

Optical Wireless Communications for Beyond 5G Networks and IoT
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Lecture - 03
Optical Wireless Communications for Beyond 5G Networks and IoT

Hello everyone. So, today we are going to discuss about optical sources in the last class we had discussed about lighting basics of lighting systems like radiometry, photometry and calorimetry. So, today now we are going to discuss about optical sources which are which will be used in optical wireless communication.

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Optical Sources (LED and LASERS)

Size

Power output proportional to electrical input

High E/O conversion

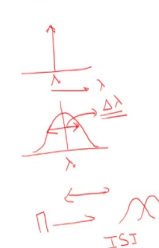
Spectral width ($\Delta\lambda$) should be small



High modulation capabilities

Energy efficient

Low cost and Long life

electrical power
↓
Optical power





So, the typical optical sources which we will consider are LED and lasers. So, let us now understand the typical requirements of your optical source whether it is for optical fiber communication or it is for optical wireless communication. So, first that the size should be

very small; it should be as small as possible. The second is you want that power output the optical power output which is coming from the source is proportional to electrical current because, ultimately I am going to modulate my electrical signal.

So, I want my power output to be proportional to electrical current or electrical input. It should have high electrode to optical conversion efficiency; that means, whatever my electrical power is it should give corresponding optical power. So, you should have that conversion rate between electrical power to optical power should be high, electrical power to optical power.

The spectral width that is $\Delta\lambda$ should be small. What I mean by spectral width? Spectral width is if the light is emitting at a single color then it will have a fixed λ , suppose this is my λ and this is a single wavelength source. So, this is a λ_0 . So, we will say that light is emitting at wavelength λ_0 , but in normal practice this λ_0 is not a single wavelength.

So, it has some spread around λ_0 . So, this spread is called as a spectral width. So, we want this $\Delta\lambda$ to be as small as possible because if the light has many colors in communication this could be a problem because, each wavelength will travel in the medium with different speeds.

And when they travel with different speeds they arrive at the receiver at different times and at different times meaning there will be a you know dispersion in the pulses or the pulses will spread. So, suppose if you launch a pulse like this if your $\Delta\lambda$ is high after say 20 kilometers or 30 kilometers in optical fiber for example, this pulse will become something like this.

So, if there is a dispersion in time then it will start interfering with next pulses. So, it will result into a limited data rate or it will result into something called which is called as inter symbol interference. So, we want this $\Delta\lambda$ should be very small. We want high bandwidth capability which means I should be able to modulate the source at high speed it should not be limited I mean the wavelength the modulation bandwidth should not be kilo

hertz or even megahertz I should be able to modulate the source at high data rate or it should have high modulation bandwidth of the order of few gigabits per second.

So, these sources should have you know high modulation capability. The sources should be energy efficient the kind of power which we apply to LED and laser should be small and it should give you it should give you adequate optical outputs. So, the sources are expected to be energy efficient and of course, it should be low cost and should have long life.

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
LED principle

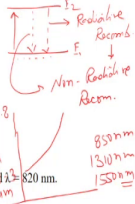
$$\lambda = \frac{hc}{E_g} \text{ (Direct bandgap)}$$

$$\lambda = \frac{1.24}{E_g(\text{eV})} \text{ (}\mu\text{m)}$$

Semiconductor material	Bandgap Energy
Si	1.12
GaAs	1.43
Ge	0.67
InP	1.35
Ga _{0.97} Al _{0.03} As	1.51

$E_g = 1.424 + 1.266x + 0.266x^2$, for $x = 0.07$ gives $E_g = 1.51$ eV and $\lambda = 820$ nm.





λ for $\text{Al}_{0.97}\text{Ga}_{0.03}\text{As}$ is 820 nm.
 λ for $\text{Al}_{0.97}\text{Ga}_{0.03}\text{As}$ is 850 nm.
 λ for $\text{Al}_{0.97}\text{Ga}_{0.03}\text{As}$ is 1300 nm.
 λ for $\text{Al}_{0.97}\text{Ga}_{0.03}\text{As}$ is 1550 nm.

So, let us first understand the basic principle of a LED. So, LED is basically a pn junction diode which is forward biased. This is p this is n and this is the depletion region and this is forward biased. So, what happens in a LED when you forward bias it some of the electrons from the lower state they go to the higher state and from that state they fall onto the lower

state and they release energy and this energy is in the form of photons. So, this is your lower energy level this is your high energy let say E_1 this is E_2 .

So, some of the electrons which are here they will go up when the current is supplied to the device by giving this forward bias and these electrons come at the higher level and then they fall down. And so this may result into two types of radiations one is a radiative recombination and another is this will result into light output and another could be a non-radiative, which may not result or which will not result into any light output non-radiative recombination.

So, this is the basic principle of LED and the photodiodes are of two types; direct band gap and indirect band gap. So, for direct band gap the light which is emitted from a pn junction is given by $h\nu$ h is Planck's constant c is velocity of light and E_g is the band gap energy.

So, the λ which is emitted is given by $h\nu = E_g$ and if I put the value of Planck's constant and now I also put the value of c here and then the λ is actually 1.24 divided by E_g and this E_g is in electron volts. So, units are important here. So, you have to put the value of E_g electron volts and then wavelength which you get using this expression will be in micrometer.

So, the λ which is emitted is equal to 1.24 divided by E_g in electron volts and the wavelength is in terms of micrometer. So, if I take different material for example, if I take silicon the band gap energy is 1.12 it will give you a different wavelength. If I use gallium arsenide the band gap energy is 1.43 electron volts and for germanium it is 0.67 for indium phosphide it is 1.35 and then you can have a different combination of gallium and aluminum and then you can get you know different value of band gap energy.

So, basically you can change the doping. So, this is $Ga_{1-x}Al_xAs$ or rather it will be $Al_xGa_{1-x}As$ because you are adding aluminum as a doping Al_xGa_{1-x} into As . So, you can change the value of this x and then you can get different band gap energy and when you change the band

gap energy essentially you are changing the wavelength of deformation. So, for example, in this case G_a is $G_a = 0.97$ aluminium 0.03 and As. So, this gives a band gap energy of 1.51 .

So, similarly if you plot for example, have different levels of x this is say aluminum mole fraction and this is the wavelength of emission. So, this will be something like this. So, this is starting point 0.3 and this could be 0.8 . So, by changing this value of x that is the doping concentration of aluminum in a material you can get different value of band gap energy and hence λ .

So, you can make your λ as per your choice because as we know there are certain wavelengths which are useful for communication for example, you know the wavelengths which are around 850 nanometer or 1310 nanometer or 1550 nanometer they all good for communication using optical fiber where they have you know low loss. So, you can generate a wavelength of your choice by changing this value of x of course, this gives you this is 1.3 I am sorry about this is 1.3 and 1.8 micrometers.

So, you can basically get a wavelength ranging from 1.3 to 1.8 by changing the value of x . So, this is another example where you have E_g actually is represented in terms of $1.4 + 1.2x + 0.2x^2$. So, this is a generic equation for E_g and depending upon what value of x you choose you can find out this E_g and corresponding to E_g there will be a wavelength formation.

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Spectral width

Probability of photon generation
 $\propto n(E_2) p(E_1)$
 $\propto e^{-(E_2 - E_1)/KT}$
 Total photons $\propto \int_{E_c}^{E_v + E_{ph}} e^{-E_{ph}/KT} dE_2$
 $\propto (E_v + E_{ph} - E_c) e^{-E_{ph}/KT}$

$\Delta\lambda$, Modulation BW



So, now we will study the two important things of source which one of them as I mentioned is the spectral width which we call as delta lambda and the other one is the modulation capability of the device modulation bandwidth of the device. So, these are the two important aspect of any optical source. So, first we will study the spectral width part. So, let us consider a pn junction of so.

So, this is for example, your valence band then this is your you know conduction band. So, there are electrons here, there will be some holes when you supply the current. So, there will be some holes and here there are different states. So, the electron is not limited to one energy line, but it has some distribution here.

Similarly, the holes are distributed here and this is the recombination radiative recombination part radiative recombination part and similarly you have the another part which is non

radiative recombination. So, both are happening in the semiconductor and this is valence band. So, this will energy E_1 and this E_2 and E_2 is not a sharp line it can be you know is a broad line for these energy states.

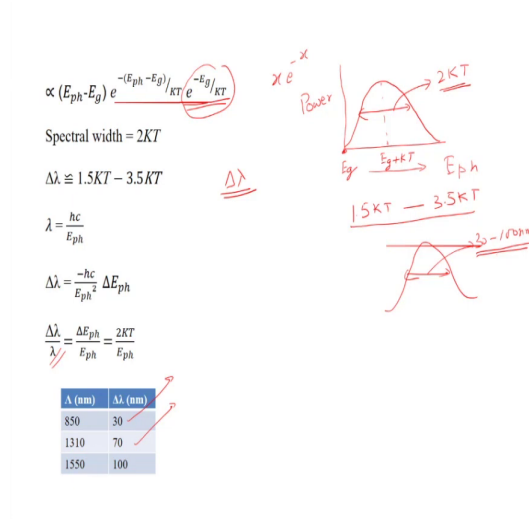
So, let us find out what is the probability of photon generation. So, it is proportional to $n E_2$ into $p E_1$ $n E_2$ means the probability of electrons in the conduction band. So, this is probability of electrons in the conduction band and $p E_1$ is the probability of holes in the valence band. So, the probability of photon generation will be you should have you know electrons and holes when they combine they either give you radiative combination recombination or non radiative

So, for simplicity we will assume that all electron hole pairs they result into some photon generation. So, there is no non-radiative recombination. So, this probability of photon generation is given by this and this expression for $n E_2$ and $p E_1$ which takes into account the Fermi level also, but that Fermi level part is constant. So, I am absorbing that constant into that proportionality sign and this is proportional to e raised to power minus E_2 minus E_1 divided by KT K is the Boltzmann constant and T is the temperature.

So, this is the probability of photon generation and the total photons if I see here and I am assuming that all the electron hole pairs are combining radiatively they are emitting photons. So, basically this will be an integration of because you know this electron may combine with this hole for example, this electron may combine with the different hole similarly there are other holes also.

So, basically I need to integrate all possible you know combining of these electron and hole pairs. So, I will be integrating this from E_c to E_{ph} and E_{ph} is the photon energy which is emitted which is given by between this point and this point. So, this is E_{ph} and this E_{ph} is actually varying for you know different pairs. So, this total photon and it is integrated from E_c to E_v plus E_{ph} and if I simplify this integration this becomes E_v plus E_{ph} minus E_c into e raised to power minus E_p E_{ph} by KT .

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And if I multiply by e raised to power plus E_g by KT and e minus E_g by KT I somehow simplify this expression. So, this is the constant this part is a constant this part. So, if you see this part this is nothing but you know x into e minus x kind of form. So, if I plot this function then this will give me something like this. So, this is for example, E_{ph} I am plotting this and this is the power because this is the photon generated.

So, this is a optical power this will give me something like this, this is the E_g point where E_{ph} this power is 0. E_{ph} is equal to E_g is 0 and it has a maximum at E_g plus KT and if I see this distance or this width this is nothing but $2KT$. So, this is actually the spectral width of the light emitting diode. So, I have taken so many approximations. So, that is why it is $2KT$, but in actual practice this is $1.5KT$ to $3.5KT$.

So, the spectral width is $2KT$ as I mentioned here and $\Delta\lambda$ for typical you know experimental device experimentally it is found that this $\Delta\lambda$ is between $1.5KT$ and $3.5KT$. So, now let us find out the value of $\Delta\lambda$.

So, we know $\Delta\lambda$ is equal to hc by E_{ph} this is the wavelength which is generated because of this E_{ph} and if I sort of differentiate $\Delta\lambda$ with respect to ΔE_{ph} I get $\Delta\lambda$ is equal to minus hc E_{ph}^2 ΔE_{ph} and if I replace you know write in a different fashion that is $\Delta\lambda$ by λ is nothing but $2KT$ by E_{ph} .

So, we can get some values or $\Delta\lambda$ for different wavelengths. So, if I put λ is equal to 850 nanometers what I get $\Delta\lambda$ as 30 nanometers. And if I increase the wavelength to 1310 nanometers then I get 70 nanometers for 1550 it is about 100. So, typical LED gives you know 30 depending on the wavelength 30 to 100 nanometer.

So, it is quite high actually for communication from communication perspective in optical fiber particularly it is quite high. So, because it lead to lot of dispersion right.

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LED Power and Efficiency

Internal Quantum Efficiency

$$= \frac{\text{No. of photons generated}}{\text{Total no. of e-h pairs}}$$

External Quantum Efficiency

$$= \frac{\text{No. of photons emitted}}{\text{Total no. of photons generated}}$$




Now, we have understood about the spectral width, let us understand about the LED power how much power is emitted. And also we will try to understand the efficiency part. So, let us take the first the efficiency. So, there are two types of efficiency which is defined; one is internal quantum efficiency which is defined as number of photons which are generated inside the device divided by total number of electron hole pairs are which are generated inside the device.

So, if you know number of electron hole pairs are all converted into photons the efficiency is 1, but that is not the case most of the time. So, this is how internal quantum efficiency is defined external quantum efficiency is actually ratio of number of photons which are emitted out of the device and number of photons which are generated inside the device. So, this is called as the external to the external world or external quantum efficiency.

So, these are the two important parameters what is your internal quantum efficiency and what is your external because ultimately the photons which are generated they are going outside to either getting coupled to the fiber or traveling to a different destination at some point b. So, that is also important. So, these two quantities both are important for sources.

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$$\frac{-dn}{dt} \propto n$$

Av. life time against recombination

$$n = n_0 e^{-t/\tau}$$

$$\frac{dn}{dt} = \frac{-n_0}{\tau} e^{-t/\tau} = \frac{-n}{\tau}$$

dep. region
n → charge carrier density

Net rate = Externally supplied + Depletion rate due to recombination

$$= \frac{J}{qd} + \frac{dn}{dt}$$

$$= \frac{J}{qd} - \frac{n}{\tau} = 0 \quad \text{under equilibrium steady state}$$

$$n = \frac{J\tau}{qd}$$

Steady state electron density in the active region when a constant current is flowing through it.



So, let us try to understand these efficiency parameter. So, as we know in a photodiode in a light emitting diode there is a p junction there is a n junction and there is a depletion region here. So, this rate minus dn by dt because the n is the carrier density charge carrier density it is getting depleted. So, minus dn by dt is actually proportional to n this is what is happening inside the LED.


And if I solve this so, this n is equal to $n_0 e^{-t/\tau}$ where τ is the average lifetime or against recombination. So, n is equal to n_0 n_0 becomes an initial charge carrier density is

e raised to power minus t by τ and τ is the average lifetime against recombination and if I differentiate this expression this becomes $\frac{dn}{dt}$ minus n_0 by τ e raised to power minus t by τ .

And I can replace this with n . So, this becomes $\frac{dn}{dt}$ is equal to minus n by τ . So, what is a net rate you are externally supplying the current and there is a depletion happening because of the recombination. So, the net rate is externally supplied current plus depletion rate due to the recombination. So, externally supplied is J by qd , J is the current density and q is the charge and d is the width of the depletion region plus the depletion is happening at the rate of $\frac{dn}{dt}$ that is a depletion rate due to recombination.

So, this becomes J plus qd plus $\frac{dn}{dt}$ and under equilibrium state steady state this is 0 and I have replaced here this is $\frac{dn}{dt}$ by n minus τ using this expression. So, this becomes J minus qd minus n by τ and net rate will be 0 under a steady state. So, this gives me the steady state electron density in the active region when the constant current is flowing across the device which is given by n is equal to $J \tau$ divided by qd .

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$$\begin{aligned}
 \eta_{\text{int}} &= \frac{\text{Radiative recombination rate}}{\text{Total recombination rate}} \\
 &= \frac{R_r}{R_r + R_{nr}} \\
 &= \frac{\frac{dn}{dt}_{\text{rad}}}{\frac{dn}{dt}_{\text{total}}} \\
 &= \frac{-\frac{n}{\tau_r}}{-\frac{n}{\tau} - \frac{n}{\tau_{nr}}} \\
 &= \frac{1}{1 + \frac{\tau_r}{\tau_{nr}}} = \frac{\tau}{\tau_{nr}}
 \end{aligned}$$

Bulk Recombination time
Non rad. recombination time



So, that was the n the current part current part. So, now, let us see the internal efficiency of the device the internal efficiency is defined by η_{int} this is how I am designating this. So, this is a ratio of radiative recombination rate divided by the total recombination rate. So, this is a useful light which is getting generated inside the device.

So, that is called the internal quantum efficiency and this can be represented as R_r that is a radiative recombination rate and total combination which consist of radiative as well as non-radiative. So, it will be R_r plus R_{nr} and this is equal to d by dn by dt radiative this is the rate and divided by dn by dt total including both radiative as well as non-radiative. So, as from earlier expression dn by dt is equal to minus n by τ and for radiative it will be minus n by τ_r .

So, this is the average carrier lifetime against radiative recombination and the dn by dt will be total. So, this will be minus n by τ minus n by τ_{nr} . So, this can be little simplified here and gets canceled and what you get n internal η internal that is efficiency internal as τ by τ_{nr} . So, τ is the actually bulk recombination time this is the bulk recombination time and this is the non-radiative recombination time non-radiative time.

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$$\left. \frac{dn}{dt} \right|_{\text{total}} = \left. \frac{dn}{dt} \right|_{\text{rad}} + \left. \frac{dn}{dt} \right|_{\text{non rad}}$$



$$-\frac{n}{\tau} = -\frac{n}{\tau_r} - \frac{n}{\tau_{nr}}$$

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

Generally $\tau_r = \tau_{nr}$


$$\eta_{\text{int}} = 0.5 \text{ or } 50\%.$$



Also we know dn by dt another expression which will connect τ and τ_r and τ_{nr} . So, dn by dt total is com is addition of dn by dt radiative plus dn by dt non-radiative. So, this we know minus n by τ is equal to minus n by τ_r minus n by τ_{nr} and this n will get canceled. So, we get a simple equation 1 by τ is equal to 1 by τ_r plus 1 by τ_{nr} . So, generally what happens τ_r is equal to τ_{nr} these two times are same.

So, if I calculate the efficiency by putting these values τ_r τ_{nr} in the η_{int} initial expression I get 0.5 or 50 percent; that means, 50 percent of the energy is only radiatively emitted the other part is there is no radiation it is absorbed or lost in the device. So, internal efficiency for a typical diode this is the order of 50 percent.

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$$\eta_{int} = \frac{R_r}{R_r + R_{nr}}$$


$$= \frac{R_r}{I}$$

$$= \frac{\frac{P_{int}}{h\nu}}{I} = \frac{P_{int} q}{I h\nu}$$

$$P_{int} = \frac{\eta_{int} I h\nu}{q}$$

$$= \eta_{int} \left(\frac{hc}{q\lambda} \right) I$$

Handwritten notes in red:
 - $3.5 \times 10^{15} \text{ m}^{-1}$ (pointing to $h\nu$)
 - $1.9 \times 10^6 \text{ cm}^{-1}$ (pointing to $h\nu$)
 - 1.1 eV (pointing to $h\nu$)
 - 1.1 eV (pointing to $h\nu$)
 - 1.1 eV (pointing to $h\nu$)




Let us also try to understand the relationship between the internal efficiency and the power which is generated that is P_{int} or $p_{internal}$. So, this is η_{int} is defined as η_{int} is defined as radiative recombination divided by total recombination and this is $I / (I + R_{nr})$ divided by total recombination will be I by q and R_r that is a radiative is the P_{int} that is a power generated because of this radiative recombination divided by $h\nu$, $h\nu$ is the photon energy divided by I by q .

So, the total becomes $P_{\text{internal}} = q \text{ divided by } I \text{ into } h \text{ into } \nu$. And so, the P_{internal} the power which is generated inside the device is $\eta_{\text{internal}} = I \text{ into } h \nu \text{ divided by } q$. And it can be little written in a different form. So, this is the efficiency part, this is the Planck's constant this has units joule second.

So, while calculating you should be very careful with the units and c is in velocity of light meter per second, λ is in meters and q is a charge in coulomb and I is the current in amps and you will get the power in terms of watts. So, if you are using mks system units these are the units you have to follow.

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Example

InGaAsP LED ($\lambda = 1310 \text{ nm}$)

τ_r and τ_{nr} are 30 & 100 ns respectively,

Drive current is 40 mA.

Find:

- Bulk recombination time
- η_{int}
- Internal power level

$$\tau = \frac{\tau_r \tau_{nr}}{\tau_r + \tau_{nr}} = 23.1 \text{ ns}$$

$$\eta_{\text{int}} = \frac{\tau}{\tau_r} = 0.77$$

$$P_{\text{int}} = \eta_{\text{int}} \left(\frac{hc}{q\lambda} \right) I = 29.2 \text{ mW}$$


So, let us do an example and try to get some you know idea about the typical values. So, if I assume a in gas phosphide LED which is emitting at say $\lambda = 1310 \text{ nanometer}$ and it has radiative recombination time as 30. And τ_{nr} non radiative as 100 millisecond here it is

shown different, but normally these times are you know similar and the drive current which is the I is actually is 40 milli amperes and we will try to calculate what is the bulk recombination time which is nothing but τ and what is the internal efficiency of the device and how much power is generated inside the device.

So, if you put all these values in the expression which I have discussed what you get τ as 23 nanosecond what you get efficiency as 0.77 and the power which is generated inside the device is the order of 29.2 milliwatts. So, these are the typical values which are used in you know optical communication using optical fiber.

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External Quantum Efficiency

$$\eta_{ext_1} = \frac{\int_0^{\phi_c} 2\pi \sin\theta \, d\theta}{4\pi}$$

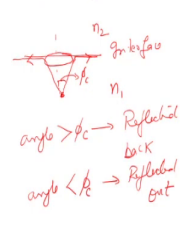
$$= \frac{1}{2} [1 - \cos\theta]_0^{\phi_c}$$


$$= \frac{1}{2} [1 - \cos\phi_c]$$


$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\cos \phi_c = \sqrt{1 - \sin^2 \phi_c} \cong 1 - \frac{1}{2} \sin^2 \phi_c$$

$$\eta_{ext_1} = 1 - \frac{1}{2} \frac{n_2^2}{n_1^2}$$







So, now let us try to understand the external quantum efficiency. So, for understanding the quantum efficiency external quantum efficiency suppose I have this interface I mean this is my device and this has some interface and this is air. So, this is a n_2 reflective index here is n

2 and inside the device it is n_1 this n_2 can be anything, but if it is here then n_2 will be 1 actually. So, there is a interface and this is suppose the photon is generated here.

And then you know there is only certain cone here and let me put this as angle as ϕ_c . So, if the your angle is greater than ϕ_c then the light gets reflected back into the medium it gets reflected back into the medium which is not useful and if your angle is less than ϕ_c the light is reflected out what I have shown here is a critical angle where the light is just grazing the interface.

So, reflected out right. So, all the light or all the photons which are in this cone only will come out the other will be reflected back and will be absorbed. So, if I you know put mathematically this is a solid angle where if the photon is confined to this solid angle only then it is reflected out. So, this can be expressed as this is this part is from the solid angle and the limits are from 0 to ϕ_c and 4π is a total solid angle.

So, the external efficiency will be you know integral 0 to ϕ_c $2\pi \sin \theta$ into $d\theta$ divided by the total solid angle which is 4π and I can write this $\sin \theta$ I can you know solve this integral and limits from 0 to ϕ_c and also. So, this gives me half into $1 - \cos \phi_c$ and we know from Snell's law that $\sin \phi_c$ is equal to n_2 by n_1 the other angle is $\sin 90$ which is 1.

So, $\sin \phi_c$ can be converted into $\cos \phi_c$ and put in this expression by putting this expression using $\cos \phi_c$ is equal to $\sqrt{1 - n_2^2}$ and doing some approximation here assuming that ϕ_c is very small. So, this half can come here what we I get as n external efficiency as this. So, this is my n external $1 - \frac{1}{2} n_2^2$ by n_1^2 square. So, as the outside medium is air. So, n_2 will be 1.

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$$\eta_{\text{ext}_1} = \frac{1}{2} \left[1 - \left\{ 1 - \frac{1}{2} \frac{n_2^2}{n_1^2} \right\} \right]$$

$$= \frac{1}{4} \frac{n_2^2}{n_1^2} = \frac{1}{4n_1^2}, [n_2 = 1]$$

$$= 0.0193 \text{ i.e. } \cong 2\% \text{ for } n_1 = 3.6.$$



And effectively I will get $n_{\text{external } 1}$ as $1/4$ and $1/n_1^2$ assuming n_2 is equal to 1 and n_1 for a typical semiconductor material is 3.6 and if I put this 3.6 value in here then $n_{\text{external } 1}$ is 0.0193, which is roughly 2 percent. So, the external efficiency because of the light which is going back to the medium because of the total internal reflection and only part of the light is coming out of the device which is about 2 percent of the light which is generated inside. So, this part gives you 2 percent efficiency.

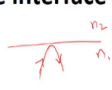
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Partial reflection at the interface

$$\Gamma = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Reflection

$$\tau = 1 - \Gamma$$



NPTEL



So, there is another thing which is happening here at the interface not everything which goes here and is inside the solid angle will go out of it some part will be reflected back right because the refractive index is changing at the boundary. So, even if the light is inside that cone, but when it is trying to come out of the device there is some part which will be reflected back.

So, we need to calculate this part also which will be reflected back. So, this is a reflection coefficient this tau is the reflection coefficient which is given by $n_2 - n_1$ divided by $n_2 + n_1$ whole square and we need to find out the transmission coefficient. So, transmission coefficient will be $1 - \tau$ the transmission coefficient will be $1 - \tau$.

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Partial reflection at the interface

$$\Gamma = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Reflection

$$\tau = 1 - \Gamma$$

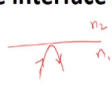
$$= \frac{4n_1 n_2}{(n_2 + n_1)^2} = 0.68$$

$n_2 = 1$

$$\eta_{ext_2} = 0.68$$

$$P_{emitted} = \eta_{ext_1} \eta_{ext_2} P_{int}$$

$$\cong 0.01 \text{ (1\%)}$$





And if I put this value of tau here and do some calculation what I get is $4 n_1$ and $2 n_1$ plus n_2 square and again assuming n_2 is equal to 1 I get 0.68. So, this is a transmission coefficient I mean the light which is going out of the device. So, n_{ext_2} earlier was n_{ext_1} which we saw was 0.2 percent 0.02 rather and n_{ext_2} is 0.68. So, the total power emitted will be multiplication of these two efficiencies into the power generated inside the device.

So, if I do this. So, basically it comes out to be 0.01 1 percent. So, only 1 percent of the light which is generated inside the device is able to come out.

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Frequency Response and Bandwidth

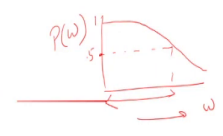
Optical Bandwidth


$$P(\omega) = \frac{P(0)}{\sqrt{1+\omega^2\tau^2}}$$


$$\frac{P(\omega)}{P(0)} = \frac{1}{2}, \text{ when } \omega^2\tau^2 = 3.$$

$$\omega_{3\text{ dB}} = \frac{\sqrt{3}}{\tau}$$

$$f_{3\text{ dB}}(\text{opt}) = \frac{\sqrt{3}}{2\pi\tau}$$







So, now, we will study about the frequency response and bandwidth of the device. So, the optical bandwidth let us first try to understand the optical bandwidth first and then there is other another concept of electrical bandwidth for the device. So, we will try to understand the relationship between the optical bandwidth and the electrical bandwidth.

So, as we know or you can find in some standard books of optical fiber communication when you are modulating your source with a light of frequency ω so, as you increase the ω the power output decreases. So, which is given by this expression $P(\omega)$ is equal to $P(0)$ is actually at dc power at dc divided by $1 + \omega^2\tau^2$; τ is the average lifetime of the photon.

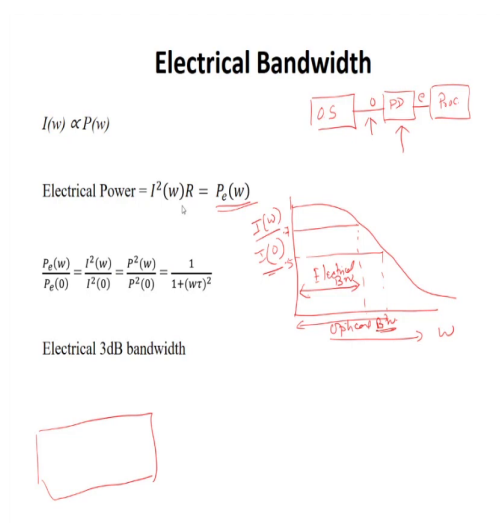
So, $P(\omega)$ is the power at frequency ω . So, if we if I plot this. So, this is for example, the my frequency or ω and this is the power say $P(\omega)$. So, as the frequency decrease

increases it follows this trend. So, this decreases with change in frequency. So, what is the tip the how do you define the bandwidth of the device so, where the power has fallen by a factor of half.

So, if somewhere here say this is 1 this is 0.5 corresponding to this whatever frequency you get that is the bandwidth of the device optical bandwidth of the device. So, this is how we define. So, this P_{ω} is equal to P_0 plus 1 by $\omega^2 \tau^2$. So, as we know this bandwidth depends on this factor τ which is the bulk recombination time.

So, for this to happen P_{ω} I am trying to calculate the optical bandwidth. So, P_{ω} divided by P_0 will be half when this is you know 3 then 1 plus 3 is equal to 4 square root of 4 is 2. So, this will be half. So, at this point the we will have 3 dB bandwidth. So, this can be. So, w 3 dB will be root 3 by τ . And you can write in terms of frequency which is root 3 by 2 $\pi \tau$ 2 $\pi \tau$. So, this is the 3 dB bandwidth optical bandwidth of the device root 3 by 2 $\pi \tau$ by 2 $\pi \tau$.

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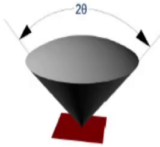


Now let us try to calculate the electrical bandwidth as we know the current which is produced in the device to calculate the electrical bandwidth actually see this is your optical source. And then you have the photodiode which actually converts this optical energy to electrical energy and then there is some processing. So, the current which is produced in this photodiode as a result of this optical input is proportional to the light falling onto it. So, there is a linear relationship.

So, $I(\omega)$ is proportional to $P(\omega)$. And electrical power if I calculate will be $I^2 R$ which is this is electrical power $P_e(\omega) = I^2(\omega) R$ and if I take the ratio of $P_e(\omega)$ divided by $P_e(0)$ that is electrical power at ω and electrical power is 0 this will be ratio of $I^2(\omega)$ by $I^2(0)$ and this can be related to optical power $P^2(\omega)$ by $P^2(0)$.

So, as you see the I and P they are directly proportional if it is optical bandwidth and for electrical power the this power is proportional to I square. So, this becomes equal to 1 by 1 plus omega tau square and this will be half when omega tau is equal to 1. So, the electrical 3 dB bandwidth is whenever this omega t is equal to half and by doing the calculation in similar way we find out that the 3 dB electrical bandwidth is nothing but 1 by 2 pi tau 1 by 2 tau.

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Lambertian Source


Total flux in a 2θ cone:


$$\Phi = 2\pi \int_0^\theta I(\theta') \sin \theta' d\theta'$$


$$\Phi = 2\pi \int_0^\theta I_0 \cos \theta' \sin \theta' d\theta'$$

$$\Phi = I_0 \pi \sin^2 \theta$$

The total flux of a Lambertian source of 1 cd is π lumens.







So, if I plot just to explain you. So, this is your omega and this is how the power is falling with rising omega. So, this is the power or let me now write in terms of current because that is a you know quantity which is common there. So, I let me plot this as I omega I 0.

So, from the expressions which I have mentioned here if you take there will be two points; one for the electrical bandwidth other will be for the optical bandwidth. The optical

bandwidth will be somewhere 0.2 and this will be for the electrical bandwidth 0.7 because this becomes 1 by 1 by root 2.

So, this will be 0.7 points and the corresponding bandwidth is depicted here. So, this is the electrical bandwidth this is the electrical bandwidth and this is the optical bandwidth. So, there is a difference between the electrical bandwidth and the optical bandwidth and this is how they are related.

So, earlier I had shown you the expression for the earlier I had shown you the expression for electrical bandwidth and similarly for the optical bandwidth and this is how it is the difference is shown in this diagram. So, I will stop at this point and from next class we will work we will discuss about the laser and we will try to calculate similar quantities for the laser also.

So thank you very much.