Optical Wireless Communications for Beyond 5G Networks and IoT

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Lecture - 13
Part - 4
Cyclic Prefix (CP), OFDM with Cp, BER of OFDM System

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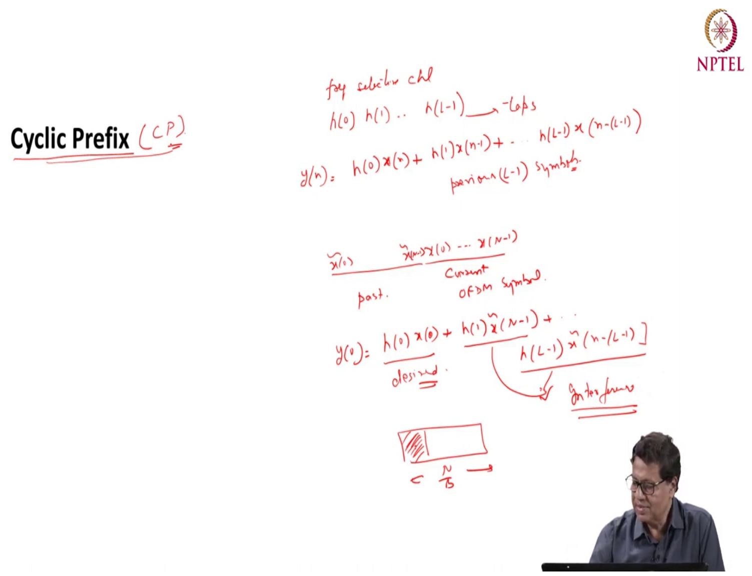

Hello everyone. So, now we will introduce the idea of Cyclic Prefix or CP. So, any frequency selective channel can be modelled as using channel taps or like h 0, h 1 or h, L minus 1, say upto lth. So, this is any frequency selective channel and these are the taps. So, what happens because of this channel? If you see any signal at time stand n y n then it will be h 0, x n and x h 1, x n minus 1 plus so on and 0073o, forth and the last one is h l minus 1, x n minus L minus 1.

So, you have you also have the contribution of previous L minus 1 symbols which is the interference term or if you have you know 2 2 O FDM symbols in say this is your current OFDM symbol which is say these are the samples x 0 so on and so forth x N minus 1 and say this is a past which will be x n minus 1 and this is x 0.

So, if I calculate y 0 this will be given by h 0, x 0 plus h 1 N minus 1 so on and so forth h l minus 1, x n minus L minus 1. So, basically this is your desired signal, but it also has interference. This is interference this is a desired signal. So, how to get rid of this interference? So, what happens your initial signals will be actually affected by the previous signals.

So, this is N B and this these are parts of initial signals the dash ones they are affected by your previous signal. So, how to get rid of this you know this interference? So, in order to get in order to do away with this interference we come out we come with an idea of cyclic prefix.

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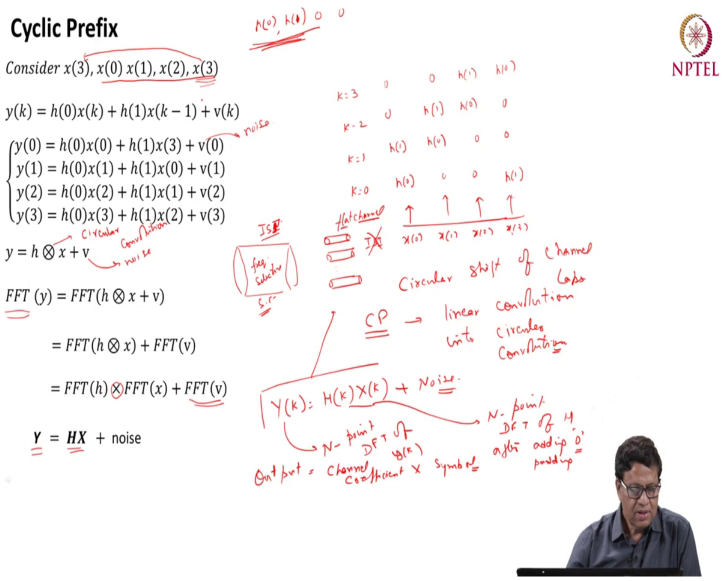

So, what actually is done in cyclic prefix? Suppose these are your samples x 0, x 1, x 2, x 3. Now, in some part of the signal is actually repeated in the beginning. So, this is actually cyclic prefix I have shown here only 1, but actually it depends on the delay spread and other factors how many will have to be prefixed with the symbol OFDM symbol. So, right now I have done only x 3 which is prefix. So, this is called a cyclic prefix and let us now try to understand how you know this will prevent this inter symbol interference.

So, if I take these samples say this was x 0 sample or this was x 1 sample, this was x 2 sample this is x y 3 sample they have you know different values. And suppose h 0 and h 1 they are the taps the channel is modelled using x 0 and x1 then let us see for k is equal to 0 for example, or k is equal to 1 or k is equal to 2 k 3. So, this k is equal to 0 will be this will be h 0 here and it will be affected by x 3. So, h 1 will be multiplied by x 3 and for k1 it will be h 1 and h 0 and others are 0.

For k is equal to 2 it will be 0 h 1 I mean you can write these equations or I have written here h 1 this is k is equal to 2 h 0 0 and this is h 1 this is h 0 this is 0. So, y k in this case when it is there is a prefix here will be given like this and for y 0, 1, 2, 3 and this is the noise part noise sample will be given by h 0 x 0 h 1 x 3 and so on and so forth. So, this is what I have drawn here and if you see here there is a circular shift of channel tabs there is a circular shift of channel taps.

So, it is actually h 1 0 h 1 0. So, this is circular shift of you can see that trend happening h 1 0 h 1 0 then h 1 0 this is a circular shift. So, effect of cyclic prefix is to make convert this into a circular convolution earlier it was a linear convolution. So, CP cyclic prefix actually has converted linear convolution into circular convolution. So, this is what the cyclic prefix has done and this is clear from this simple example where I have used x 3 as the cyclic prefix to the samples x 0 x 1, x 2, x 3.

So, y is actually this is circular convolution this is circular convolution this is different from linear convolution and this is the noise part. So, y has become H circular convolution plus x plus v. Let us take FFT because we are doing the FFT operation on the receiver. FFT of y will be FFT of whole thing and this can be broken FFT of g circular convolution x plus FFT of noise.

So, this is the property of circular convolution this can be broken into FFT of H into FFT of x plus FFT of y. So, this gives you Y is equal to HX and this is some quantity noise. So, this can this will be direct multiplication in case of circular convolution. So, this is actually. So, this becomes Y is equal to HX or YK for each subcarrier is actually HK XK plus of course, some noise which we are just adding here.

So, this is nothing but N point DFT of YK this is N point this HK N point DFT of H after you have added padding after adding 0 padding because it is h 0 and h 1. So, I am pointing others as 0. So, you have added them as 0 and then after adding this N point DFT after adding you know after adding padding with 0.

So, this basically gives you the output. So, basically the output is nothing but the channel coefficient in the symbol. So, what essentially channel coefficient? That is HK into symbol this is capital YK. So, what actually has happened here earlier we started with something which is frequency selective single carrier frequency selective single carrier. Now, we have broken this into N parts now each modulated by subcarrier.

So, we have made N parallel streams and all those parallel streams were combined and they were transmitted. So, the whole thing is converted into small parallel streams. This is what we did in MCM multi carrier modulation and this was frequency selective and from this it is very clear that each sub channel is flat channel.

So, there is no question of ISI, no ISI. The ISI was the main culprit in single carrier frequency selective channel and we have converted this into N parallel sub channels and each sub channel is now flat channel. So, the ISI problem has been resolved. So, by simply doing cyclic prefix we are able to convert the channel into a flat channel N flat channel and there is no effect of or there is no ISI now happening among the symbols.

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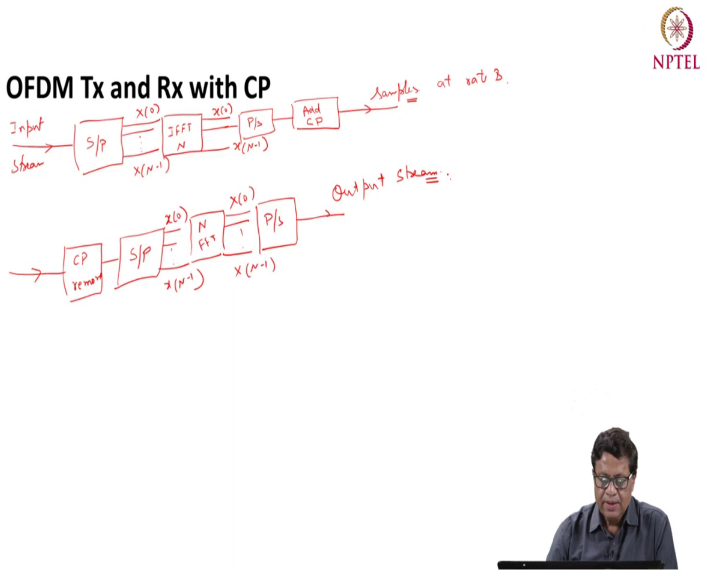

So, now we are in a position to draw the complete diagram of OFDM transmitter in receiver. So, this is your input stream. First operation what we do is serial to parallel. So, you get this different data x 0 to x minus 1 and then we do IFFT. This is quite common IDFT algorithm IFFT endpoint and then you get these symbols here.

This is x N minus 1 and then these samples are parallel to serial and once you have done the serial then you add CP. This is the add CP cyclic prefix. Now, we have understood why we want to add CP. So, these are the samples which are transmitted. Samples at rate B. Now, these samples.

So, first thing is CP is removed and the receiver CP removed. Then serial to parallel. This is serial to parallel. You get these samples out and then you do the N point FFT operation to get the data symbols out because the coefficients of FFT operation is nothing but data symbols x

N minus 1 and then you do parallel to serial to get the output stream. So, this becomes a complete by DM diagram.

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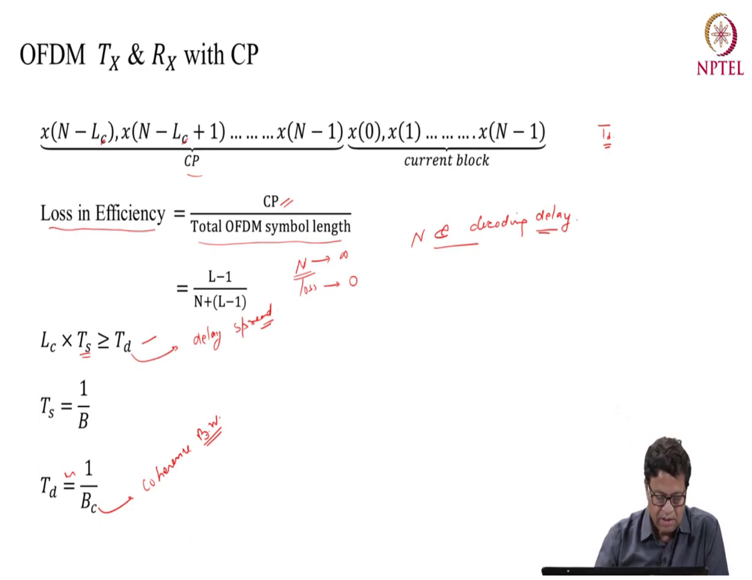

So, now let us understand OFDM transmitter and RX with CP. How normally it is done? So, this is your current block x 0 to x N minus 1. These are the samples and depending upon the delay spread T d you select those samples and prefix it in the middle in the beginning. So, this is what is the CP and I have selected L C number which are prefixed. So, this becomes OFDM symbol complete OFDM symbol. And because of this addition of CP there is a loss in efficiency.

So, this is CP length and this is a total OFDM symbol length. Though this gives you the loss in efficiency. So, the CP is actually L minus 1 here. Yeah, L minus 1. So, this C can be removed. And the total length will be N is because of these samples and L minus 1 is your CP length. So, the total loss in efficiency becomes L minus 1 and plus L minus 1. So, as N tending to infinity this loss is actually 0.

But we cannot make N infinity because if you make N infinity; that means, there are many symbols which have to be collected at the receiver before they can be decoded. So, basically the time for decoding increases. So, the overall system becomes slow. So, that is why N cannot be made very high. So, there has to be a compromise on the decoding delay. There is a trade-off between N and decoding delay. So, let us see how this number of symbols which have to be prefixed are selected.

So, basically this is this L is a number of symbols which have which are to be prefixed into T s. The duration of each symbol should be greater than or equal to T d. T d is the delay spread. So, one has to meet this condition. This is delay spread. So, we know T s is equal to 1 by B and T d we know this is approximately 1 by B C. B C is the coherence bandwidth.

So, from wireless course we should know that the delay spread is equal to one of 1 by B C. This is coherence bandwidth. This is approximately given by 1 by B C. So, L c becomes from the earlier equations T d greater than T d by T s and T d by T s is nothing but B by B C.

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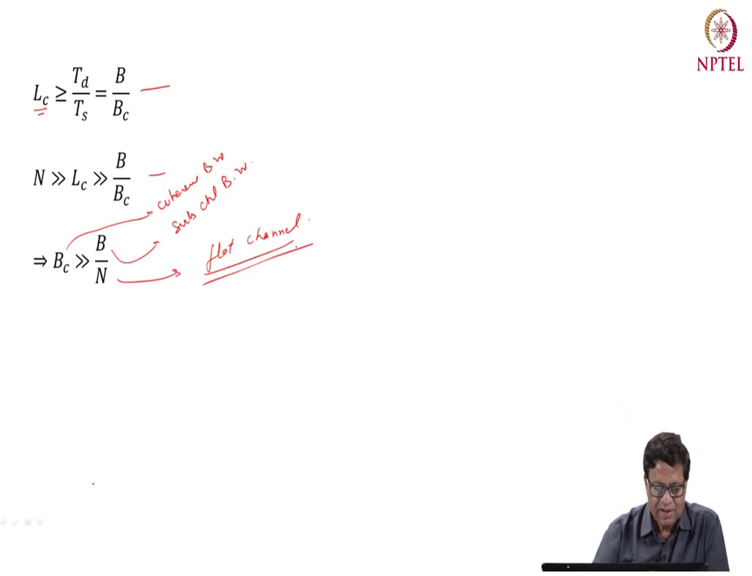

So, this gives me N should be greater than L should be greater than B by B C or the B C should be greater than B by N. Now, let us understand this. So, these two equations have come from whatever I had written in the last slide. So, this B C is the coherence bandwidth and this is your sub channel B by N is sub channel bandwidth. So, if B C is greater than this just sub channel bandwidth then there is no the system is actually flat channel this we have seen. We have come back to the same condition alright.

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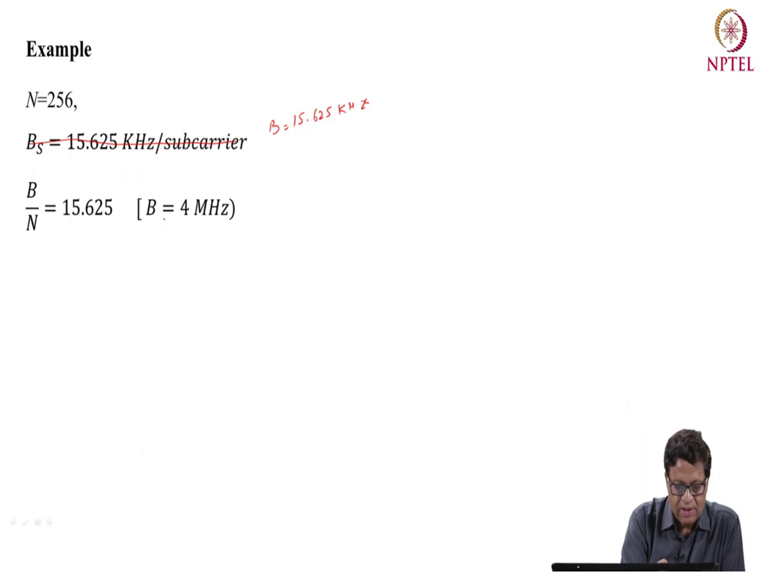

So, let us understand an example if N is equal to 256 that is the number of subcarriers and B s is say 15.625 kilohertz per subcarrier. So, B by N this is B this is actually B this is; B this is not correct. So, B is 15.625 kilohertz. So, B by N or I am sorry. So, this is your B is 4 megahertz.

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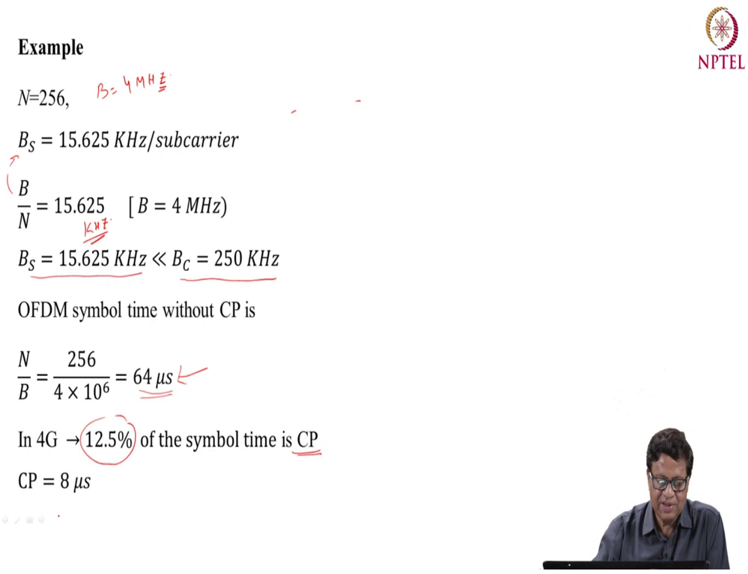

So, B by N will be 4 megahertz divided by N will give you 15.6 kilohertz. So, this is per subcarrier this is what I have written here. So, this becomes your B by N. So, if you see this B S and your B S for a wireless N is 250 kilohertz this B s is much less than B C. So, OFDM symbol time without cyclic prefix will be N by B this is symbol time N is equal to 256 and B here bandwidth is 64 micro second.

So, in 4G for example, 12.5 percent of the symbol time that is of the 64 micro second is kept for CP. So, CP 12.5 percent of 64 micro second will be 8 micro second.

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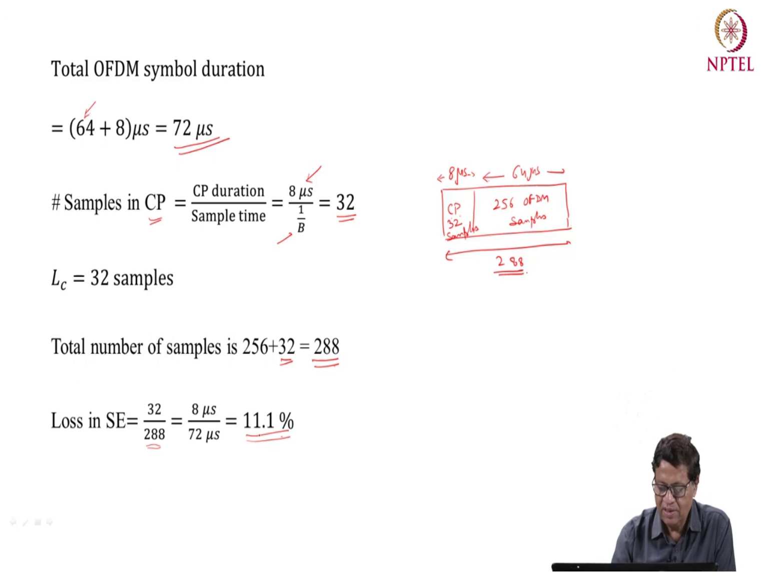

So, the total OFDM symbol duration which will be CP and the 64 which has come earlier. Let us see this 64 is N by B OFDM symbol time without C P. So, this is without C P with C P is 8. So, this is the total OFDM symbol duration which is 72 micro second. And number of samples if you see in C P the C P duration is 8 micro second this is we have seen earlier slides and sample time is 1 by B.

So, the samples in CP which have to be prefixed are actually 32. So, LC is equal to 32 samples and total number of samples will be 256 was without CP and 232 is you know the CP samples the total becomes 288. So, if you want to draw. So, this is your CP. CP this is 8 micro second this this one 8 micro second which we have taken and this was 64 micro second in this example.

And this is 256 OFDM samples and this is 32 samples. So, this is what is. So, this whole thing becomes total number of samples will be 256 plus 32. So, this total will be 288. So, and loss in spectral efficiency will be 32, 32 is the C P sample cyclic prefix and the total is 288 which gives you 11.1 percentage 11.1 percent.

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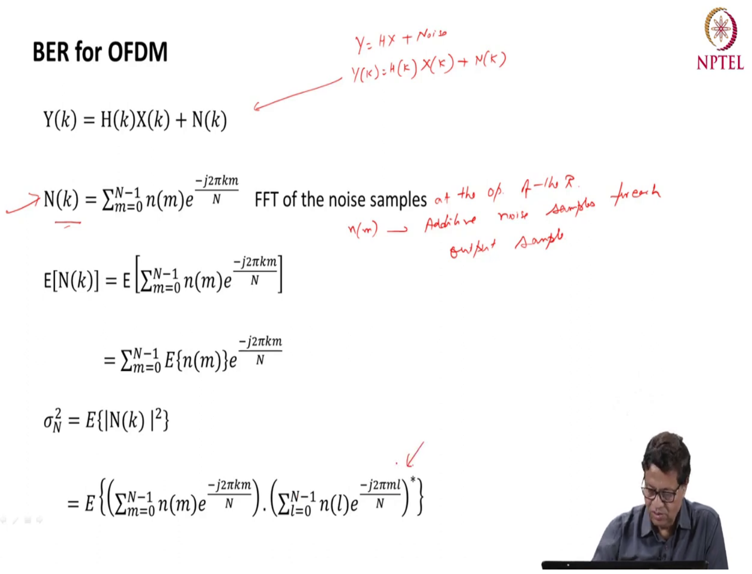

Now, let us try to calculate the BER for OFDM. Bitter a rate for OFDM the we had calculated for baseband. So, we will also calculate for OFDM. So, right now it is nothing to do with optical wireless channel right now I am discussing about the fundamentals of OFDM and how it behaves in RF channel.

So, as we have seen in OFDM Y is equal to H X plus noise and for each sub carrier Y k will be H k X k plus N k. This is what I have read. And N k we know is FFT of the noise samples at the output of the R x where these n m this n m they are the additive noise samples for each output sample. And this is N k is the FFT of the noise sample. Remember we are doing we have done FFT operation in the receiver in OFDM.

So, if you take the expected value of N k of this FFT of the noise sample at the output of the receiver expected value of this. You can take the expected operation here. This is E raise to the power minus j 2 pi k m expected value of n m. And also, if I calculate the variance of this N k the FFT of the noise samples variance of N k whole square will be given by expected value of N k is from here. This into conjugate of this. So, this is a conjugate part. So, you simplify this.

This will give you expected value of double summation n m into n conjugate l e raise to the power minus j 2 pi m minus l K by N. Now, this expected operator can go inside. Summation can come inside. And this can also is outside the expected operator. So, this gives you this gives you because each component n m into n conjugate l will give you sigma N square. This is only valid for you know L is equal to M.

And then this is summed over all the subcarriers which will give you N times sigma N square. So, this is the variance. So, the variance has increased. Its it has become N sigma N square. And let us do the same operation for H k. H k is given by these are the L paths. You know remember we had L type filter.

So, H will have value from h 0 to H L minus 1 into E raise to minus J B 2 pi k m into N. This is actually from this equation. So, I am trying to calculate the FFT end point FFT of H. So, take the square. And so, this is assuming this is these taps these taps. Let us assume that these taps. They are complex Gaussian distribution of mean 0 and variance N 1. So, this will give 1 variance actually and this sum summed over L times. So, 1 plus 1 plus L times we what we will get is L.

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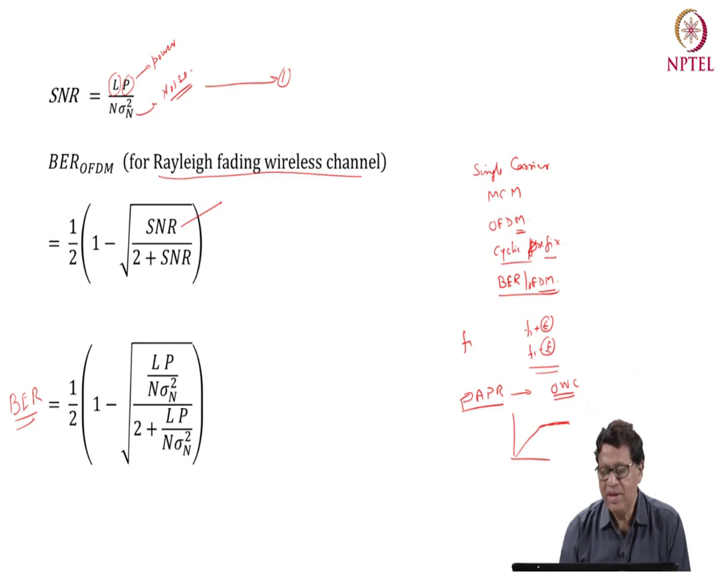

So, this total signal to noise ratio. This is the that H k component L and this is the power. This is the power. And the noise was actually N sigma N square. This was the noise. So, total noise is N sigma square N. And the total signal power is L times P. So, and BER OFDM from many standard textbook. If you assume Rayleigh fading wireless channel, BER OFDM is given by half 1 minus root SNR divided by 2 plus SNR.

Now, I can put the value of this SNR from this equation 1. So, this becomes half 1 minus root L P N sigma N square divided by 2 plus L P N sigma N square. So, this becomes the BER for OFDM. So, this is we have done the, what we have done in this class as understood about starting with single carrier.

Then we started with multi carrier modulation system and then with multi carrier modulation system we went to OFDM. And then we understood the concept of cyclic prefix. How by using OFDM we are able to convert a frequency selective channel into N parallel sub channels which are flat channels. And then we had calculated BER for a OFDM signal, OFDM system.

So, next what we want to study is see the different sub carriers. It is possible that when the sub carriers are transmitted there may be a shift. Suppose F 1 was transmitted and that F 1 during the transmission there may be some shift F 1 plus epsilon or F 1 minus epsilon. So, we want to study the effect of this epsilon. I mean how does it change the orthogonality of the system and how does it change the performance of the system.

This is what we will do in the next class and also, we will discuss about the a problem of PAPR. That is peak to average power ratio which is quite common in OFDM. This PAPR is generally high for OFDM signals and this is not a suitable thing or appropriate thing for our old optical wireless communication systems.

Because high PAPR will give will you know will make the LED system you know go into a non-linear mode because it has only limited you know dynamic range and then it is almost constant. So, high PAPR may pose some problem. So, we are going to study about the frequency offset in OFDM and also PAPR. And then we will move to the next step where we will see how OFDM can be utilized or can be made use of in optical wireless communication systems.

Thank you.