Optical Wireless Communications for Beyond 5G Networks and IoT
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Lecture - 14
Part - 1
Frequency Offset in OFDM, PAPR in OFDM

Hello everyone. So, today we are going to discuss about the effect of Frequency Offset in OFDM. In the earlier class, we understood the concept of OFDM. We also understood that all the sub-carriers are orthogonal and we also discuss about the cyclic prefix and by adding cyclic prefix, we were able to make parallel channels which were of flat fading in nature. So, now today we will discuss if there is a carrier offset, then what happens in the OFDM signal to noise ratio.

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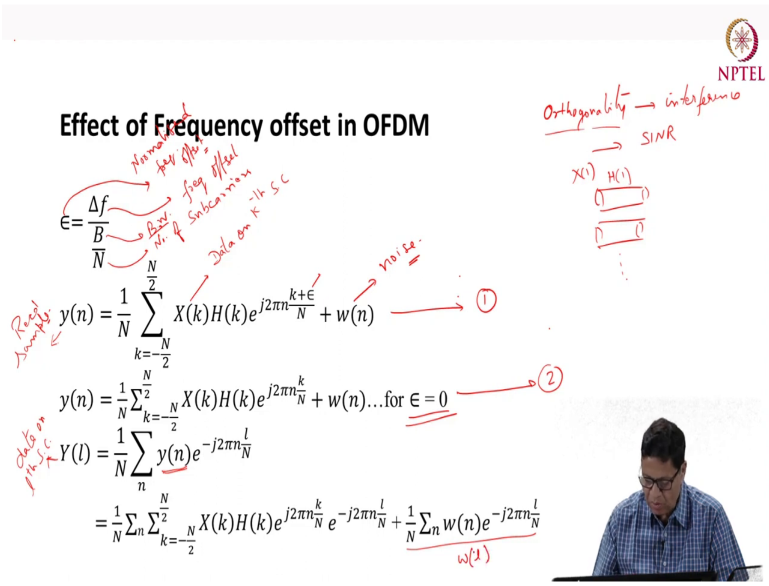

So, basically if there is a frequency offset in OFDM, then you may lose orthogonality of the sub-carriers. This will introduce interference and which is going to effect your signal to interference noise ratio SINR. This will go down. In RF, what happens when the user is moving towards a receiver or away from the receiver? There is change in the carrier frequency which is called as Doppler shift and the second possibility could be that the local oscillator at the receiver might drift.

So, in both the cases, the orthogonality of the sub-carriers is lost. The moving of user is not an issue in indoor while optical wireless communication system. So, basically the offset may occur because of offset in the local oscillator frequency at the receiver. So, let us try to calculate what will be the signal to noise ratio when there is a frequency offset. So, for that we need to define parameter which is called as normalized frequency offset.

So, let us define this as this is delta f is the frequency offset and B as we know is the bandwidth and N is the number of sub-carriers. So, I am defining a parameter epsilon which is normalized frequency offset, normalized frequency offset which is defined as the delta f that is the frequency offset divided by the spacing between the two sub-carriers which is B by N. B is the total bandwidth and N is the number of sub-carriers.

So, let us consider this particular equation. This is the received signal or received sample received sample and N is as I told you is the number of sub-carriers and X k is the data on the kth sub sub-carrier data on kth sub-carrier and H k is the corresponding channel coefficient as we have discussed that the OFDM makes the channel as frequency fading channels. So, you have basically you know parallel channels for each sub-sub-carriers defined by, for example, this is first carrier, H 1 and X k is the data are reading on it.

So, there are k such channels and this will be e raised to the power j 2 pi n k plus sigma, sigma epsilon s is the normalized frequency offset. Let us consider this now and this is the noise. Now, y n is given by 1 by N. Now, let us try to understand this particular equation, this is equation 1 when epsilon is 0, there is no frequency offset whether does it lead to situation when the channel becomes a flat-fading channel.

So, let us put epsilon is equal to 0. So, this gets reduced to 1 by N summation over all the sub-carriers X k, H k, e raise to the power j 2 pi n k by N, where epsilon is 0 plus noise. And, if I want to pick extract the data on the lth subscriber, this is lth data, on lth sub-carrier. Received data on the lth sub carrier is given by 1 by N y n, in 2 e raise to the power minus j 2 pi n l divided by N. This we have studied when we were discussing about OFDM.

So, let us now put the value of y 1 from this equation, in this equation of Y l capital Y l. So, by putting this y n from this equation number 2 into equation for capital y L. So, this becomes 1 by N. If the two summations n and summation of sub-carriers. And, this is e H k, X k, H k into e raise to the power j 2 pi n k by N into e minus j 2 pi n l by N plus this is the noise drum. So, this is actually 1 or not 1, but this will give me, this is only valid when n is equal to L. So, this will give me w l.

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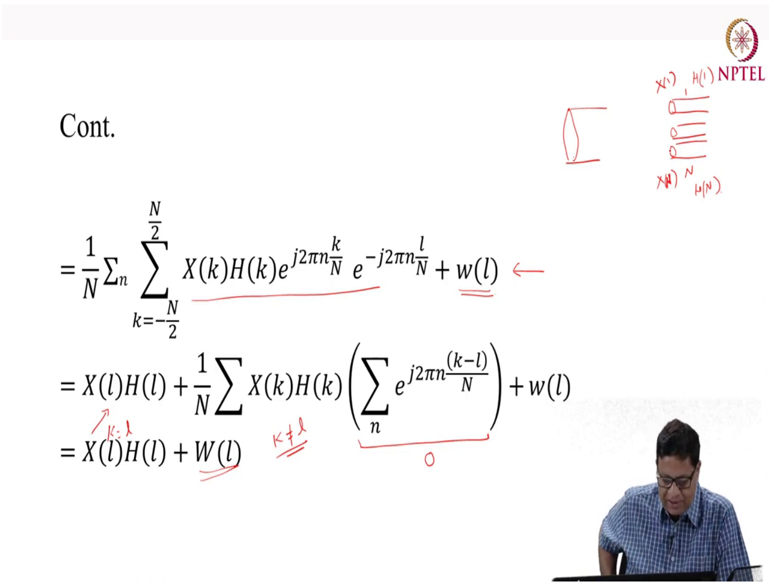

And so, I am left with this expression plus w l. And now, let us break this whole thing into two parts. One is when k is equal to l, which will give me this X l into H l and k not equal to l. So, this is k not equal to l. So, I have broken this into two parts k is equal to l and k not equal to l.

So, this particular thing is 0 because of orthogonality condition. So, I am left with X l into H l plus w l. So, the final expression becomes X l that is a data on lth subscriber, the channel coefficient for corresponding to the lth subscriber plus the noise in the lth sub-carrier.

So, this we already had seen this result when we had a frequency selective channel and then we using OFDM, we made it flat fading channels. And, these are the sub-carriers and they have channel coefficient as H 1 or 2 H N and each has data this is X 1, this is X l. So, we get back the same result as discussed earlier. Now, go back to the same equation and let us now understand the effect in the presence of frequency offset.

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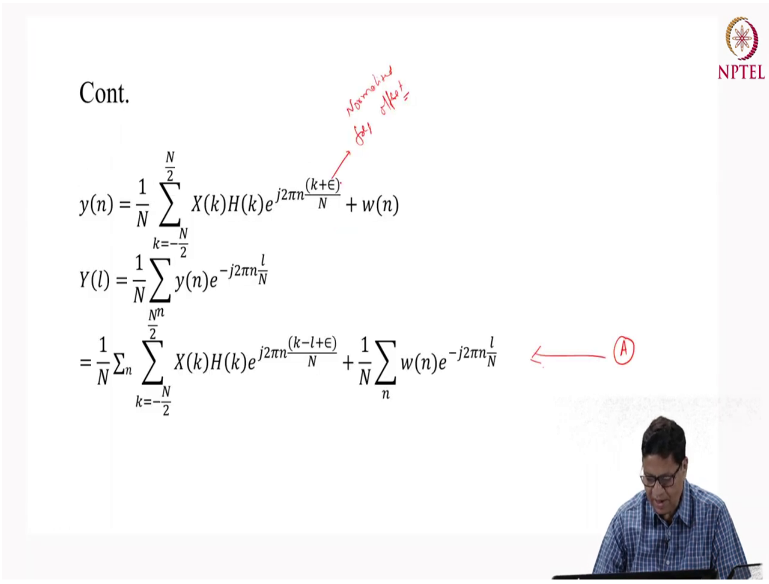

So, the y n the received sample is given by in the presence of frequency offset, this epsilon is frequency normalised frequency offset. So, we have seen this when epsilon is 0 and we got back our you know as we have discussed in the OFDM lecture. So, this is in the presence of frequency normalised frequency offset. And so, Y l the data on the lth sub-carrier can be written as 1 by N y n received signal e raised to the power j 2 pi n l.

Now, putting this y n here will give me this equation. So, I am simply putting Y l here and doing some mathematical adjustment here. So, this is what I get. So, there are two terms here. Now, in order to solve this particular thing, we will take certain approximations and see what kind of results we get. So, in order to solve this equation, let me name this as A. I will use some approximations.

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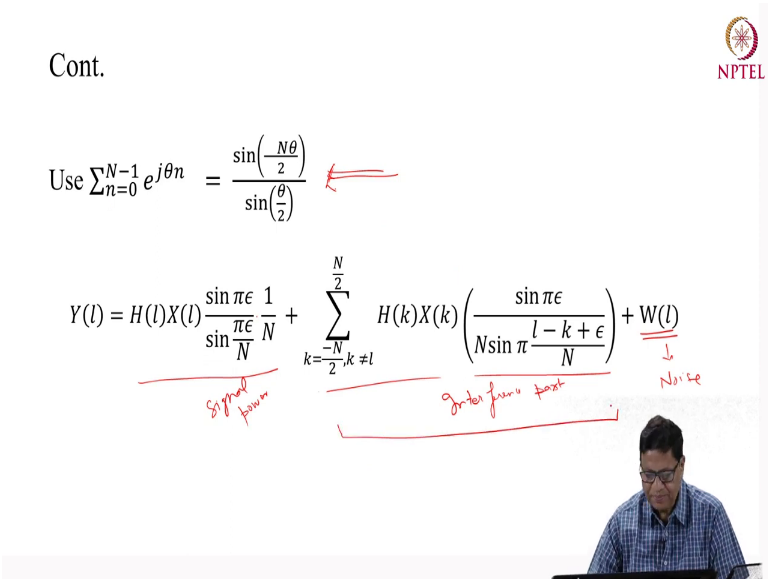

So, first approximation is or the first identity I will use is summation n is equal to 0 to N minus 1 a e raise to the power j theta n is actually equal to sin N theta by 2 divided by sin theta by 2. So, using this identity and putting in the equation which I had named as A in this equation, this equation A replacing actually e raise to power j theta here; I get H l X l sin pi epsilon divided by sin pi epsilon divided by N into 1 by this is into 1 by N plus summation and again here also I have changed using this equation and this is the noise part the W l.

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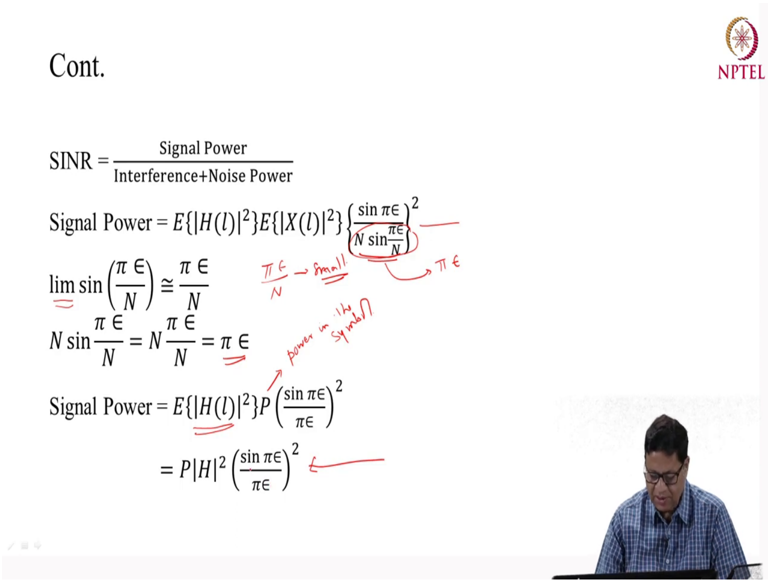

Now, in order to calculate signal to interference noise ratio, we need to calculate signal power. So, signal power is in the last equation if you see here, this is the desired signal of signal power and this is the interference part and this is the noise part. So, let us try to calculate the signal power. So, signal power is given by expected value of H l mod of H l square into expected value of E X l because H l and X l they are independent into sin pi epsilon divided by N pi epsilon by N whole squared.

So, this is a signal powered coming from the equation A, the signal part and we can approximate this value sin pi epsilon divided by N pi epsilon those by N. So, sin limit sin pi epsilon N is actually equal to pi epsilon divided by N, when you know this factor is small pi epsilon to N is small.

So, this is true when which actually is small because there will not be a major frequency offset is small. So, this this identity is valid if this pi epsilon divided by N is small. So, this will give me I multiply by N both sides. So, this N sin that is at this part is nothing, but equal to pi epsilon.

So, by signal power replacing this part by pi epsilon, I get expected value of H l square into P, P is the power in the data that is expected value of X l mod X l whole square is the power in the data. This is power in the data symbol in the symbol multiplied by sin pi epsilon divided by epsilon whole square after considering this approximation.

So, this will give me P, if I take the average value of channel coefficient as H then I am I get signal power as P; there is power in the symbol into average value of the channel coefficient into pi epsilon divided by pi epsilon whole square. So, this is the signal power we have got.

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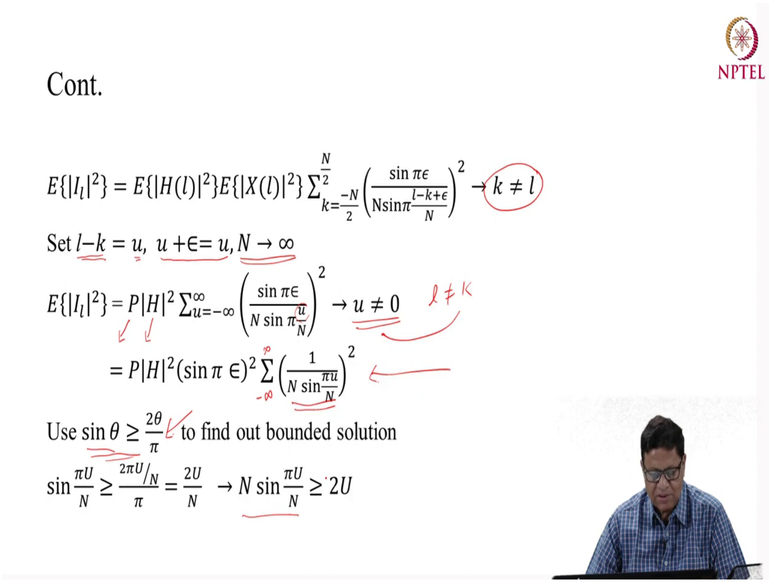

Now, let us try to calculate the interference power and then we will be able to calculate the signal to interference noise ratio. So, interference power taken from the earlier equation from this part. So, I am now trying to calculate this part, this is the interference part is given by expected value of I l mod of I l square into expected value of channel coefficient into the expected value of mod X l square multiplied by summation sin pi epsilon N sin divided by N sin pi l minus k plus epsilon into N whole square.

And, this is term actually is for k not equal to l, k is equal to l was the first signal part. So, this is the interference part. So, let us do a substitution here, put l minus k is equal to u and we can also take an assumption that u plus epsilon is u because epsilon is very small and N is very high. So, under these conditions u plus epsilon is equal to u because epsilon is high and N is high number of subcarriers are high.

Under this condition expected value of I l square, there is interference part is P, P is the again the power, power in the data symbol, H is the average channel coefficient. And, these limits on the summation they change, they become u is equal to minus infinity to infinity and because your N is tending to infinity.

And, then you have the bracket sin pi epsilon N sin pi l minus k, I have replaced by u and u plus sigma is u, this approximation I have done and this is u not equal to 0 because l is not equal to k. So, both are same. So, with this equations what we get is P into H mod square and I take sin pi sigma square outside; this is summation, this is over u minus infinity to infinity 1 by N sin pi u by N whole square.

Now, let us try to find a bounded solution to this problem and we know that sin theta is always greater than 2 theta by pi. So, I am just trying to find the bounded solution for this interference equation. So, assuming sin theta is equal to 2 theta by pi. So, I replace this sin pi U by N using this expression and then by doing this sin pi U by N will be greater than 2 pi U divided by N divided by pi which is 2 U by N. So, which means N sin pi U, the whole expression here is greater than 2 U.

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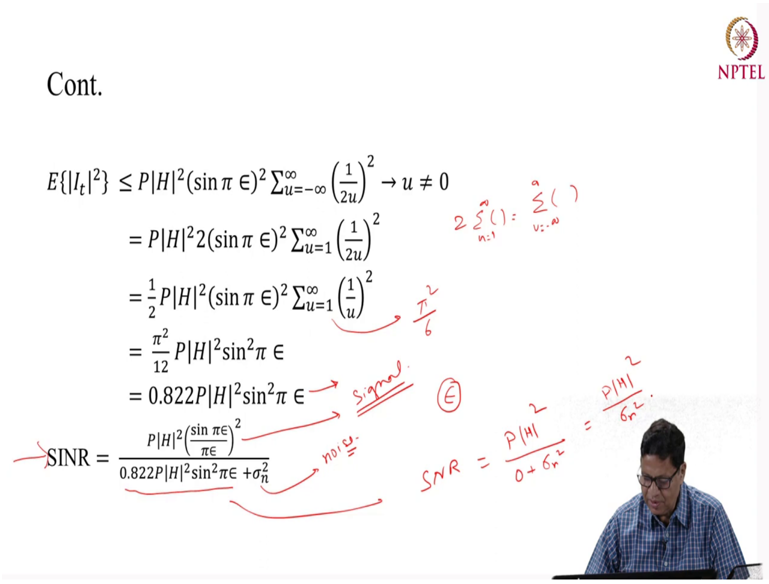

So, using this approximation and finding out the bounded solution, what I get is expected value of the interference component, interference power is less than or equal to P H square into sin pi epsilon whole square and then we have this summation and u is not equal to 0. So, I can take this 2 outside and then there is a standard expression for summation minus infinity to infinity of 1 by u whole square which is nothing, but pi square by 6.

So, this is and I can also replace this is twice because this is symmetrical. So, this is twice of u is equal to 1 to infinity, the limits I have changed. So, this is twice of this twice of this infinity u is equal to 1 and some expression is actually equal to u is equal to minus infinity to infinity this expression. So, using this I get simplified expression 1 by 2 P into mod H square sin pi epsilon whole square summation u is equal to 1 to infinity 1 by u n.

This is a standard identity and this gives a value pi square by 6. So, putting this value, the total expression becomes pi square by 12 power in the data symbol, the channel coefficient average value is the channel coefficient for different sub carriers into sin square pi epsilon. So, this can be further simplified and it is 0.822 P H square sin square pi epsilon.

So, now we have calculated the value of interference power. Now, we are in a position to calculate the signal to interference noise ratio. So, this was the signal part which we had calculated earlier and this was this is the interference part which has come from this particular equation and this is the noise part.

So, under the presence of frequency offset which we had defined through a parameter epsilon, normalized frequency offset our SNA SINR has is given by this equation. Now, let us try to understand the effect of this frequency offset on the SINR. How much is the degradation which is caused because of this frequency offset?

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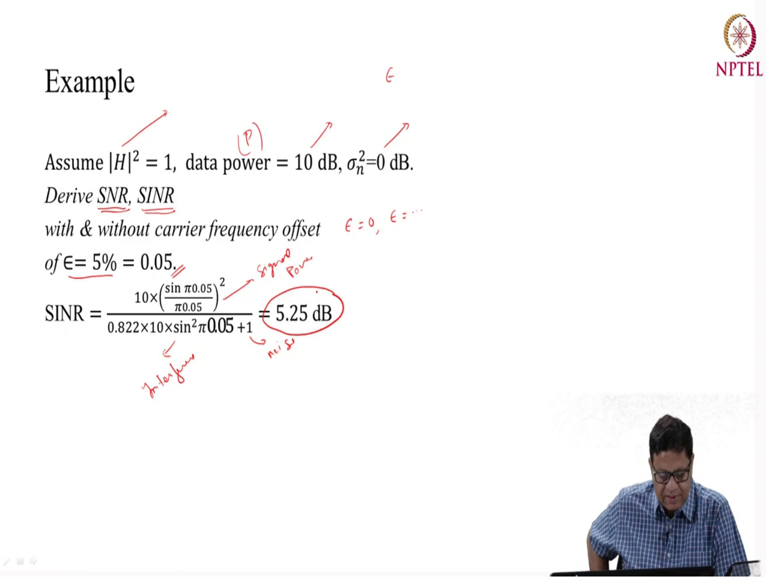

So, let us understand this using one example. So, let us assume that every channel coefficient is 1, the data power which is actually P is a 10 dB and assume the noise is 0 dB because we want to see the effect of epsilon. So, I am making some simplified assumptions here. So, let us find out what is the value of SNR, when there is no interference and what is the value of SINR when there is interference.

And we will calculate SNR, SINR for with and without carrier offset that is either sigma is equal to 0 or sigma has some value. Epsilon sorry, epsilon has some value. So, in this example, let us keep this epsilon as 5 percent or 0.05. So, for these values let us try to calculate the SINR using the earlier formula which I had derived.

So, this is given by this is a signal power part signal power. This is the interference just putting in the values here and of course, the noise was 0 dB or 1. So, this was the noise. So, this gives me 5.25 dB as the SINR.

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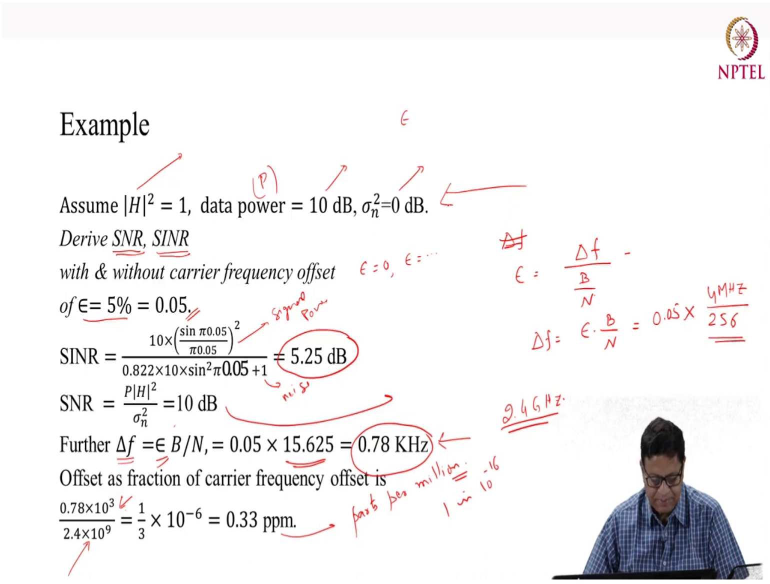

Now, let us calculate the SNR. SNR meaning there is no interference; that means, there is no offset. The epsilon is 0. So, if I so, keep epsilon is equal to 0 here what I get is SNR. So, this will be P H square, this limit will actually tend to 1. So, this will be 1 divided by this term, the interference part will be 0. So, 0 plus sigma n square. So, this is what I get as SNR. And, putting the values of P H n noise, we get SNR as 10 dB.

So, notice here that because of the frequency offset the S SNR has fallen to by another by approximately 5 dB, it has become 5.25 dB. So, that is the effect frequency offset will have in OFDM transmission. Or further just to find out what is this delta f frequency offset; so, epsilon is this is how we are defined delta. So, as epsilon is equal to delta f divided by B by N. So, I am trying to calculate delta f from here. So, delta f will give me epsilon B by N and this is 0.05 into B, let us assumed as 4 megahertz.

N number of sub carriers say I take example 256. So, this will give me 15.625 this particular thing and multiply by this will gives me a delta f of 0.78 kilohertz. So, that is the frequency offset. And, now let us understand this offset as a fraction of carrier frequency and I am assuming that OFDM is being transmitted as say 4 point 2.4 gigahertz, that is the carrier frequency. So, if I see the fraction of carrier frequency offset, it becomes 0.78 kilohertz divided by 2.4 gigahertz.

So, this gives me 0.33 ppm, ppm stands for parts per million; that is 1 in 10 to the power minus 6 is 1 ppm. So, what I get is 0.33 ppm. So, that is the offset which will give a degradation of SNR by almost 5 dB for you know these parameters. So, maintaining the local oscillator frequency and containing the offset is very, very important in OFDM. It is very challenging in cellular or RF communication, but that is not the case here because, in in our optical wireless and indoor communication the user is not moving.

So, only issue will come from drift in the local oscillator at the receiver. So, that we need to maintain exactly at that value. Otherwise, any slight offset as low as 5 percent of 0.05 will degrade the SNR performance. So, this is the effect of frequency offset on OFDM.

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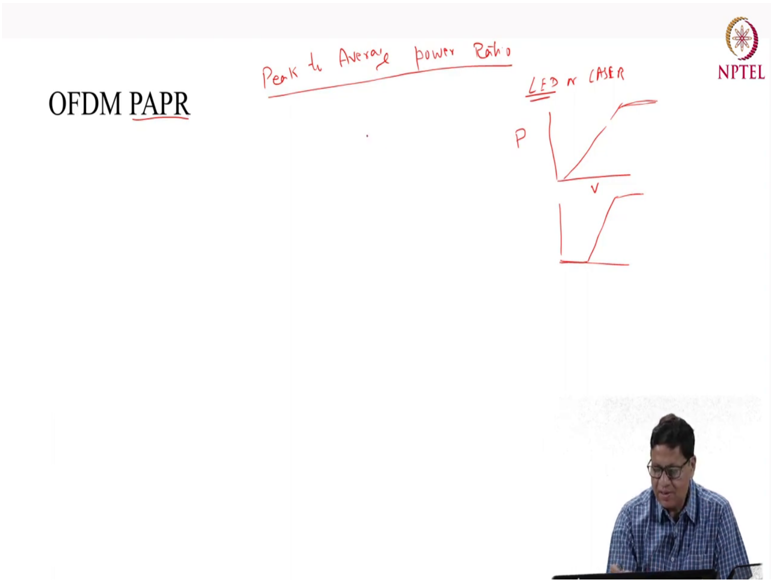

Now, let us try to understand another issue which is there in OFDM which is called as PAPR which stands for Peak to Average Power Ratio. This is very important in OFDM because OFDM signal has high PAPR and we know that in optical wireless communication system, we normally use LED or laser.

And, if you see the characteristic that is this is a voltage or I given to the device and this is the say power output P, then for LED it is linear and it has some dynamic range after that its sort of saturates and for laser also it is linear. So, both the devices they have some limited dynamic range. The laser is actually little different where you have say threshold device. So, then sort of saturates.

So, it has limitation and if the OFDM signal the voltage level of the OFDM output is higher than the dynamic range of these devices, then it results into distortion. So, we need to contain this peak to average power ratio. So, it is very very important when you are designing a OFDM system to you know contain the PAPR, PAPR value.

So, let us now try to understand this peak to average power ratio for OFDM. Now, before that let us understand for a signal carrier, single carrier what is PAPR that will explain the definition of PAPR.

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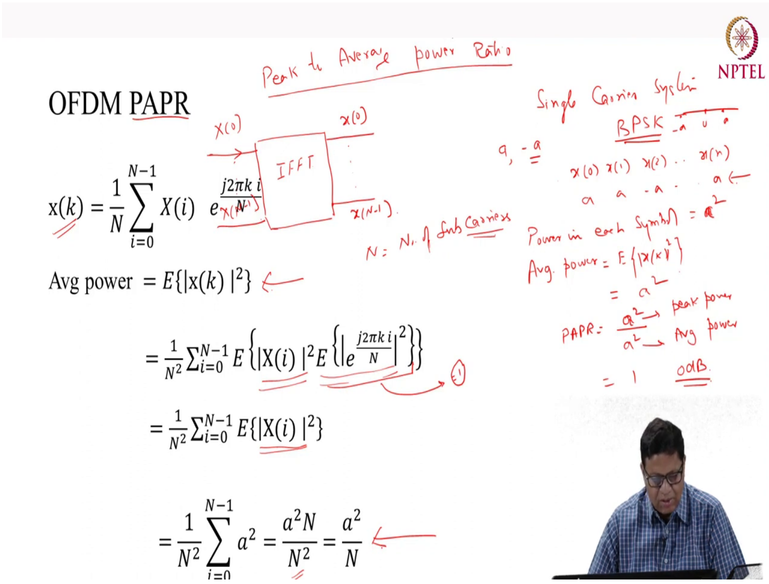

So, for example, for single carrier system say and I am using for example, BPSK; then the data will be you know x 0 for example, x 1, x 2 so on and so forth x n and this can have value either a or minus a depending upon the constellation. So, this is for example, this is 0, this is a and this is minus a. So, this can have a, this can have minus a or whatever.

So, if you see the power in each symbol, power in each symbol is a square; a is the amplitude the power in each symbol is a square and if I calculate the average power because this is peak to average power. So, this is a peak power in each symbol. So, average power will be expected value of x k whole square. So, from these for a BPS BPSK signal so, this will be is going to be a square only. So, if I calculate the peak to average power ratio that is PAPR, it will be a square divided by a square.

This is the peak power and this is the average power which is 1 or you can say it is 0 degree. So, this is a case of single carrier system using BPSK constellation scheme. Now, let us try to understand the PAPR in OFDM. So, OFDM signal if you recall from our OFDM discussions. So, I am not drawing the total OFDM clock diagram, just drawing the IFFT part. So, where you have these are the data symbols or the constellation.

So, this is X N minus 1 and this is in the frequency domain and then IFFT converts into time domain which are these are the samples. So, these are the samples. So, x 0 to x n minus 1, where n is a number of subcarriers. Now, the x k the sample is given by 1 by N summation i is equal to 0 N minus 1 data on the X i into e raise to power j 2 pi k i divided by N. Average power for the sample which are x 0 to x n minus 1 is given by expected value of mod value of x k x k square.

Now, let us put this value of x k here and try to evaluate this average power. So, this will become 1 by n square just putting the value of x k here and squaring it 1 by N square summation. And, then this becomes power X i mod value of X i square and expected value of this and this is 1, this is equal to 1. So, I am reduced to 1 by N square summation over all the subcarriers expected value of X i mod square and this value expected value of X i because samples at the output are changing.

So, I am assuming it to be a. So, I am just trying to find out the maximum average power. So, this will give me 1 by N square summation a square and then doing summation for all the subcarriers, subcarriers it will give you a square plus a square plus a square N times. So, the total a square into N divided by N square which is nothing, but a square by N. So, this is the average power which is given by a square by N.

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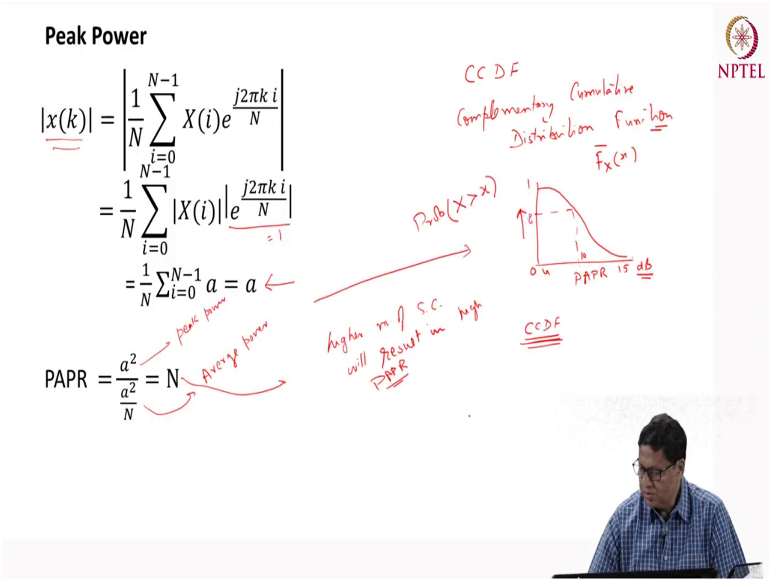

Now, let us try to calculate the peak power. So, the peak power is given by mod value of x k that is a sample given by this equation and so, this 1 by N can be taken out. Then, we have X i into e raise to the power j 2 pi k i by N and this will give me this is 1. So, this is equal to 1. So, this 1 by N into summation i is equal to 0 and minus 1 into a and this is a plus a N times N a divided by N gives me a. So, this is the mod value of the sample here.

And, if you take the power, this will be a square the power. So, if I calculate the peak power, peak power we had calculated just now as a square and the average power I had calculated the last slide. This was the average power a square by N. So, basically it gives you N.

So, high higher the number of subcarriers, higher is the P PAPR, higher numbers of subcarriers will result in high PAPR. This may not be as high as N, because we have taken some because sample amplitude is changing. So, the average value will change to some extent. So, this will be little less than this.

So, this is the PAPR for OFDM and for this is how it is defined in practice about the PAPR. So, normally one mentions one uses CCDF that is Complementary Cumulative Distribution Function, because this is not constant so, distribution function CCDF. So, if you plot say this is PAPR value and this is your say the probability that X is greater than x that is the value of PAPR is greater than some threshold that probability which is defined by this complementary cumulative distribution function which is can be written as F X x and this is the PAPR.

So, the curve will be something like this. So, this is the result you will get for PAP PAPR measurements. So, this is starting from 0 to 1, this is a 0.8 and this could be for example, 10 dB, this is say 4 dB and this these are say 15 dB. I am just putting some approximate values.

So, when a when the PAPR threshold is considered as 10 10 dB, then the 20 percent of the signals or 0.2 or the signals will be more than 10 dB, otherwise 80 percent or 0.8 part signal will be will have PAPR less than 10 different data symbols will have PAPR less than 10 dB.

So, the correct definition or the way it is defined for OFDM is to find out the CCDF or PPR PAPR. So, basically it tells you what is the probability that the value is more than certain threshold. Right. So, higher the PAPR, there will be more degradation in the performance of the system because the devices which are being used, they have limited dynamic range. So, if they exceed the dynamic range, the signal OFDM signal exceeds the dynamic range, then it results into distortion.

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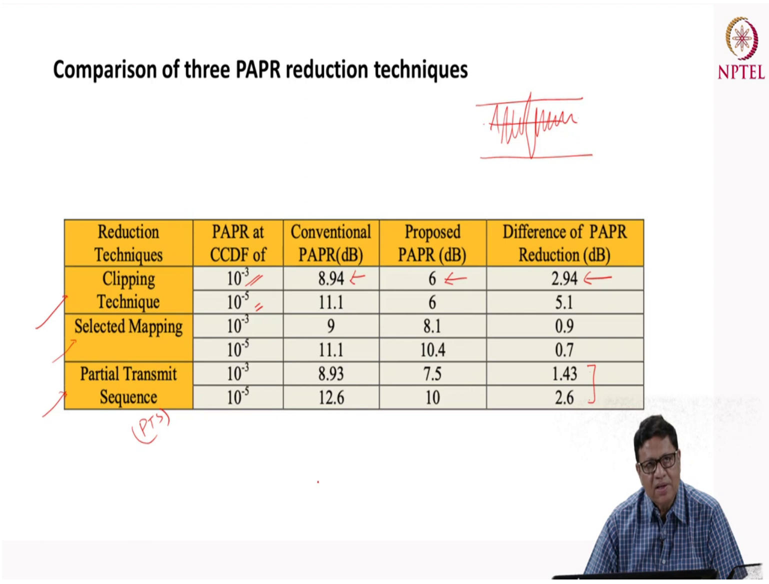

Reduction Techniques	PAPR at CCDF of	Conventional PAPR(dB)	Proposed PAPR (dB)	Difference of PAPR Reduction (dB)
Clipping Technique	10^{-3}	8.94	6	2.94
	10^{-5}	11.1	6	5.1
Selected Mapping	10^{-3}	9	8.1	0.9
	10^{-5}	11.1	10.4	0.7
Partial Transmit Sequence	10^{-3}	8.93	7.5	1.43
	10^{-5}	12.6	10	2.6

So, this was about PAPR and there are lot of techniques which have been explored for the reduction of PAPR. So, we will not go into the details of these techniques, but some of these techniques like clipping technique, you clip the signals because if you see the OFDM signal, it is you know noise. So, you can you know clip the signal at both the ends or at one end. So, using clipping technique and then you want to calculate the PAPR at certain BeR. So, in this table, it is 10 raise to the power minus 3 and minus 5.

So, if you do not do any clipping and calculate the PAPR, it is for example, 8.94 dB in case of 10 raise to the power minus 3 BeR. But, if you use clipping technique, you know, clip at either ends or at both the ends, it gets reduced to 6 dB. So, there is an advantage of 2.94 dB which actually improves the overall performance of the system. The second method is selected mapping. Again, you get certain advantage using this and then there is partial transmit sequence or there is also what is PTS.

So, again you see some examples there are lot of techniques for reducing the PAPR. I have mentioned here three, some of these important techniques which have been used literature. So, with this, we have good background about OFDM and we started with the fundamentals of OFDM. And, then we discussed about the how the frequency selective fading channel got converted into flat fading channel and then we also saw you know using cyclic prefix.

And so, after that, we have discussed about what happens if there is a frequency offset or the sub-carrier frequency, how the SINR degrades. And, then also we understood about PAPR, which is the main issue in OFDM transmission and the effect it has on the performance and what are different PAPR techniques.

So, now we move to the next topic which we will try to use this OFDM in visible light communication. And, understand what changes are required if I want to use OFDM in for example, visible light communication, for indoor communication or for outdoor communication.

Thank you.