there is a phosphorus layer on top of it. So, this actually gives you white light.

But the issue with this is; that it has a very slow relaxation time or it slows down the response. If I want to modulate this LED because of this phosphorus, it will slow down the response. So, basically such LEDs which are combination of you know blue LED with phosphorus coating, they are not ideal candidate for modulation because they limit the response. The response of the signal is limited.

The other type of LED is RGB LED which is it is basically there are three LEDs inside. One is red LED and green LED and blue LED and when it gets combined, it gives you a white light. So, both of them give white light. But the response is not limited because there is no phosphorus layer here.

So, you can separately modulate the red light. You can separately modulate the green light as well as blue light. And basically the there is increase in terms of data rate if using I mean if you are using WDM technique because you have three colors here and there is no limitation coming because unlike in in blue LED with phosphorus coated.

So, why the CSK schemes actually makes use of will use RGB as the LED. So, this is listed also or this is discussed or this is also a standard in VLC stand VLC of the optical wireless communication standards which is I triple E 802 dot 15 dot 7. It discusses about color shift keying as one of the modulation technique for indoor communication or outdoor communication as well.

So, this is discussed in 5, 3 layer 5, 3 layer. So, before we understand this CSK let us understand about a about a color diagram which will explain how different colors can be used for communication.

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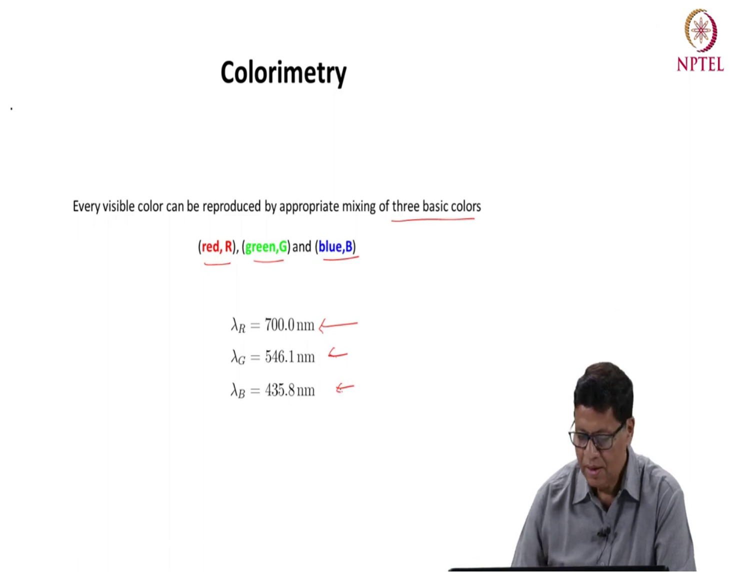


So, as we know every visible color can be reproduced by mixing three basic colors. So, if you have red color, the green color, the blue color you can basically generate any kind of color. And here I have given the values of the wavelength for red it is 700 nanometers, for green is 546.1 nanometer and for blue is 435.8 nanometers.

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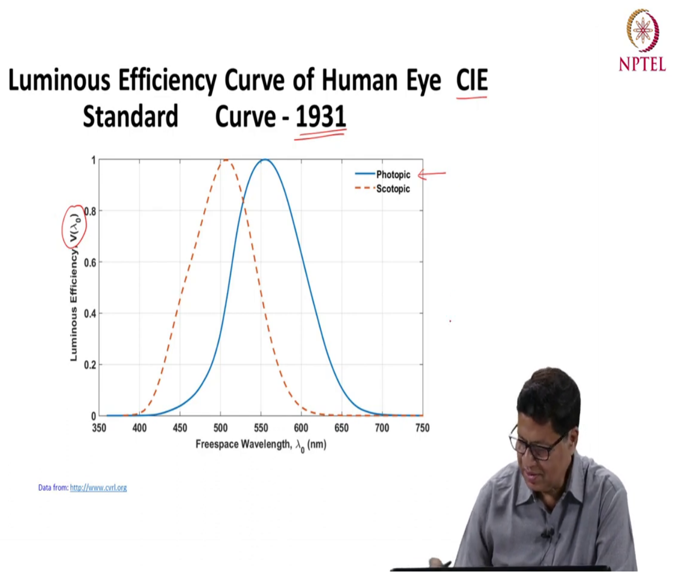


And this is a; this is a curve of luminous efficiency curve of human eye and this is standard is actually called as CIE 1931. So, this is shown for two regions that is photopic regions and scotopic region. So, this is basically when you have enough light which is actually photopic region and when you have less light to the light is not you know in high intensity then it is scotopic region.

So, this is the response of the luminous efficiency curve of the human eye for two different regions which is also you know represented as V lambda. So, this is efficiency curve versus lambda which is called as V lambda.

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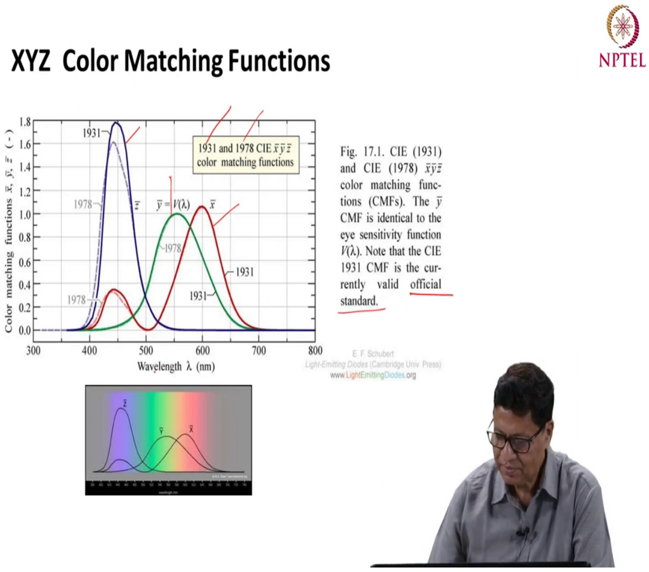


And these are the color matching function. These are the color matching function and, in this diagram, if you see there are two color matching function two curves are given where one correspond to 1931 other is 1978. So, both are part of the standards, but 1931 is the official standard is the current official standard which is followed.

So, we will focus our discussion only for 1931 CIE 1931. So, these are the color matching function x bar y bar z bar. So, for blue for this is the for blue it is z bar and then you have for the red it is x bar and for the green this y bar. So, this is the wavelength versus color matching function for different colors x bar y bar and z bar. So, these are color matching function.

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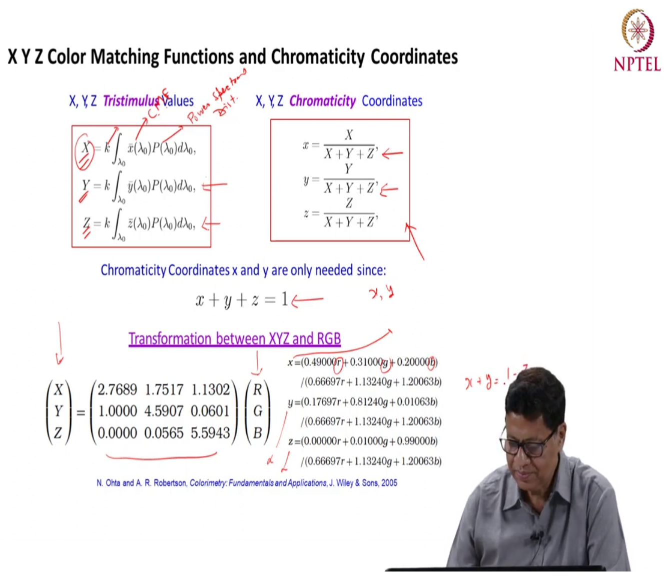


And how you can translate any color to some chromaticity coordinate? So, this is what this particular slide explains. So, you have tristimulus values which is capital X capital Y capital Z and this is some constant and this is the spectral distribution power spectral distribution, this is power spectral distribution of the light and this is the color matching function color matching function CMF.

So, this power spectral distribution is weight weighted with the color matching function corresponding color matching function you get the value of capital X which is a tristimulus value. Similarly, you can calculate capital Y and some Z and if you want to convert this into some coordinate x, y, z. So, x is given by capital X divided by this is the total, this is the total I mean the total and similarly for z. Now, we see from here the sum of these chromaticity coordinates x and y this is x, y, z if you see it is 1.

So, even if I know x and y the z can be found out. So, basically, I need only two values the third can be found out because the sum of these chromaticity coordinate is 1x plus y plus z is equal to 1 and this is the transformation which is used. So, this is the capital X tristimulus value and this is the R, G, B and this is the matrix.

So, this is a transformation between X, Y, Z and R, G, B and these x, y values are given here in terms of R, G and B. And if you see here in these equations x plus y will be 1 minus z. So, basically, I need to calculate only x and y value and I can map any color to x, y coordinate using color matching function.

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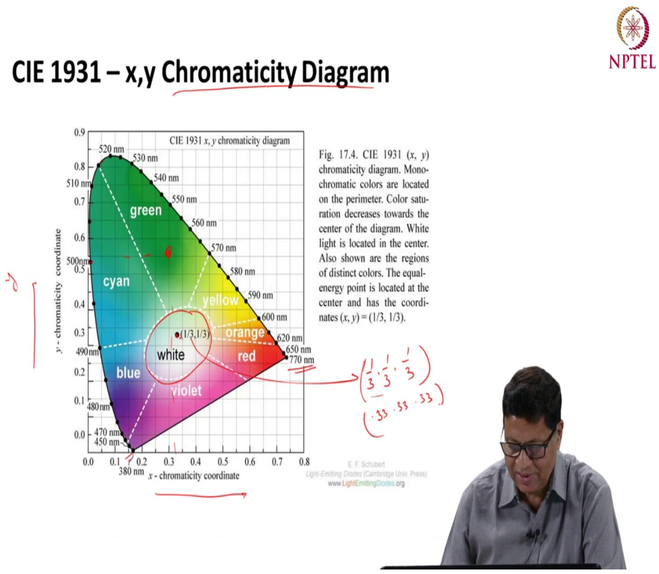


So, this is the diagram we get which is called as chromaticity diagram. So, you see on this coordinate is x coordinate and this is y coordinate and the diagram looks like this. So, at the top you will see some you know specific wavelengths by 5 to 20 nanometer or 770 nanometer or 450 nanometer and there are other wavelengths you know they are denoted by these dots here.

And I mean if you want to choose any color for example, this color then the corresponding value of x will be this. So, any color can be you know written or explained or shown in terms of these x, y coordinates. And if you see in the middle for example, this area this gives you different tones of white light because here the contribution of red green and blue they are all the colors are present and then they combine they give you a white tone.

So, this particular thing which is 1 by this 1 by 3 x value 1 by 3 and this z will be 1 by 3 because the sum has to be 1 gives you know this particular coordinate and this has some value of x and some value of y. So, this is sorry, this has a value of x is 0.33, 0.33 and 0.33 and it gives a white tone if you have you know this particular point. So, this is you know chromaticity diagram which explains that any color can be written in the form of some x, y coordinates.

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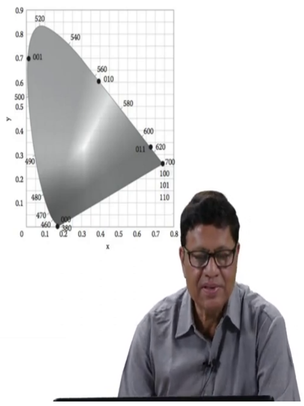
(x,y) Color coordinates of the seven bands

| Band (nm) | Code | Center (nm) | (x,y) |
|---|---|---|---|
| 380–478 | 000 | 429 | (0.169, 0.007) |
| 478–540 | 001 | 509 | (0.001, 0.597) |
| 540–588 | 010 | 564 | (0.402, 0.597) |
| 588–633 | 011 | 611 | (0.669, 0.331) |
| 633–679 | 100 | 656 | (0.729, 0.271) |
| 679–726 | 101 | 703 | (0.734, 0.265) |
| 726–780 | 110 | 753 | (0.734, 0.265) |



So, standards have defined the seven bands. So, this is the 7 these are the different bands here and they are they are given by some code triple 0 0 001 and so on and so forth.

And if you see the center wavelength it is given by here that is the center wavelength of this band 380 to 478 is 429 and if you see the corresponding x, y coordinate is given here 0.169 and 0.007. So, the standards have defined these 7 bands.

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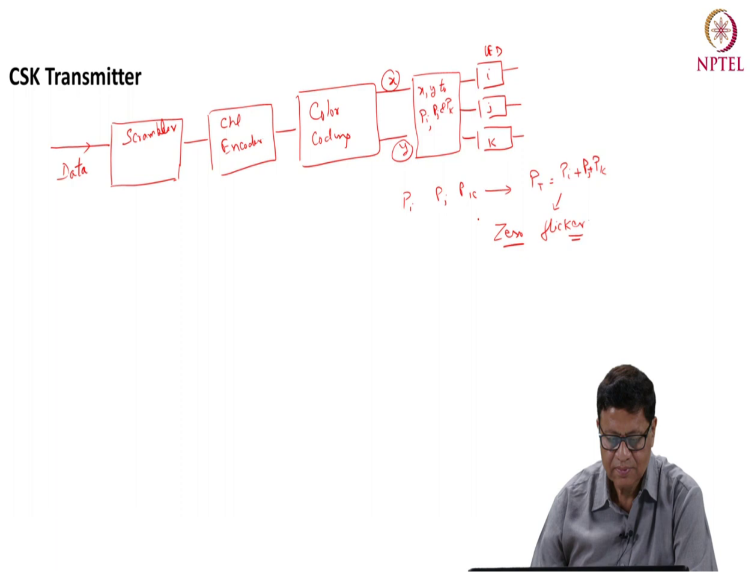


And now, let us before yeah let us now make a how a TSK transmitter a CSK transmitter will be designed. So, you have a data here and normally a data may have you know continuous runs of run of 0's and all 1's. So, normally you scramble the data before giving to for the processing. So, this could be some kind of scrambler. And then we have the standard channel encoder and then we do the color coding.

And it will give you some value corresponding to that particular color in terms of x and y. And then this x and y will give different power or different intensity with different power and intensity or power which are referred for different LEDs which are red, green and blue.

So, x, y to P i, P j and P k. So, this is the intensity, this is the power which is to be given to different LEDs. So, this is the LED i, this is LED this is LED j and k. So, these are actually optical sources or LED and this is the power which is you know transmitted. So, this is how a typical CSK transmitter works.

And the best thing is that there may be a different value of P i, P j and P k corresponding to these two x, y values. But if you see the total value the sum will remain the same this is P T will remain as same. So, basically you will not notice any flicker coming here because the power is constant only the relative power is changing. So, this is 0 figure.

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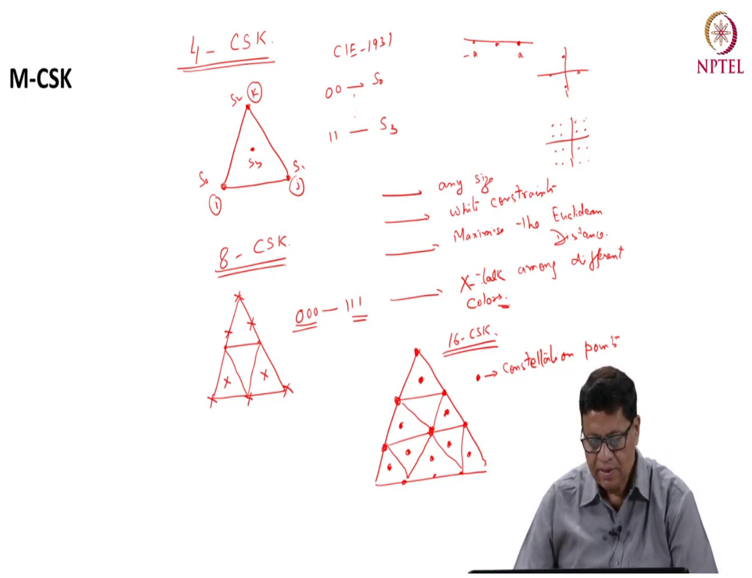


Now, let us understand how do we make constellation diagram in CSK. We have read what constellation diagrams in RF where we have for example, this is BPSK, there is one here, one here this is a plus a this is minus a and also BPSK or you can have 16 QAM, this is 4 here, this is 4 here, this is 4, this is 4, this is 4. So, this is what we have studied in any digital communication or in in in in RF domain.

So, let us see how we can design a constellation diagram for optical wireless communication system. So, suppose I want to design for 4 CSK for example. So, this probably one point can be here, another point can be here; this point can be here I mean inside that CI 193 diagram and the fourth point could be in the middle. So, this is say k this is i, j, k, this is i, this is j and this is say S0, S1, S2, S3 and they all represent some data symbols. So, for some 0 0 will correspond to say S0 and say 1 1 will correspond to say S3.

So, there are various algorithms which have been designed and for designing such algorithm one has to consider few things in mind that you should be able to design algorithm for any size or whether it is you know 16 CSK or 64 CSK and the algorithm should be such that ultimate light which comes out comes out is white. So, there are some white constraints you have to maintain. And also, you have to design these constellation points in such a way that the distance between two adjacent constellation points is maximized.

So, maximise the Euclidean distance and also because you are transmitting three colors. So, you have to also see that the cross talk among these colors is also minimised. So, one has to also consider the cross talk among different colors. So, designing any constellation size for optical wireless communication will require all these constraints which have to be tackled cross talk among different colors. So, this is example of 4-CSK and 8-CSK for example, I want to design a 8-CSK.

So, this is say middle part, this is a middle part. So, one point could be this I am representing each constellation point by a cross here could be this and this is another point and one point can be somewhere here. Somewhere here and another point would be this. So, there are 8 such points. So, this is they represent you know for example, triple 0 to triple 1.

So, this is 8-CSK and if I want to design a 16-CSK probably I can consider something like this, but there are other algorithms which can give you much better results. So, this will be this is for 16-CSK. So, this is 1. So, this is this a constellation point this is in this diagram, constellation. So, there will be one here, one in the middle. So, this could be a constellation diagram for a 16-CSK.

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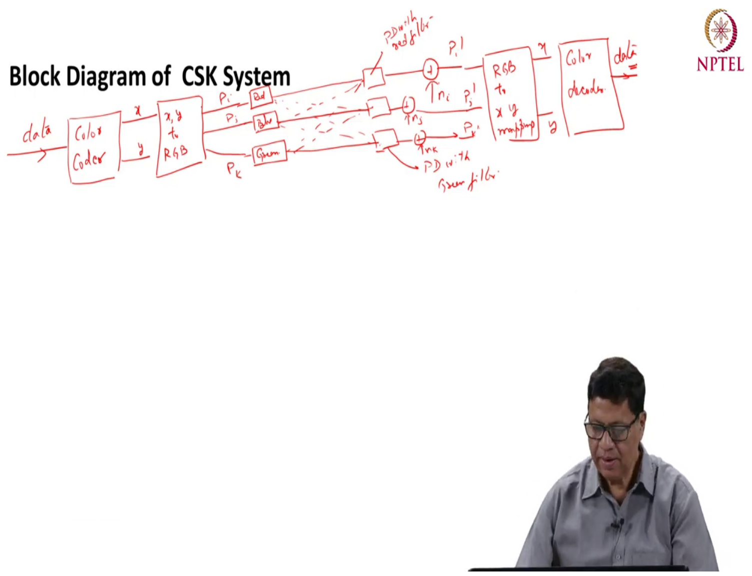


So, let us now draw the complete block diagram of CSK system including the receiver. So, this will be that data. So, I am skipping the scrambler and then source encoder, those are standard blocks which you have in a CSK system. This is color coder and it is mapped to some x, y values.

And this is x, y to RGB which is given by that transformation matrix which we had discussed when we were discussing about CI 1931. So, this will give me a P i, P j and P k. So, this is given to red LED red and this is blue and this is green.

And then you have the PD's here with filters. So, this PD is PD with the red filter and the PD with the middle one this PD with blue filter and the last one is photodiode with green filter. And when the light is getting transmitted, it should be sensed by this photo diode ideally, but you know it may go here as well which is actually the cross talk may go here and similarly the light ID should go here, but it goes here. So, these are the interference here. Similarly, for green.

So, actually you would have a matrix here, channel matrix and these are the interference, you know these dash lines they are the interference. So, once you have this and then after the PD you know that noise which is say for P i's and i, this gets added and then the power is say P i dash and similarly the noise this is n j and the power is P j dash and similarly n k power is P k dash.

And then these powers are given to RGB to x y mapping. And once you have got this x y from here, you can use color decoder. This is a color decoder color decoder and you get your data from here. So, this is a complete block diagram of CSK system.

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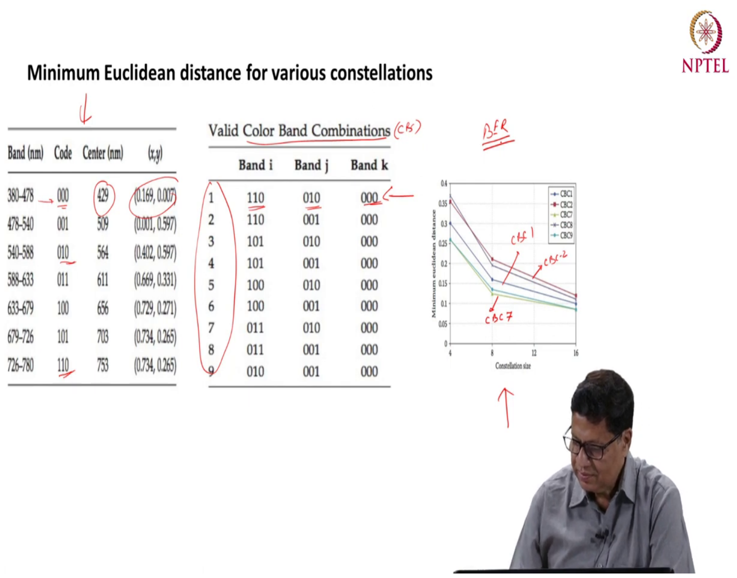

Also, we had discussed about these bands in this table, these are different bands defined seven of them which are represented by codes and their central wavelength is this and these are the x, y coordinate. Now, so, you can have when you are transmitting your CSK signal, you can have some combination of these color bands. So, for example, for band i band j, you can have combination of for example, 110 is somewhere here or 010 somewhere here and triple 0 is somewhere here.

So, you can have different combinations. So, the standards they have defined some nine combinations. So, in total there are nine such combinations. So, here and they all differ in terms of constellation you know constellation diagram and also the minimum Euclidean distance also varies for different combinations. So, this particular plot actually gives a shows the Euclidean distance for different constellation size and for different color band combinations.

So, CBC stands for color band combination this is CBC. So, for example, CBC 1 this particular one is represented by this color, this is CBC 1. And if you see for example, this one the last one which is CBC 2, this is CBC 2. So, the min minimum Euclidean distance for example, for CBC 7 is for 4 the values are less as compared to CBC 1 or CBC 2.

And if you see the for higher the same trend is valid for higher constellation diagram. So, basically you know this will give different performance in terms of you know BER. So, one can try you know different color band combinations and you will get different minimum Euclidean distance which will actually decide what is the performance of the system.

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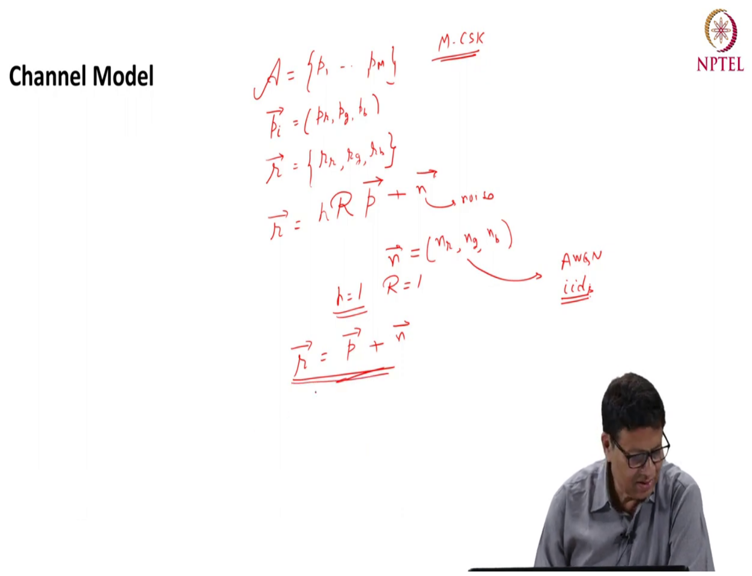


So, let us understand about the channel model. So, channel model let us see we have a set and represent by p 1 to p m and this is for say M-CSK. So, these are the constellation points from p 1 to p m. And each p 1 is represented by a by a by a set of colors. So, each p 1 let me write as p i which is actually a vector now is represented by some red color, some blue color, green color, blue color. And the received vector r will be represented by r r, r g and r g. So, this is a received vector.

And the received vector can be written as the channel matrix h which will be a 3 cross 3 matrix in case of 3 colors r g b h into responsibility into p vector plus n vector. This is a noise. And noise will be for different colors. It will be a set which is n r, n g and n b for different colors. So, for the sake of simplicity I can take h is equal to 1, the gain is 1 and also, I assume that there is no cross talk between that among the colors.

So, h can be taken as 1 and let us assume also responsibility as 1. So, basically the received vector r will be p plus n. And these noise are AWGN, treated as AWGN and they are iids. So, so, this is the received vector. So, this is how the channel model is represented. Now, let us try to understand the constraints and what is the detection mechanism which we use.

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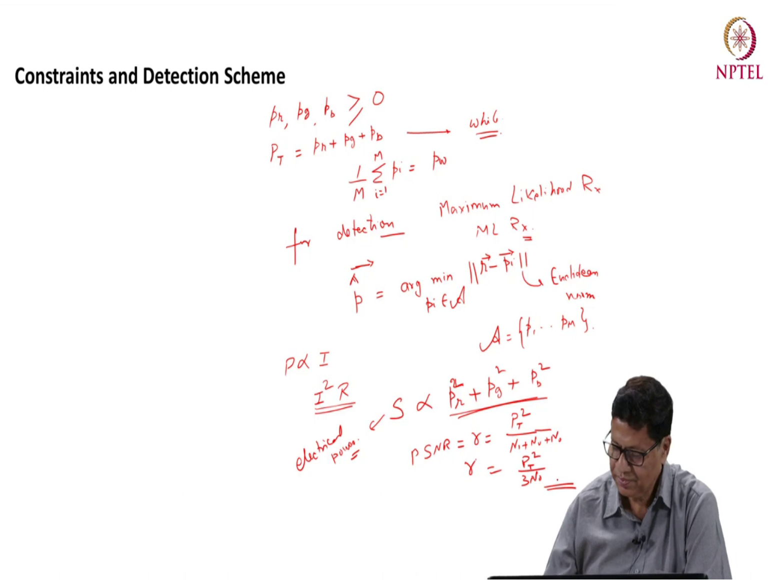


So, the constraint will be these different colors that is p r, p g and p b. They are either 0, greater than or equal to 0. It cannot be non-negative. It cannot be negative. So, and the total power is actually sum of these powers.

So, the relative power is changing, but the sum remains the same. So, so that is why we discussed earlier that there is no 0 figure, there is no figure. Because the output power remains constant. And so, the total color of elimination will be white, if you combine all these colors so you will get white. And this this basically one has to ensure that is i is equal to 1 to M p i should give power which is corresponding to white p w.

And for detection for detection, we use maximum likelihood receiver or ML RX. Which shows that the estimated value and this will be vector estimate value will be argument. Suppose r is the received vector, this should be minimum that is this Euclidean norm r minus p i should be minimum.

So, this is the Euclidean norm. And this is for all values belonging to that set which is actually currently A. And remember this A was p 1 to p n. So, the power actually is proportional to I. The power which is falling out on to the detector will give some corresponding current which is proportional to the incident power. And if I calculate the electrical power, it will be basically I square by R R is a load resistance. So, it will be square of the current that will be the electrical power.

So, if I write electrical power which is say S electrical power that will be proportional to the square of the power in red blue and green. So, S will be proportional to p r square. Sorry, not p r, p r red, p g square plus p b square. So, there is a total power. So, the SNR for such a system is defined as gamma which is P T square is a total power square because of because I am considering the electrical power. And the noise is basically coming from all the sources.

So, this is sometimes called as CDO SNR as if only one fluid diode is there. So, the equivalent noise which will come from different sources will be actually N naught plus N naught for different colors plus N naught. So, this is represented in P T square over 3N naught. So, this is the gamma or the CDO SNR for such a system.

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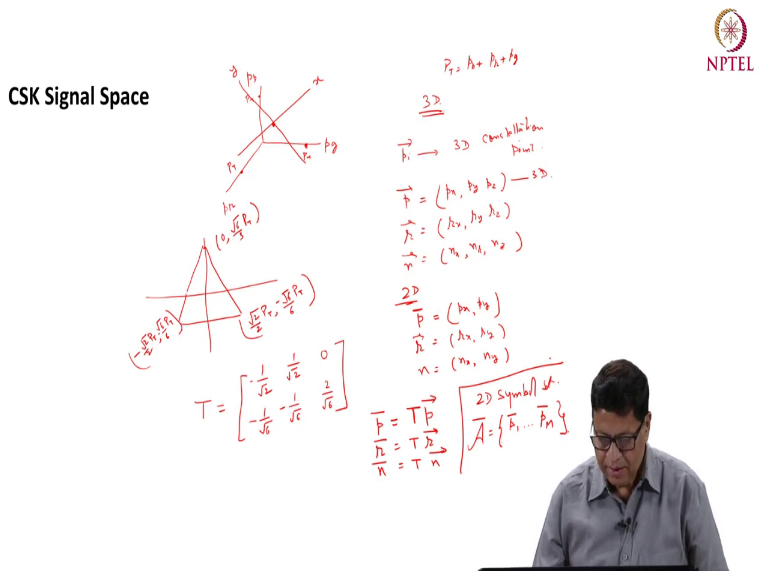


Now, try to in order to calculate probability of error for such a system, we need to understand the concept of CSK signal space. So, in CSK signal space suppose I represent those three colors by three coordinates. So, this is a p g, this is p b and this is p r. And let us see this is a total point P T, this is P T because we know P T is equal to p b plus p e power in the red or power in the green.

So, this point actually is the power total power is in p g. So, this is other such 0. So, this will be P T here, similarly, this will be P T here. Now, I want to this is a 3D CSK signal space. So, I want to convert this 3D into 2D then I will do some transformation and this will be basically x and y and this point. So, this is your x, this is y.

So, this is I am trying to convert this 3D into 2D. So, let us represent p i vector each component here as a 3D constellation point. So, p will be I mean p x or p y, p z and similarly r will be I mean this x, y, z are actually for red blue and green. So, I am writing some x, y, z.

So, this will be r x, r y or r z and noise and n z. So, if I convert this into 2D it will be represented by you know x, y coordinates 2 coordinates., So, which I represent for PD for 2D I am representing p with a line here is different from a 3D where I show an arrow here this is 3D. So, p for 2D system will be p x, p y similarly r will be r x for r y and n will be n x, n y. So, if I do this transformation from 3D to 2D I will get something like this.

So, and these points are 0 root 6 by 3 P T and this particular point will be root 2 over 2 P T root 6 over 6 and this is minus P T and this point will be minus root to 2 P T minus root 6 to 6 P T. So, these are the points I will get when I map this 3D system to 2D systems and this is actually done by a transformation matrix and in this case the transformation matrix T will be 1 by root 2, 1 by root 2, 0 minus 1 by root 6 and this is 2 by root 6 this is the transformation matrix, that is p 2D the single line is T this is a matrix into p which is a 3D constellation point.

So, similarly for you have r and similarly for n. So, this is the transformation I have done from 2D space to from 3D to 2D. So, I have got 2D symbol sets which is A I represent this like this. So, this will be p 1 this is 2D so on and so forth p m this is the mth constellation point this is the 2D. So, this is the 2D symbol set I have got from that 3D which I had red blue and green.

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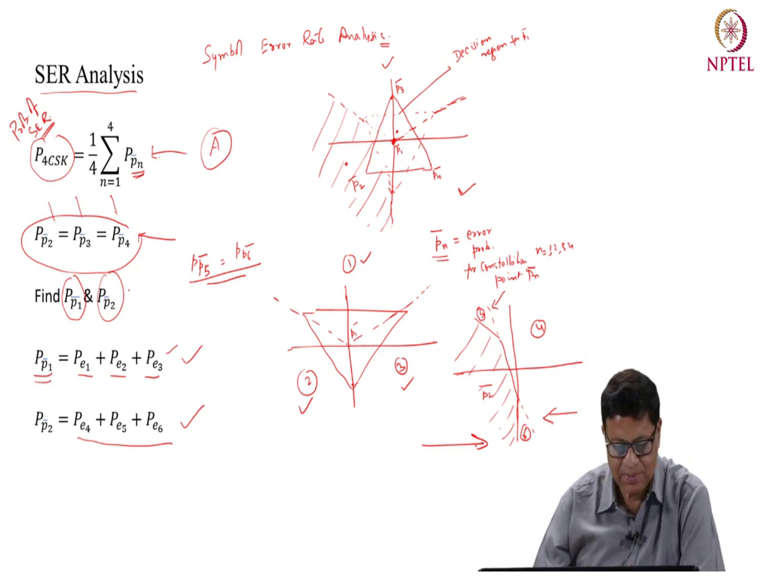


So, now let us try to do the SER analysis Symbol Error Rate analysis for CSK system symbol error rate analysis. So, for doing this symbol rate analysis let us draw a 2D constellation diagram and try to understand the decision regions. So, let me again draw this. So, this is say one point let me let me call as p 1 here this is the 2D point p 1 this is let me say p 3 and this is p 4 and this is p 2.

Now, what is the decision region for different points? So, for p 1 this is the decision region which is inside the inside the triangles. So, this is the decision region point the decision region rather for p 1. And similarly, we can find out the decision region for other 2 points. So, let us for that we will have to draw a triangle which is. So, for the decision region for p 2 for example, will be this is a single line here sorry, I have drawn two lines here.

So, for the decision region for p 2 will be actually this which I am showing as shaded. And inside this is the for decision region p 2 similarly there is a decision region for p 2, p 4 here and similarly for p 3 here. So, any point which is falling in these decision regions will be estimated as that particular constellation point.

So, if your point is falling for example, somewhere here will be and the detected point or the estimated point will be p 1 bar. And if your point is falling somewhere here in this decision region then will be p 2 p 2 bar or similarly for p 3 bar or p 4 bar. So, let us write now as p n where n is 1, 2, 3, 4, 2 dimension is the error probability for constellation point p n.

So, n so p 4 PSK which is a probability of SER here this is a probability of SER for 4 PSK will be given by 1 by 4 summation of the all these probabilities there is P p 1 plus capital P. So, p 2. So, this is the constellation point this is the probability P. So, this will be some of these and divide by 4 that will be the probability of SER.

And if you see that decision region here that is 2, 3 and 4; 2 for corresponding to p 2 bar p 3 bar and p 4 bar that decision region is identical, it is same. So, that probability will remain the same. So, P p 2 is equal to P p 3 is equal to P p 4. And so, and P probability P 1 of p 1 bar is we need to find out and we have we need to find out any one of these probability since they are all equal.

So, I need to only find out P p 1 and P p 2 and then I will be able to calculate p 4 PSK. And if you notice this P p 1 actually, let me draw another diagram to explain about this P p 1. So, this will be. So, this is a. This is a region say 1 this is the region say 3 and this region is 2. So, P p 1 is given by and this is for say p 1, this is p 1. So, if the point which was actually p 1 the transmitted was p 1 and it has fallen onto either 1 or 2 or 3 that is an error, right.

You should have been detected estimated as p 1 bar, but it is falling in one of these regions. So, the total error probability will be P p 1 will be given by the probability of error falling in 1, falling in 2, falling in 3. So, which are represented by this P e 1, P e 2 plus P e 3. And to understand this P p 2 in this we have discussed about the p 1 part let us now discuss about the p 1 part. So, for that let me draw another diagram here maybe in this space let me draw here.

So, this I am drawing this particular region that is say p 2 region I am drawing or we can see I am just drawing another p 2 region here. So, this p 2 region will be something like this, just concentrate on the p 2 region p 2 region will be. So, this is a p 2 region this this one is a p 2 region I have re drawn here this just to make it more clear. And I have additional I denote this as say 5 and I denote this as 4 and this region is say 4 here.

Sorry, this region is this is 6, this is 4 this is 6, right. Then for P p 2 this is p 2 will be if the point falls either in 4 or 5 here or this is 6 then it will be an error. So, P p 2 will be P e 4 plus P e 5 plus P e 6. So, P p 1 was when the when the point is falling in other places which are p 1, p 2, p 3 denoted here, then these are the probability of error for p 1 and then probability of error for p 2 is if the point is falling in not in p 2, but any region which is 4, 5 and 6 in this diagram.

So, these are the two probabilities we have. So, now I can put this value this P p 1 and P p 2 in this equation, let me name this as A. And when I put this equation and I use these identities that is P p 1 is equal to P p 3 is equal to p 4 and also, I know that p p 4, 5 so sorry, this is 5 is equal to p p 6. So, this region 6 and 5 here you know this one and this one 5 and 6 they are same. So, using all these identities we can calculate what is P 4 P s P 4CSK.

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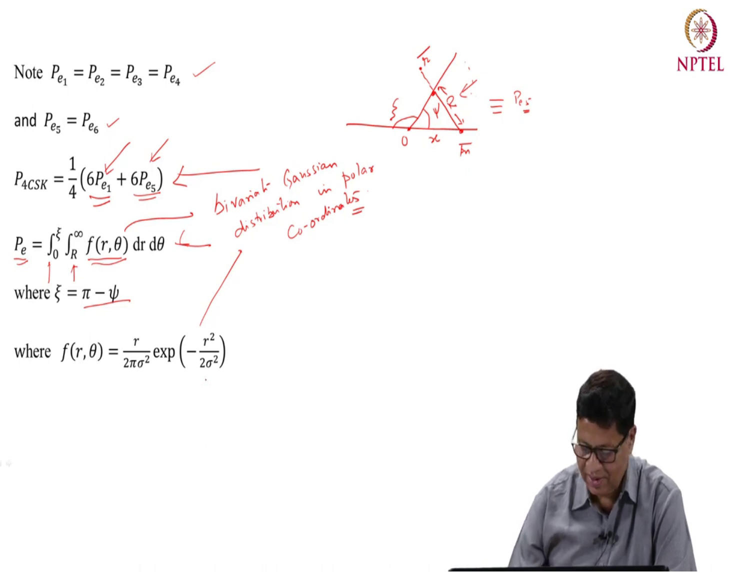


So, this we know and this is also equal to P e 5 equal to P e 6. So, this will give P 4 4 CSK basically I will put all these quantities in my earlier equation because named as A. So, I will get 1 by 4 6 P e 1 plus 6 P e 2. Now, I need to calculate P e 1 and P e 2 to get the total probability of error.

So, how do we calculate P e 1 and P e 5? So, for calculating P e 1 and P e 5, let me draw a decision region of the form which is equal to P e 5 and P e 5, P e 1 and then we can calculate the values of P e 1 and P e 5. So, it is something like this if you carefully see that decision regions are. So, this is say p n point and this is say x this is O, let me write this as psi and this is say xi and this suppose the received point is somewhere here r, right. And then this makes with the p n this and this distance of through this point is say r, this is r.

So, this if you see this is region equivalent to P e 5 it will have some value of psi and xi, but this is equal to 5. So, we will put the you know limits of psi and xi and try to calculate this 6 P e 1 and 6 P e 5. So, for calculating this probability of error because this r can be anywhere, it can it can be from r to you know r to infinity.

If it is in this region in the region which, is you know in this region here the region which is shown here then it is not error in error, but if it is r changes from r to any other point infinity it is an error, right. So, so these limits are from r to infinity and this is angles are 0 to xi, this xi can change from 0 to some angle psi and this is the distribution which is Gaussian distribution which is in given in polar coordinates.

So, this is bi variate I mean this is a distribution which r will have. So, by bivariate Gaussian distribution in polar coordinates. So, this is given by this equation the probability of error and when I calculate for P e 1, P e 2 the values of psi and x and xi they will be different.

So, we will put those different values you know to calculate P e 1 and P e 5. And as you see from this diagram xi is equal to pi minus xi is equal to pi minus psi and f r theta is actually that the bivariate Gaussian distribution in polar coordinates which is represented by r divided by 2 pi sigma square exponential minus r square by 2 sigma square.

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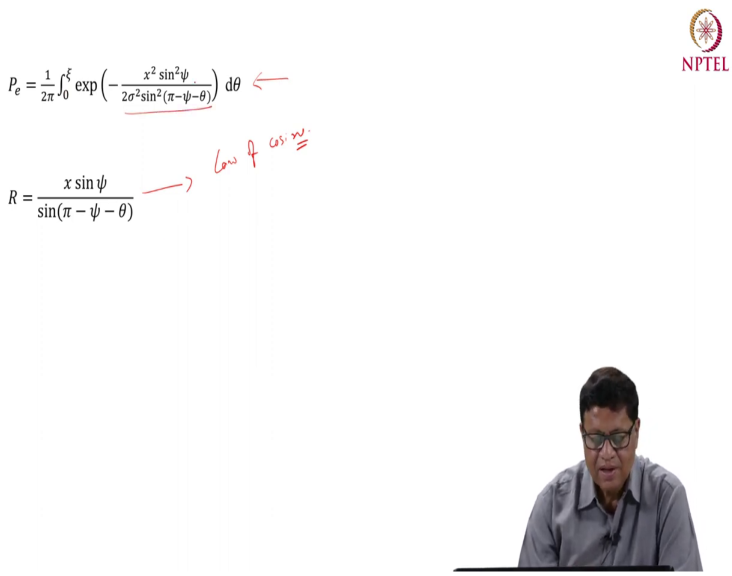


So, putting these values and using this law of cosines; this is law of cosine, law of cosine and putting the values here then I can get that expression for P is equal to 1 by pi 0 to xi exponential and even whatever is given in the bracket into d theta.

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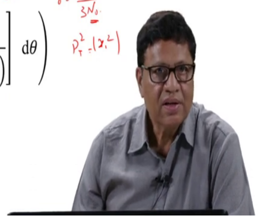
For 4-CSK

| | $x_i$ | $\psi_i$ | $\xi_i$ |
|---|---|---|---|
| $i=1$ | $\frac{\sqrt{6}}{3}P_T$ | $\frac{\pi}{6}$ | $\frac{2\pi}{3}$ |
| $i=5$ | $\frac{\sqrt{6}}{3}P_T$ | $\frac{2\pi}{3}$ | $\frac{\pi}{3}$ |

$$P_{4CSK} = \frac{3}{4\pi}\left(\int_0^{\xi_1} \exp\left[-\frac{x_1^2 \sin^2\psi_1}{2\sigma^2\sin^2(\theta+\psi_1)}\right] d\theta + \int_0^{\xi_5} \exp\left[-\frac{x_5^2 \sin^2\psi_5}{2\sigma^2\sin^2(\theta+\psi_5)}\right] d\theta\right)$$

$$= \frac{3}{4\pi}\left(\int_0^{\frac{2\pi}{3}} \exp\left[-\frac{\gamma\sin^2\frac{\pi}{6}}{\sin^2\left(\theta+\frac{\pi}{6}\right)}\right] d\theta + \int_0^{\frac{\pi}{3}} \exp\left[-\frac{\gamma\sin^2\frac{2\pi}{3}}{\sin^2\left(\theta+\frac{2\pi}{3}\right)}\right] d\theta\right)$$



Now, for four CSK these are the values of x i's psi i's and xi i. So, i is equal to 1 this is P e 1, we will calculate x i is given as root 6 pi 3 P T, si i is pi by 6, xi i is 2 pi by 3. Similarly, for P P for to calculate P e for P E 5 for i is equal to 5 these are the values of x i and psi i.

So, putting these values in that expression we get P four probability for P 4 CSK which is given by 0 to psi 1 and this this is corresponding to 2 pi by 3 and this psi 1 is xi 1 is corresponding to 2 by 3 and this psi 1 is corresponding to pi by 6. And similarly for i is equal to 5 and this is i is equal to 1.

So, this is a P 4 PSK probability of error. Now, whole thing can be written in terms of gamma and gamma remember it was P T square divided by 3 N naught where P T square is x 1 square. So, you can replace this x 1 square and sigma square with the corresponding gamma value, this is the gamma value we have introduced. So, this is the probability of error expression for P 4 P CSK.

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