Optical Wireless Communications for Beyond 5G Networks and IoT
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Lecture - 13
Part - 2
Digital Pulse Interval Modulation

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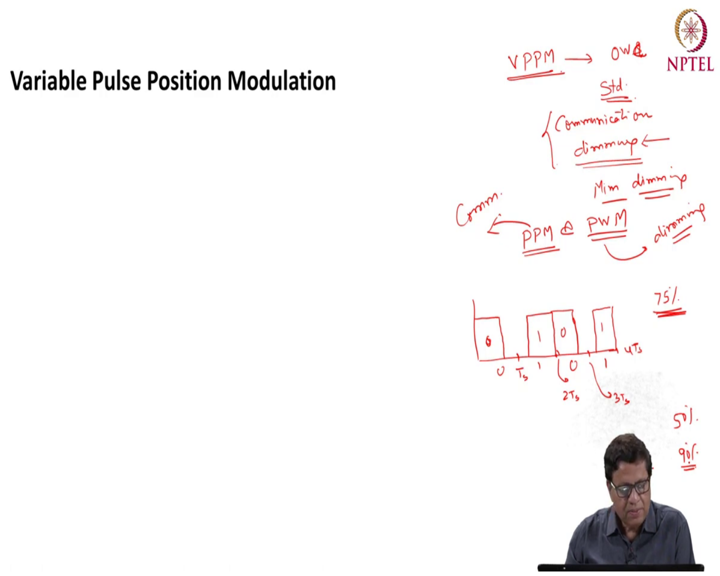

Hello everyone, another modulation which is used as pulse position modulation, PP, a variable pulse position modulation; so, this is also a part of OWC standard this is part of OWC standard. So, when you are using a light as communication then also you need to you should be able to dim the light and even then, the communication should happen.

So, this kind of modulation scheme is for both which can have for both communication and it can meet some dimming requirement also because the LEDs may not be on all the time. So, sometimes you will have to dim it; so, we need to understand or we need to have a modulation scheme which can support dimming.

So, standards have identified this and they have recommended modulation scheme which is good both for communication as well as you should as far as well as dimming. And under dimming conditions your communication should not go your communication should be maintained. Now, we need to understand what is the minimum dimming you can have to get good performance.

So, this is what we will understand in this part of the lecture; so, we will try to do analysis for VPPM; so, first let us understand variable pulse position modulation. So, for example; so, this is actually combination of PPM and pulse position modulation and pulse width modulation which will meet both dimming criteria as well as communication criteria.

So, let me draw a one VPPM pulse to make it more clear; so, suppose I have 0 1 0, 0 1 0 this is 0 1 and this is say T s this is 2 T s and say this is 3 T s and this is 4 T s. And for 0 let me have a pulse which is say cover 75 percent of the time slot and 25 percent is left vacant; so, this is will be taken as 0 and this is actually 75 percent dimming.

I mean your light is down by 25 percent, 75 percent there is a there is elimination there this is what it means. And whenever one is there you do not start from the beginning of T s, it will be starting, because it is also based on pulse position to know for the communication. So, this is for the communication requirement this is communication requirement and this is the dimming requirement.

So, in order to meet both we need to combine these two modulation schemes. So, for 1 it will be like this, this is 75 percent area which is you know it is 1 and 25 percent it is 0. Then again you have 0; so, the pulse will start from this point itself and it will end somewhere here. And then you have 1; so, 1 for 1 again it will start from here and it will be 0 here; so, this is 0 1 this is 0 and this is 1; so, this is 75 percent.

Similarly, you can draw for 50 percent; so, it will be half and you can draw for 10 percent. So, we will see when we analyze the if the dimming is 10 percent what is the performance of the system and if the dimming is a 90 percent then what is the performance of the system. So, this is a variable pulse modulation position modulation; now, let me draw a diagram of the VPPM; so, let me erase this and then; so, let us draw the diagram of VPPM.

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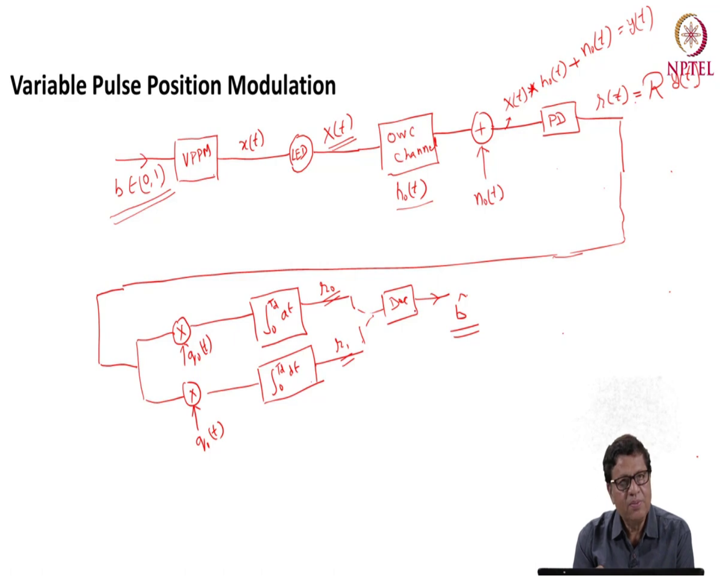

So, this is your input, this is b which either 0 or 1 and this is some binary data and this is some VPPM. And the output signal is x by t and then this LED is modulated this is a LED which is modulated by this signal, this is a LED and the output of this is say x t capital X t. And then you have the channel optical wireless channel, optical wireless communication channel and this has impulse response as h 0 t and then this is the additive white Gaussian noise which gets added; let me write this as n 0 t.

So, the signal out here is X t convolved with h 0 t plus the noise is added; so, this is a signal at this point. And then you have the photodiode at the receiver and then what you get is r t received signal which will be responsibility into y t; so, this I can write as y t. And on the receiver side once you have got received signal r t then it goes to; so, this I will explain what is this q 0 t, this is q 0 t, this is q 1 t.

And then q integrate from 0 to T it is rather T d we will also see the T d is actually the duration of the pulse plus the guard band. So, Td dt and similarly you have dt dt and then you get basically two inputs outputs r 0 r 1 and then there is a decision circuit here which will decide whether it was 0 was transmitted or 1 was transmitted. So, this is the decision circuit; so, this was b here and this will be b estimate; so, we will discuss more about the receiver part little later.

But just to recap this there is a input here which is 0 1 then you have the v P P M as I explained earlier. The output is x t and then there is a LED which gets the x t modulates the LED, we get capital X t and then it this is optical power it goes through a optical channel which is characterized by impulse response is 0 t the noise is n 0 t. So, the total signal will be x t capital X t convolved with h 0 t plus n 0 t; so, that is the y t.

Then it goes to photo detector which has a responsibility of capital of R; so, the received signal r t will be R into y t. And then this signal is actually is multiplied by two basis function these are modified basis function q 0 2 when q 1 this is q 1. And then integrate from t d and t d consist of the slot duration as well as the guard, after each pulse you have some guard time as well.

So, t d consist of the pulse duration and the guard and then you get r 0 and 1 then depending upon your decision circuitry you select either r or r 1, r 0 or r 1. Or a difference between r 0 and r 1 can be seen and whether the difference is more than 0 or the difference is less than 0 depending upon that you decide what was your b which was transmitter; so, this is the block diagram of variable pulse position modulation.

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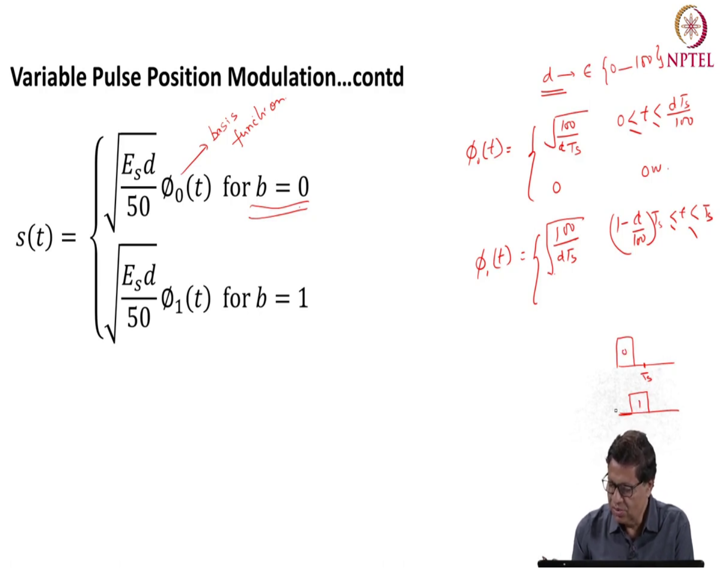

So, the s t which is actually transmitted is given by root s is the energy in the pulse and d is the dimming range and the d value can change from or is 0 to 100; so, this is this actually characterizes the dimming value. So, s t which is transmitted for b is equal to 0 it will be E s d into 50 and these are the basis function and this basis function depend upon the value of d.

So, these are the basis function and which is which depends on the value of d. And let me define this basis function phi 0 t is 100 over d T s when 0 it is 0 otherwise and the basis function phi 1 t the other function which characterizes the 1 will be 100 over d T s and this is between 1 minus d 100, T s less than t T s. So, these are the basis function as you see they are function of d, the dimming value which was from 0 to 100.

So, phi 0 t root 100 by d T s in this time frame this is we also saw the waveform for VPPM where the waveform was you know for 0 it was starting from like this and this was the T s and depending on the value of d. So, I had assumed the case of 75 percent and for 1 it is something like this; so, it starts late and ends at 0; so, this is for 0 this is for 1.

So, this is how the basis functions are defined for different value of dimming range phi 0 2 and phi 1 t, and this is the s t the transmitted signal corresponding to b is equal to 0 and b is equal to 1.

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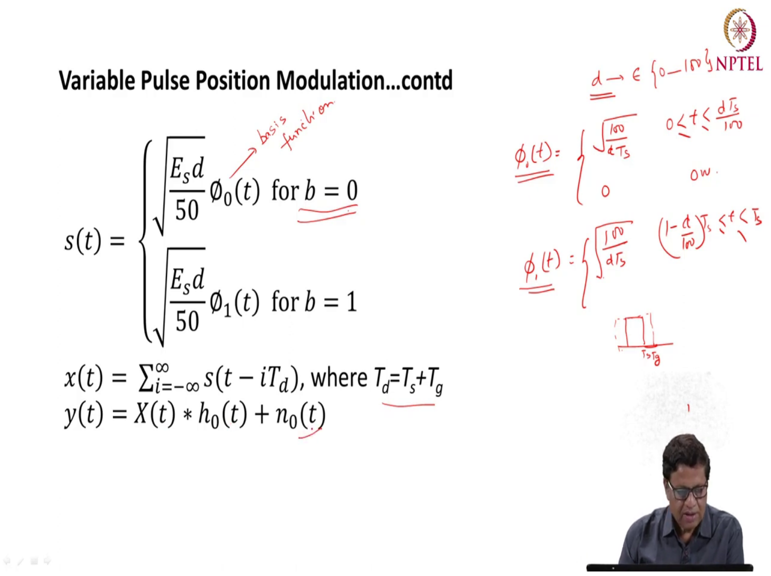

So, the total signal x t which is going to be transmitted will can be written as s t minus i T d where T d is the T s plus d g T g. So, suppose you have this is your T s, this is your T s and then you add some T g here guard band; so, this becomes your total duration T s plus T g. So, this is the transmitted signal and from my earlier diagram as I explained that y t is equal to X t capital x t which is coming after the LED convolved with h 0 t impulse response plus the noise.

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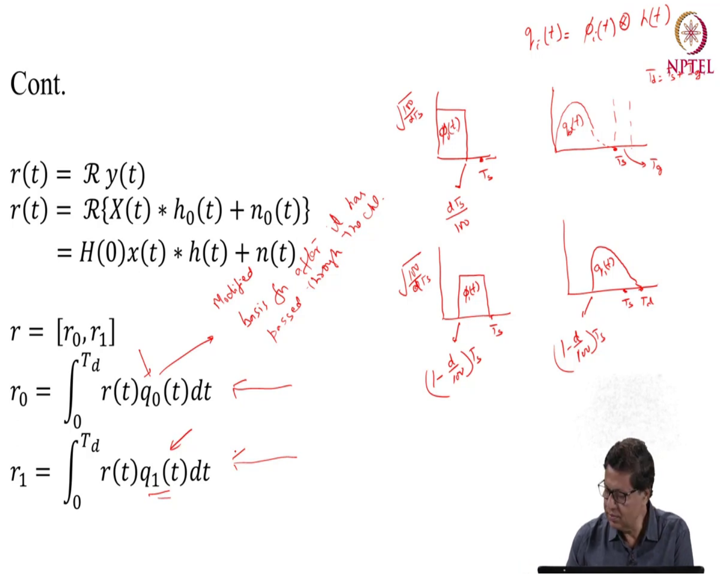

And r t will be whatever y t multiplied by the responsibility of the photodiode which is R into y t; so, that becomes the received signal. And received y t we can put y t which is x t into channel impulse response plus noise; so, this gives you a d c gain or if you multiply this becomes d c gain h 0 into x t which is nothing but responsibility into x t into convolved with h t and plus n t.

So, you get two values of r's r that is r 0 and r 1 which was there in the diagram. So, this is r 0, this is r 1 after you know multiplying with the modified template of the basis function. Because this basis function will also be corrupted by the noise and the impulse response which will become q 0 t and q 1 t instead of phi 0 t and phi 1 t and then you integrate over the whole full period 0 to t d you get a value of r 0 and r 1.

So, there are two possibilities you can subtract these two signals and see whether it is greater than 0 or less than 0 and take a call whether b is equal to 0 was transmitted b is equal to 1 was transmitted. So, r 0 is can be written as this from the block diagram 0 to td that received signal multiplied by the modified or modified basis function.

So, this is basis function rather modified basis function after it has after it has passed through the through the channel. So, r 0 is equal to given by this and similarly r 1 is given by 0 to T d r t q 1 t dt; so, these are the two r 0 and r 1's we have got. And now let us see let us draw the basis function and see how this q 0 and q 1 look like which is actually if you see q i t is nothing but that original basis function convolved with the impulse response; so, this is a modified; so, let us see how it look like.

So, this is my basis function for the say phi 0, this is for phi 0, phi 0 t and this is here T s and the energy of this basis function is equal to 1. So, this amplitude will be 100 divided by d T s and this is d T s d is the dimming range d T s divided by 100. So, this is phi 0 2 t and if you convolved with h t it look something like this, because it is a dispersive channel.

So, this will something this is T s here something like this and then you have this is the T g part guard band which I have kept here T g and the whole total duration is T d T d is equal to T s plus T g. So, this is q 0 t, this is the template which I have got which is used here and similarly I can write for phi 1; so, the phi 1 this is this is T s.

So, phi 1 will be this is phi 1 t and this is same as 100 over d T s and this is nothing but 1 minus d by 100 T s. And the modified basis function will be; so, let me first draw this is T s and this will be T g and this will be T d; so, this will be something like this. So, this is q 1 t and these points are same 1 minus d by 100 T s and this is the T d.

Because I have kept the guard band; so, even if there is some signal going there; so, it is in the guard band only it is not affecting the next symbol. So, this is how r 0 and r 1 which I have received which will be used for decoding.

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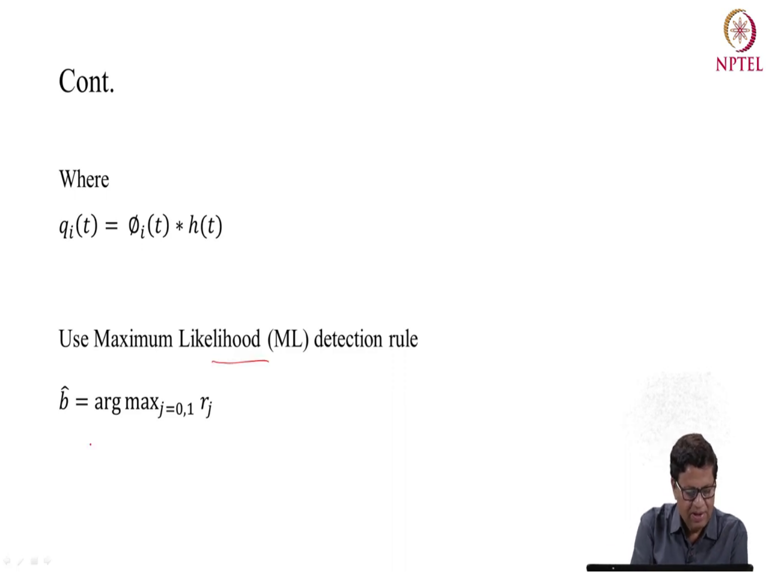

So, as I mentioned this q i t is phi t original basis function convolved with h t. So, and let us use maximum likelihood detection scheme for estimating of the transmitter signal that is b estimate, the maximum value of r j; where, j is equal to 0 and 1.

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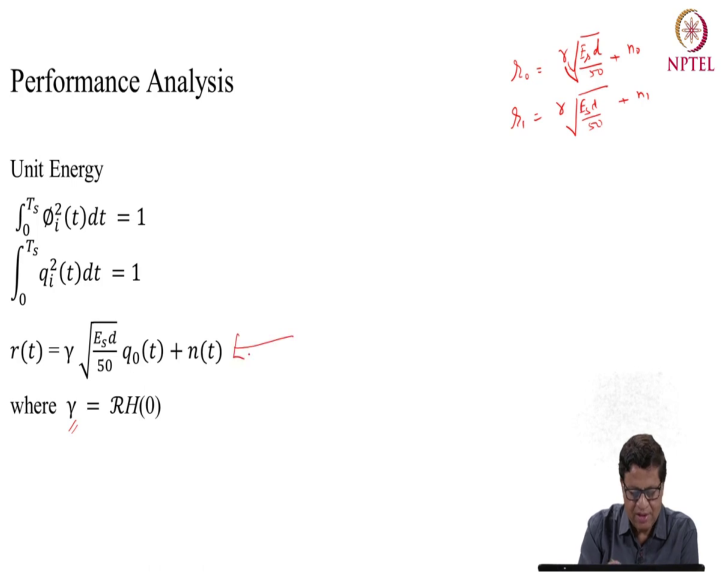

So, let us now do the performance analysis for such a system, how the performance varies with respect to the dimming range; so, these basis functions they are all unit energy that is 0 to T s phi i squared t dt is equal to 1. Similarly, the modified template or modified basis function is also the energy unit energy remains the same and the r t is given by gamma.

So, I have combined the h 0 the gain and the responsibility into a single factor gamma. So, my r t which is received is gamma root E s d by 50 q 0 t plus n t and this actually has come from r 0 was equal to gamma E s d over 50 plus n 0 and similarly r 1 was gamma; so, this is whole 50 E s d over 50 plus n 1.

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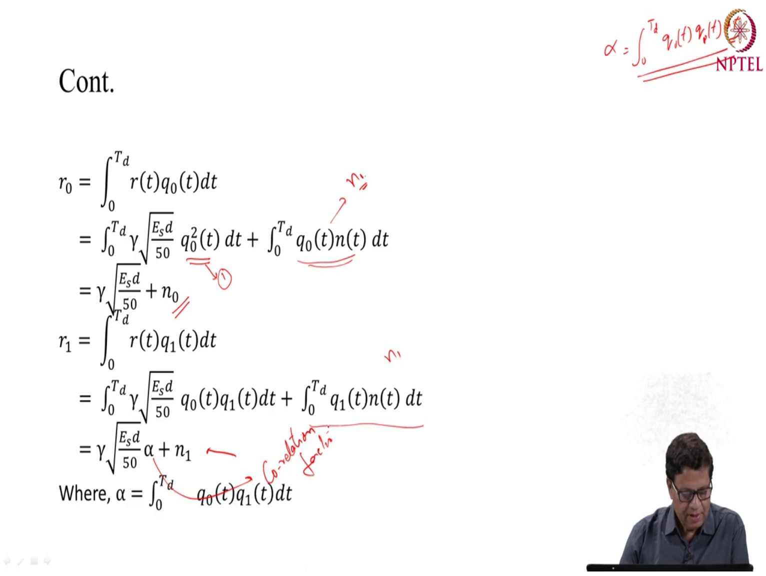

So, r r 0 is equal to 0 to T d t as this is we have seen earlier; so, if I put the value of r t here which I had given just now here r t here this one. Then I get 0 to T d this gamma which is the responsibility and the channel gain; root E s d by 50, this is q 0 square and this is q 0 t and t. So, the noise also gets modified; so, this becomes the new noise q's integrate integration from 0 t d q 0 t into n t.

And this can be written as n naught because this energy is 1 this is unit energy basis function. So, this is 1 when you integrate from 0 to T d and this is the new energy with new noise, I have got n 0; so, this expression for r 0 becomes gamma root E s d divided by 50 plus n naught. Similarly, I can calculate the value of r 1 the other received signal which is multiplied by another basis function which is q 1 t; so, this becomes r t q 1 t and integrated from 0 to t.

So, this will give you putting the value of r t here; so, you get this is n 1 here and this I call this as alpha. So, my alpha I have introduced one term which is alpha and this alpha is nothing but 0 to T d which is a correlation factor q 0 t, q 1 t d t So, this is a correlation factor, this is an important factor that we will understand the implication of this later; so; so, this becomes gamma root E s d by 50 into alpha alpha is a correlation factor.

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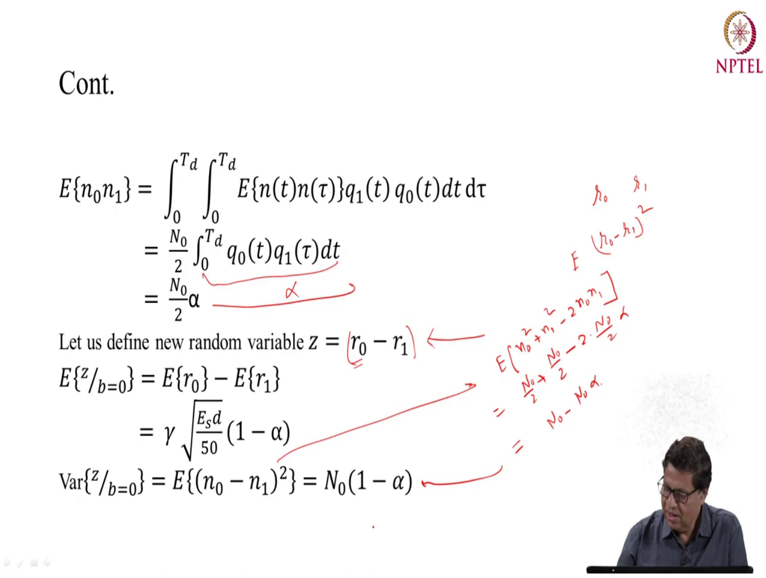

So, now let us try to calculate what is expected value of these n 0 and n 1 which I have got. So, putting the value of n naught which is n t q 1 t d t integrate integrated over 0 to t d and similarly n 1 and I am trying to calculate the expected value of this. So, expected operator is here; so, this will give me because the expected value of E n square n t is n 0 by 2 which will come out and then you are left with 0 to t d q 0 t into q 1 which is nothing but alpha this is alpha.

So, this whole thing becomes n 0 alpha by 2; so, this is n 0 n 0 by 2 into alpha; so, this is the expected value of 2 noise. Now, as I mentioned earlier let us define a new random variable z which is actually a difference between r 0 and r 1, the 2 received vectors. Then the expected value of z when b 1 b is equal to 0 was transmitted assuming b is equal to 0 is transmitted will be expected value of r 0 minus expected value of r 1.

And if I put expected value of r 0 from my earlier expression and expected an r 1 from the earlier expression then I get gamma into E s d is the dimming divided by 50 1 minus alpha. So, this is when b is equal to 0 was transmitted; so, this is the expected value of z when z is defined as r 0 minus r 1. Also try to calculate the variance of this z; so, variance of this z will be you one knows about r 0 and r 1 is known.

So, basically you have to take r 0 minus r 1 whole square, put the value of r 0 here and r 1 here square it and then take the expected value. So, there will be some constant value which for which the expectation operator does not mean anything. So, you will be left with expected value of n 0 minus n 1 whole square and expect and for calculating this basically you have to expand this.

So, this will be n 0 square plus n 1 square minus n 0, n 1 and expected value this expected value can go inside. So, this will give you expected n 0 square is N 0 by 2 this will give you also N 0 by 2. And this we have calculated just now which is 2 into n 0 2 by alpha which is calculated here. So, this is N 0 by 2 alpha; so, what we get is N 0 minus N 0 alpha which is nothing but N 0 into 1 minus alpha.

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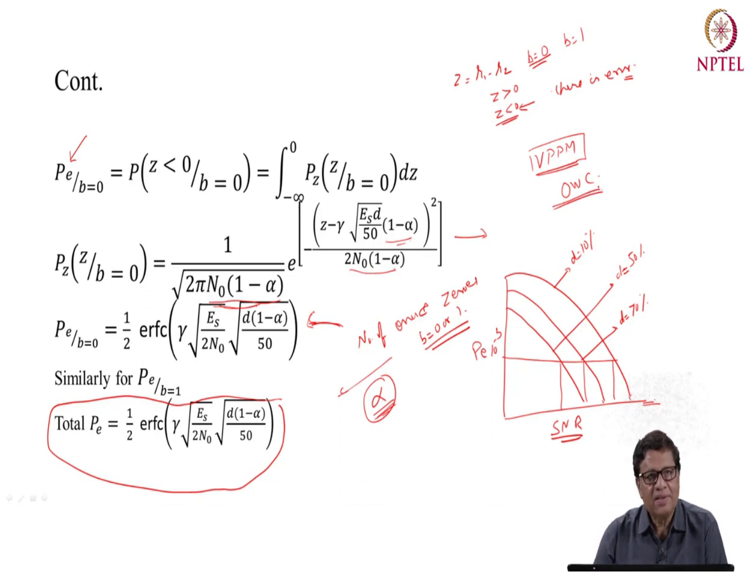

So, now I am in a position to find out the probability of error when b is equal to 0 was transmitted. So, b is equal to 0 will be when the z that is a difference between the r 1 and r 2 that is less than 0 is less than 0 the z was r 1 minus r 2. So, either z can be greater than 0 or z will be less than 0 right; so, if you transmit for example, b is equal to for example, 0.

So, maybe it will correspond to and if z is less than 0, suppose you transmit b is equal to 0 and z is less than 0 then there is a error right. And if you transmit b is equal to 1 and z is greater than 0 there is an error; so, this is how you will calculate both these errors. So, probability of error z less than 0 when b is equal to 0 was transmitted this will be given by minus amplitude 0 p z and we already have calculated the mean and the variance of z the difference r 1 minus r 2.

So, this will give me the value of probability of error and this can be written because I know the value of mean and I know the value of variance sorry mean is somewhere here and this is the variance part this is the variance part. So, this is Gaussian distribution function; so, this is the value of probability of error when b is equal to when b is equal to 0 was transmitted.

And similarly, you can calculate I mean this can be converted into q function, this we have done when we were doing on off-key how to convert this Gaussian into other complementary function or q function. So, this can be converted into error complementary function which is given by this.

And similarly for P probability of error when b is equal to 1 was transmitted, we can on the same you following this doing the same analysis we can find out this probability which will be actually same as this and you add this total probability.

So, total probability and assuming that number of 1's and 0's that is b is equal to 0 or 1 they are same under that condition you can submit and the total probability error will be given by this. So, as you see this probability of error actually depends on alpha which is the correlation factor and which actually depends on the noise whether 0 was suffered with noise or 1 suffered with more noise.

So, this alpha the total probability of error is actually function of alpha and if I plot this probability of error for example, this is probability of error and I take this SNR here I mean this E s by n naught is some sort of SNR. So, if I take SNR and if I plot for different values of d; so, this will be d is equal to 50 percent and this will be d is equal to 10 percent and this will be d is equal to 70 percent.

So, actually for d is equal to 50 percent when the dimming is 50 percent, it gives you best result as compared to d value which is more than 50 percent or less than 50 percent. Because, if it is less than 50 percent the received signal is very very low because the dimming is high. So, you hardly receive anything after the attenuation; so, the performance goes down

And on the other side if the d is 70 70 percent or 75 percent, then the this value of alpha actually increases; so, because of this correlation factor there is again a degradation. So, the degradation happens on both sides of 50 percent and 50 percent gives you the maximum on one side is amplitude other is on the other side it is the correlation factor.

So, this is how one can see you know suppose your requirement is 10 to the power minus 3, then these are the different SNR's which are required for 10 to the minus 3 for if you are for different dimming ranges. So, this is a VPPM that is Variable Pulse Position Modulation which is actually a part of standard for optical wireless communication systems which will supports both communication as well as dimming.

So, we will stop at this point for baseband modulation techniques. And next we will go to multi carrier modulation techniques which are called which is called as MCM. And we will also study other forms of MCM like OFDM optical orthogonal frequency division multiplexing etcetera.

So, thank you very much.