

Optical Wireless Communications for Beyond 5G Networks and IoT
Prof. Anand Srivastava
Department of Electronics and Communications Engineering
Indraprastha Institute of Information Technology, Delhi

Lecture - 13
Part - 1
Digital Pulse Interval Modulation


Hello everyone, so as you recall last time we had studied about on-off-key, we had studied about pulse amplitude modulation, we also studied about pulse position modulation and we did extensive analysis and we understood these modulation techniques with respect to bandwidth efficiency and power efficiency.

Today we will do another type of or try to understand another type of modulation scheme which can be used in optical wireless communication which is called as digital pulse interval modulation. Digital Pulse Interval Modulation which is also called as DPIM.

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Digital Pulse ^{Interval} Modulation (DPIM)

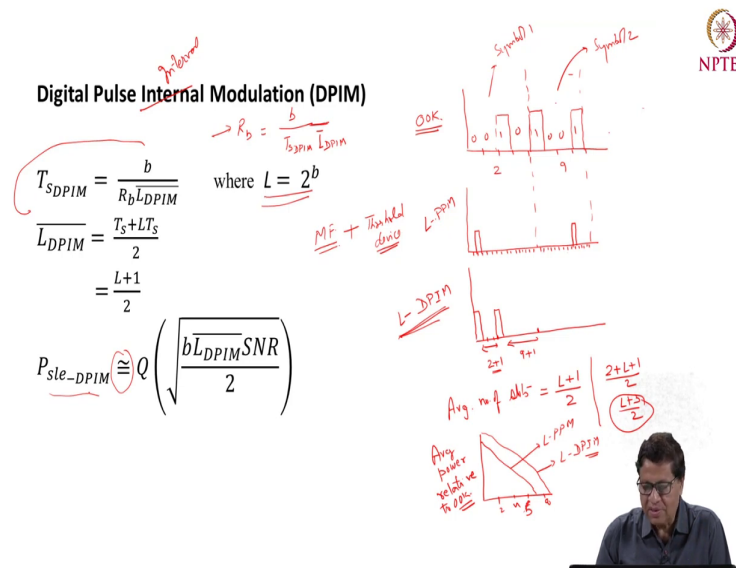
PPM → Better PE
But ↓
Not / Symbol Synchronization



So, recall that we studied PPM earlier which has better power efficiency, but bandwidth efficiency was poor; bandwidth efficiency was poor and also PPM required some slot and symbol synchronization. So, these were the issues with the PPM. Now, we will understand a scheme which is digital pulse interval modulation which will give which can give higher transmission capacity and also synchronization is not required because you know there is always a one when you start a symbol.

So, that itself is in synchronization symbol orbit. So, let us first understand the DPIM how it is different from normal OOK and normal PPM. So, if I take OOK signal which is say. So, let me draw area OOK signal first.

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So, I am drawing OOK and I am taking slots like this. So, this is for example, one symbol and say suppose this is one others are 0 0 1 0 the other one is 1 double 0 1. So, let us take this other another symbol as 1 0 0 and maybe we can take one here. This is another symbol this is symbol 1, this is symbol 2 this is symbol 2 this is symbol 1.

So, this is normal on off key and now if you see the decimal value of this symbol 1 is 2 and decimal value of the symbol is 9 and now let me draw the first the PPM and then we will draw the DPIM. So, the PPM will be this is L PPM. So, in this case it will be 16 PPM because there are 4 bits in a symbol 2 raise to power 4 PPM.

So, this is a symbol 1 and then you have to divide this whole thing into 16 parts. Similarly, this so this is decimal 2. So, you will have a pulse after 2 slots, so you will have a pulse here. This is decimal 9, so you will have a pulse after 9 slots.

So, let me count again 1, 2, 3 somewhere here. So, this is the case of L-PPM and now let us try to draw DPIM how this is L-DPIM how this digital pulse interval modulation look like. So, so the decimal value is 2, so first it starts with a pulse and then after 2 slots the another symbol will start, but normally you keep a guard band between the 2 symbols. So, it will start after say 3 pulses.

And then the second is 9 the symbol value is 9. So, or then it will be 9 plus 1 pulses that is 1 is for the guard band. So, this will be somewhere here. So, this is actually 2 slots because the symbol value is 1 and 1 you keep for the guard band and similarly this will be 9. So, this will be 9 plus 1, 9 empty slots and 1 for the guard band.

So, this is how L dash DPIM look like and if you see the average value of slots in average number of slots will be if you do not give any guard band then there may be issue of inter symbol interference, then the maximum value can be L the minimum value can be 1 time slot. So, the average value will be 2 and if you keep a guard band the minimum value will be 2 T_s that is 2 plus L plus 1. So, this is L plus 1 and then divided by 2.

So, this is in case of when you keep a guard band after every symbol, so this will be L plus 3 by 2. So, this is average number of slots and if you see the power the average power in L-DPIM is going to be more as compared to L-PPM. So, there is there will be higher average power in case of L-DPIM because pulses are coming frequently quite frequently as compared to this. So, if you take the average power will be high, but it can give you a high transmission capacity or high transmission rate.

So, and also if you see the average power if I plot this is a number of bits 2, 4, 6, 8 and if I plot for PPM this is average power. So, for PPM L dash PPM will be something like this and for L-DPIM will be something like this. So, this is L-PPM and this is L-DPIM. So, the

average power this is relative to OOK average power relative to OOK you always take some reference. So, in this case we have taken the reference of on off key.

So, now let us see what is the bit rate for a DPIM. So, the bit rate for a DPIM R_b will be B that is the number of bits per symbol divided by the time slot D_s let me qualify this as DPIM into the average number of time slots in a symbol. So, which is actually L average DPIM. So, this is the rate R_b and I can write this in this form R_b can come down and so the time slot is equal to b divided by R_b into average length of average length of slots number of slots in a symbol and here L is equal to $2 M b$.

So, this is what I had explained the average number will be one time slot is the minimum in the case of L-DPIM and the maximum can be for the highest value of decimal number it will be $L T_s$. So, the average will be 1 plus L divided by 2 if you are not considering any guard band or guard slot between the symbols. So, if you consider that then it becomes L plus 3 by 2 .

So, now this composite signal the L-DPIM can be fed to a matched filter and we can have some threshold device in order to recover empty slot or the slots which or the slots which are full. So, as we had calculated the slot error probability in earlier cases on the similar fashion, we can calculate the slot error probability for DPIM.

And here I have assumed that number of 1s and number of 0s are same which actually is not the case in DL DPIM because if they are same then I can have the threshold exactly in the middle, but here the number of empty slots and number of slots they vary. So, the optimum value of threshold will not be exactly in the middle it will be somewhere else depending upon how many else empty slots you have and how many full slots you have.

So, but this I have taken an approximation here this is an approximation value of slot error probability this is Q function again you have to recall from our earlier discussions. So, this will be root of b average value of number of slots in a symbol divided into SNR by 2 this is the probability of error for DPIM.

The bandwidth now we have seen the transmission rate has increased for DPIM, but at the cost of poor power efficiency. So, depending upon the situation depending upon the application you one can select this modulation scheme also for an optical wireless communication systems.

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NPTEL

Differential Amplitude Pulse Position Modulation (DAPPM)

$M = \log_2(A \times L)$

$s(t)_{DAPPM} = \sum_{k=0}^{\infty} \left(\frac{P_p}{A} \right) a_k p(t - kT_s)$

$a_k \in (0, 1, 2, \dots, A)$


Handwritten notes:
 \rightarrow for L pulses
 \rightarrow a_k amp levels

Handwritten diagrams:

For $L=4$, $A=1$ (PAM):
 $\Delta_1(t)$, $\Delta_2(t)$, $\Delta_3(t)$, $\Delta_4(t)$

For $A=2$, $L=2$ (PAM):
 $\Delta_1(t)$, $\Delta_2(t)$

$M = 2 \text{ bits/Symbol}$
 $2 \rightarrow$ different waveforms



Now, let us go to another type of modulation scheme which is called as differential amplitude pulse position modulation. So, this is sometimes referred as DAPPM. So, DAPPM is actually this combination of two modulation schemes. So, in this case you have combination of PAM and PPM.

So, we will see the advantage of such a scheme. So, DAPPM is combination of pulse amplitude modulation and pulse position modulation. Now, let us try to understand how a symbol will look like in this. So, suppose I take L-PPM where L is equal to 4 and I take PAM

where I use only one amplitude. So, that is I use A is equal to 1, so I use only one amplitude we will see when the amplitude is more than 1.

So, there will be 4 types of wave form for L is equal to 4 and A is equal to 1. So, let us divide this into 4 parts. So, this will be 1, this will be another this will be. So, this is as 0 t s 1 t, s 2 t, s 3 t. So, there is one amplitude A which is a you can say this is P , this is P , this is P and this is P . So, this is L is equal to 4 and A is equal to 1. So, it is actually similar to LPPM, but now let us try to have A is equal to 2 and L is also is equal to 2.

So, I am trying to make a DAPPM for this. So, let us once again draw the waveforms for this combination. So, these are S_{02} , S_{01} . So, this will be 1 level. So, let this be P by 2 instead of P and this could be another P by 2 and this is say P this is another level and this is. So, I have used both position and amplitude to define my symbol. So, this is A is equal to 2 and L is equal to 2 and there are in both the cases M is equal to 2 bits per symbol.

So, this will have high transmission capacity and so there are actually 2 raised to power M disjoint waveforms. So, M will be $\log_2 A \times L$ where A is the number amplitude and L is as defined by the slot that is the positions in the symbol. So, M is equal to $\log_2 A$ into L . So, if I take this case, so this becomes $\log_2 2$ into 2 that is $\log_2 4$ $\log_2 4$ is 2. So, M is equal to 2 bits per symbol and similarly in this case $\log_2 4$ into 1 which is $\log_2 4$ which is again M is equal to 2 bits per symbol.

So, the composite signal which is transmitted using DAPPM scheme digital amplitude pulse position modulation scheme is given by this expression where this is the peak power these are the number of the levels number of amplitude levels and a k can have value from 1 to A or 0 is written here a k can have value from 0 to A and this is unit pulse and this is a time slot.

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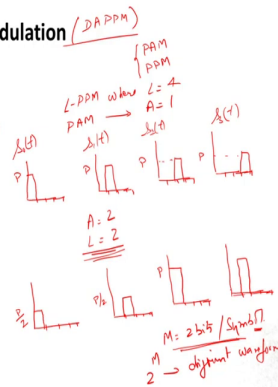
Differential Amplitude Pulse Position Modulation (DAPPM)

$$M = \log_2(A \times L)$$

$$s(t)_{DAPPM} = \sum_{k=0}^{\infty} \left(\frac{P_p}{A} \right) a_k p(t - kT_s)$$

$$a_k \in (0, 1, 2, \dots, A)$$

$$\overline{L}_{DAPPM} = \frac{L+1}{2M}$$



So, this is the composite signal which is transmitted using DAPPM and if I assume that the average number of pulses or the slots in case of DAPPM will be $L+1$ divided by $2M$. So, 1 is minimum L is maximum divided by 2 and if I consider that all the M is equally likely, then the average value becomes $L+1$ divided by $2M$.

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Cont.

$P_{se} = p^0 Q\left(\frac{\theta_1}{A} SNR\right) + p^A \sum_{i=1}^{A-1} \left[Q\left(\frac{(i-\theta_1)}{A} SNR\right) + Q\left(\frac{(\theta_{i+1}-i)}{A} SNR\right) \right] + p^A Q\left(\frac{(A-\theta_A)}{A} SNR\right)$

$(\theta_1, \dots, \theta_A)$

$PER = 1 - (1 - P_{se})^{L_{DAPPM} D/M}$

$\cong \frac{L_{DAPPM} D P_{se}}{M}$

$r_k \rightarrow \text{received signal}$

$\hat{a}_k = 0 \text{ if } r_k < \theta_1$

$\hat{a}_k = i \text{ if } \theta_{i-1} < r_k < \theta_i$

$\hat{a}_k = A \text{ if } r_k > \theta_A$

$i = 1, \dots, A-1$


$D \rightarrow \text{pkt length}$

$(1-x)^a$

$1-ax$

$PPM, DPTM, DAPPM$

Packet error rate



So, now let us try to calculate the slot error probability for DAPPM. So, this is the expression for slot error probability. So, as you notice there are three expressions; one of them is this where now let us see how you basically decode the signal. So, the received signal for example, is say r_k right. So, if a_k is 0 it will be 0 if and only if the received signal r_k is less than some threshold θ_1 which is θ_1 is a threshold and these are different thresholds for different amplitudes.

So, the received signal will be estimate value of a_k will be 0 if and only if r_k the received signal this is a received signal is less than some θ_1 which is here. And this a_k the estimate of received signal a_k is i if and if the r_k lies between θ_{i-1} plus 1 and θ_i and the third cases the estimated value will be taken as A if and only if the r_k the received signal is greater than θ_A and here i goes from 1 to A minus 1.

So, these are the three decision criteria for decoding the signals. So, this p_0 is the probability that it is an empty slot and it is given by our standard equation in terms of Q function which is $\frac{\theta}{A \sqrt{\text{SNR}}}$ and the second is that the received slot has amplitude A this is let me write a probability that slot is empty this is probability that slot of amplitude A probability that slot is of amplitude A.

So, this first expression is because of the empty slot which is actually comes from this condition, the second comes from this condition second one which is given by Q again this is $\frac{1 - \theta}{A \sqrt{\text{SNR}}}$ and $\frac{\theta + 1}{A \sqrt{\text{SNR}}}$ this is in terms of this is for the second one and for the third one this one a k is equal to A if and if the received value is greater than theta if the this is a probability that there is a slot and A minus theta a divided by a SNR.

So, these are the three terms which are added to give you the probability of slot error a in case of DAPPM and if I want to calculate the bit error rate or the packet error rate because symbol has slots and which are you know empty slots as well as there may be a sequence there of one sequence of 1 and 0 in a symbol and if I assume that packet length is D this is my packet length because if one chip is in error the whole packet is in error. So, this is packet length.

So, packet error rate will be given by $1 - p_e^D$ raised to power the average value of chips in the symbol into the packet length divided by M. So, we can use some approximation formula you know p_e is very small. So, this the power can come I mean the I have used this formula $1 - x$ if x is very small say this is say a. So, this can be written as $1 - ax$. So, this is what I have used after using this approximate value is L average DAPPM into packet length into probability of slot error divided by M.

So, that becomes the packet error rate. So, this is the packet error rate. So, if you see the average power requirement that we can maybe in the next maybe we will we can compare different schemes like you know PPM and we have also studied about DPIM we also studied about just now digital pulse DAPPM Digital Amplitude and Pulse Position Modulation.

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Mapping of 3 bits OOK words into PPM, DPIM and DAPPM



OOK	PPM (L=8)	DPPM (L=8)	DAPPM (A=2,L=4)	DAPPM (A=4,L=2)
000	10000000	1	1	1
001	01000000	01	01	01
010	00100000	001	001	2
011	00010000	0001	0001	02
100	00001000	00001	2	3
101	00000100	000001	02	03
110	00000010	0000001	002	4
111	00000001	00000001	0002	04

1, 2 → Am

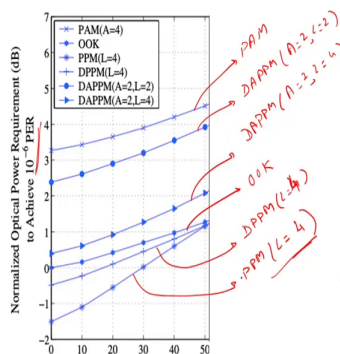


And this next this slide actually tells you how the sequence will look like this is for 3 bits. So, on OOK you will have from 000 to 11, PPM will have 2 raised to power 3 8 combinations. So, starting with 1 all 0s to all 0s and 1 this is DPIM LDPIM. So, you will have less number of bits per symbol and DAPPM I have taken two examples here where A is equal to 2 and L is equal to 4 and DAPPM another example where A is equal to 4 L is equal to 2.

So, these are the values and this 1 and 2 actually this and 1 and 2 they are the amplitude in this case and in this case the amplitudes are there are more number of amplitudes 1 there are 4, 4. So, this is another case. So, these are the waveform which will look like for different modulation schemes PPM, DPPM and DAPPM of two different varieties. So, a well designed DAPPM will give you a high transmission capacity, but the average power requirement is actually high.

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Average power requirement PAM, OOK, PPM, DPIM and DAPPM versus bit rate R_b on a dispersive channel.



So, if you want to see a comparison of optical power requirement to achieve a 10 raised to the power minus 6 BER for different schemes for example, this is PAM which has highest optical power requirement and this one is DAPPM with A is equal to 2 and L is equal to 2 and this one is DAPPM A is equal to 2, L is equal to 4.

So, the power requirement is lower as compared to PAM, but it is higher as compared to other modulation schemes for example, this modulation schemes which is OOK and this modulation scheme is DPPM and this is L is equal to 4 and this is the PPM L is equal to 4.

So, the power requirement is minimum in case of PPM and the power requirement for example, in DAPPM of different kinds is higher than PPM, but it can it has a high transmission capacity and it is definitely less than PAM. So, depending upon the power

requirement and what kind of transmission capacity you want what kind of bandwidth efficiency you want one can select a module specific modulation scheme.

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BANDWIDTH REQUIREMENTS, CAPACITY AND PEAK-TO-AVERAGE POWER RATIO (PAPR) OF PPM, DPPM AND DAPPM, WITH M BITS/SYMBOL.



Modulation scheme	PPM	DPPM	DAPPM
Bandwidth requirement (Hz)	$\frac{2^M R_b}{M}$	$\frac{(2^M + 1) R_b}{2M}$	$\frac{(2^M + A) R_b}{2MA}$
Capacity	M	$\frac{M 2^{M+1}}{(2^M + 1)}$	$\frac{MA 2^{M+1}}{(2^M + A)}$
PAPR	2^M	$\frac{2^M + 1}{2}$	$\frac{2^M + A}{(A + 1)}$

Capacity =
No. of bits
which can be
transmitted during
the time required
to transmit M bits
Using OOK mod.

Peak to Average
Power ratio



So, this slide basically compares three factors one is what is the bandwidth requirement for different schemes and what is the capacity let me define here capacity this is number of bits which can be transmitted; which can be transmitted during the time required to transmit M bits using OOK modulations. So, these are with reference to OOK modulation.

So, the number of bits which can be transmitted during the time required to transmit M bits using OOK. So, this is the definition of capacity and here the capacity has been calculated and these are the bandwidth requirement where you know capital M is number of bits per symbol R b is the rate a is the amplitude in case of DAPPM and then we have also given a

comparison of peak PAPR; PAPR is peak to average power ratio sometimes this becomes critical because the LEDs which are used they have limited dynamic range.

So, if your peak to average power is high then there may be some sort of degradation in the performance because of the nonlinearity in the characteristic of the LED. So, PAPR becomes important and there are techniques for reducing PAPR that we will understand in subsequent lectures.

So, these are the three main things which need to consider bandwidth requirement and the capacity and the PAPR and it has been given here. So, depending upon the value of MNA you can play with these modulation schemes and depending upon the application you can select the suitable modulation scheme for your application.