Optical Wireless Communications for Beyond 5G Networks and IoT

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Lecture - 12
Part - 2
BER of M-PPM, BER of L-PPM

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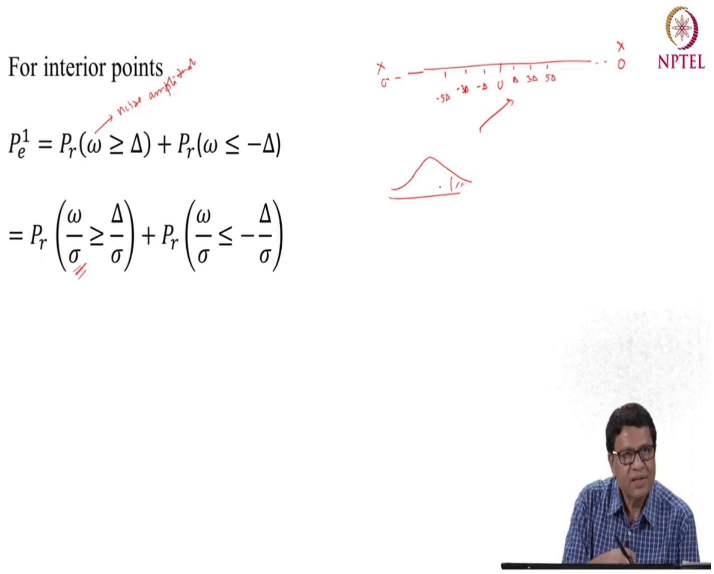

Now, as I let me again draw this. This is 0, this is say delta, this is you know 3 delta, this is 5 delta and so on and so forth, minus delta, minus 3 delta, and say minus 5 delta and the end there, end points here. So, there are two types of constellation points here. One is you know the points which are inside that is you exclude the point in the last at the either end. This other points are same. So, if the noise affects any constellation point plus minus delta, this way or that way, there will be an error.

It is different for end points. So, we will separately handle the end points. But right now, I am trying to calculate the probability of error for constellation points which are in the mid in the interior that is excluding the end points all points are actually similar. So, you can calculate for one and then you can get the total probability of error. So, I am trying to calculate for interior points which I called as probability of error 1.

So, the probability will happen when the noise variance is greater than or equal to delta or it is plus it is less than minus delta to. So, you pick up any point here in this constellation diagram. The probability, the error will occur if that noise is more than delta or noise is more is less than minus delta.

And I can divide this by sigma, noise variance. Actually, this is the noise amplitude. So, this is a noise amplitude. So, error will happen when the noise amplitude is greater than delta or noise amplitude is greater than minus delta, for any of the interior constellation points. So, I can divide this by sigma. So, this will remain the same.

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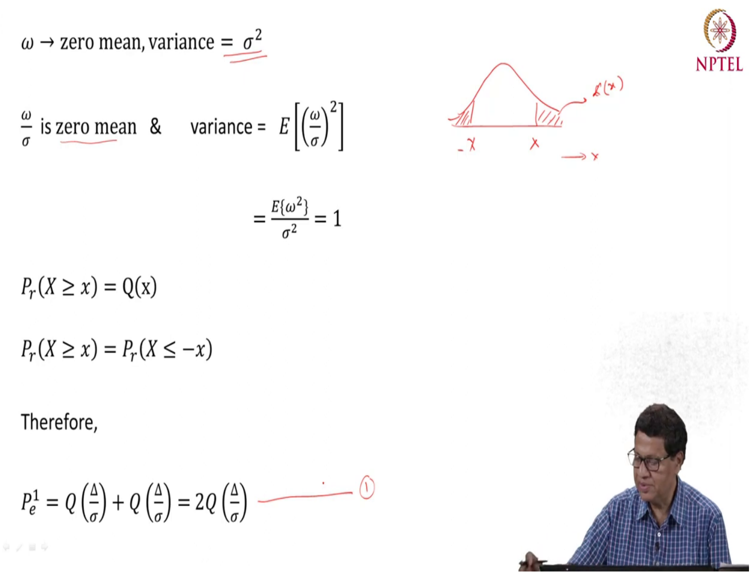

So, this noise amplitude actually is 0 mean and it has some variance sigma square. So, if I divide by sigma, so it will still be 0 mean and if I want to calculate the variance this will be the expected value of you know w by sigma square. And this is nothing but 1, because if I take expected value of noise amplitude is nothing but sigma square, sigma square divided by sigma square is 1. So, w by sigma has 0 mean and variance is 1.

So, using this now in a Gaussian distribution, if this point is for example, this is say x and the probability that x greater than x, that means, this area this is defined by Q x. And this area this is all symmetrical, this curve is symmetrical. So, this area if I see here is same as this area, and this is say x, this is x or minus x.

Therefore, the P e 1, the probability because for the interior points will be Q delta by sigma plus Q delta by sigma. So, when you add them, it becomes 2 Q delta sigma.

So, if you see the expression here, here, and using this the area concepts for this Q function. I can write this as Q delta by sigma and similarly Q delta by sigma. So, this is how I have got. So, this is the probability of the interior points. So, this is one probability we have calculated for the interior points.

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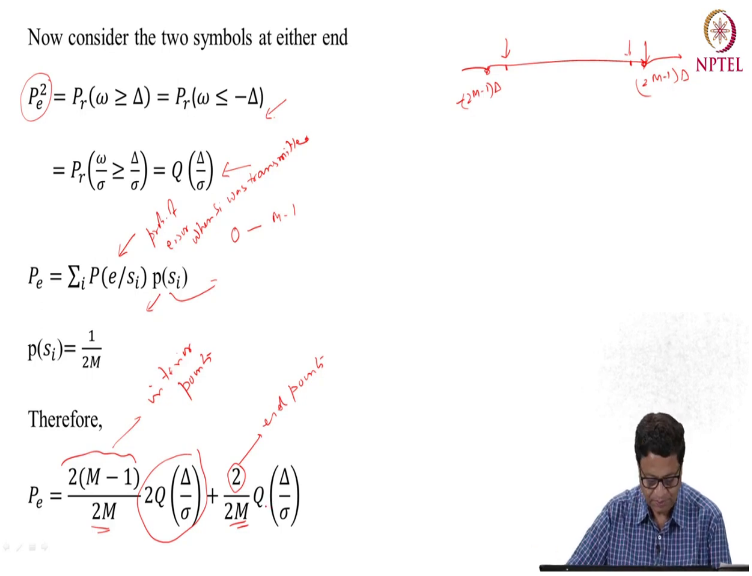

Now, consider the two points at the either end that is this point and this point which is 2 M minus 1 delta, and this is minus 2 M minus 1 delta. And here there will be neighbouring point. So, anything which is beyond this noise suppose it has shifted this it always be you know decoded at this point.

Similarly, if the noise is in this direction, this will be decoded as this point. Whereas, if it is on the other side the noise amplitude then there may be an error, it may be you know decoded as this point in this case or it may be decoded as this point in this case. So, the scenario is different for the end points which is which was not the case for the interior points.

So, I am trying to calculate the probability of error for these two points only, rest are other identical that we have calculated. So, the noise amplitude greater than or equal to delta on this side, on one side, and then on the other the left hand constellation points will have this condition. So, this is P e 2 which is for the end points on either side. So, this is second probability of error.

So, this by the same logic I can write this as divide by sigma both ways and I get this is the probability of error in terms of Q delta by sigma. And the total probability of error is defined as summation over all points over all points i which is changing from 0 to minus 1.

This is probability of error when s i was transmitted. This is probability of error when s i was transmitted and this is the probability that s i was transmitted. So, this is the P e, the total probability. And the total p s i's are 1 by 2 M. So, the probability assume that all the points are occurring equally likely. So, the p s i the this probability that s i was transmitted is 1 by 2 M.

Therefore, the probability of error; I have added both those probabilities, so remember we came we had this 2 Q delta by sigma and also, we had Q delta by sigma. So, this is for you know two points. So, that is why this factor of 2 comes here, end points, these are end points these are end points, and these are rest of the points the interior points, this is interior points. And 2 M is the probability of s i coming 1 by s i. So, this becomes the total probability of error for a M-ary system.

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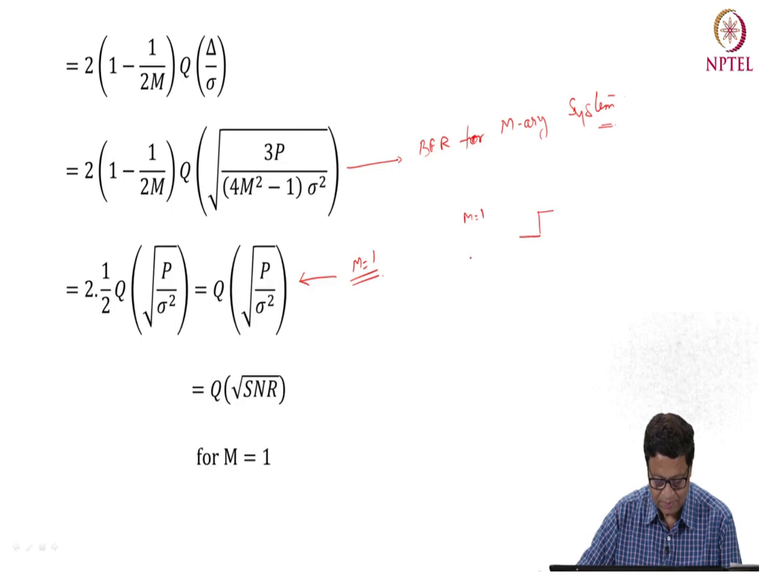

So, this can be slightly rewritten as 2 1 by 1 by 2M into Q delta by sigma. And this delta can be written in terms of power, so this becomes 2 1 minus 1 by 2M. So, here I have replaced the value of delta in terms of power. So, this is 3P 4M square minus 1 sigma square. So, this is the probability of error BER for M-ary system.

So, this can be written for example, this is for suppose M is equal to 1. So, this becomes reduces to this equations put M is equal to 1 here. So, this becomes half and this quantity becomes 4 minus 1, 3 3 gets cancelled. So, what you get is this and this is for M is equal to 1 which is nothing but signal to noise ratio.

So, M is equal to 1 is a very simple case of on-off keying. M is equal to 1 means you have 2 levels only this is 1 and the other level is this. So, this becomes a special case of on-off keying. So, this is we can you know verify this expression by using some simpler time structure.

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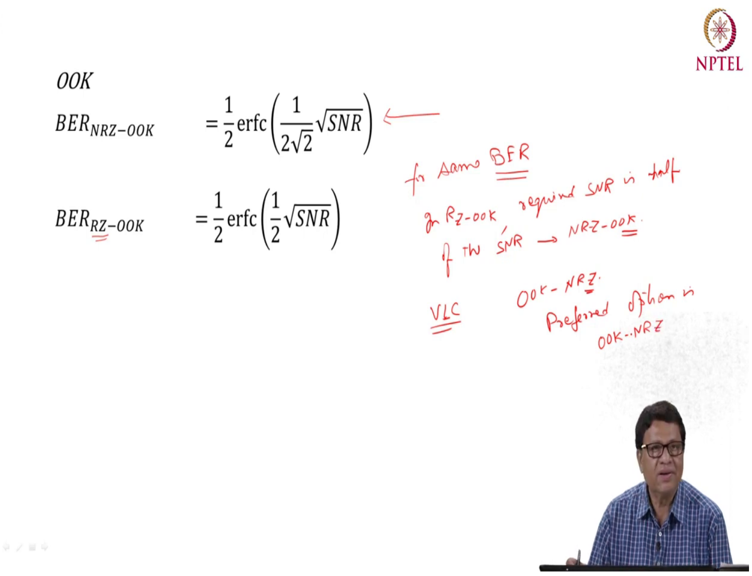

So, now let us see for on-off keying, as we had seen earlier for a on-off keying NRZ signal had considered NRZ signal, but one can also considered RZ signal and can get the probability of error in the same fashion as we had done in the last in the discussion. So, BER NRZ OOK is as calculated earlier is half error complementary 1 by 2 root 2 root of SNR.

And if I replace this by RZ, this becomes half error complementary 1 by 2 root SNR. So, as you see the in RZ, required SNR is equal to half or minus 3 dB of the SNR of the NRZ for the same BER. So, basically for same BER, for same BER, NRZ OOK, the required SNR is half of the SNR which is required in NRZ OOK.

So, you require lesser amount of power if I was transmitting RZ on-off keying to get same BER and you require more power for NRZ OOK scheme whereas, the bandwidth requirement for RZ OOK is higher as compared to NRZ OOK. So, in some sense, it has good power efficiency RZ OOK, but bandwidth efficiency is poor.

Similarly, the other way around for NRZ ok OOK, it is good bandwidth efficient, but the power efficiency is bad as compared to RZ OOK. So, these are the two expressions for NRZ and RZ. So, in VLC system or any optical you know outdoor system like free space optics, so the preferred option is you know on-off keying NRZ signal.

I mean at least in VLC because there the power is not a constraint whereas, the bandwidth is a constraint because the devices which you use for example, LED or photo editor or the channel inside the room is all bandwidth limited. So, I should have a modulation scheme which is more suitable for such scenario.

So, preferred option is NRZ because of the illumination there is enough light and there is enough power available. So, that may not be a constraint. But bandwidth is a constraint. So, the preferred option in VLC system for example, is option is OOK NRZ, if you have to choose between NRZ or RZ.

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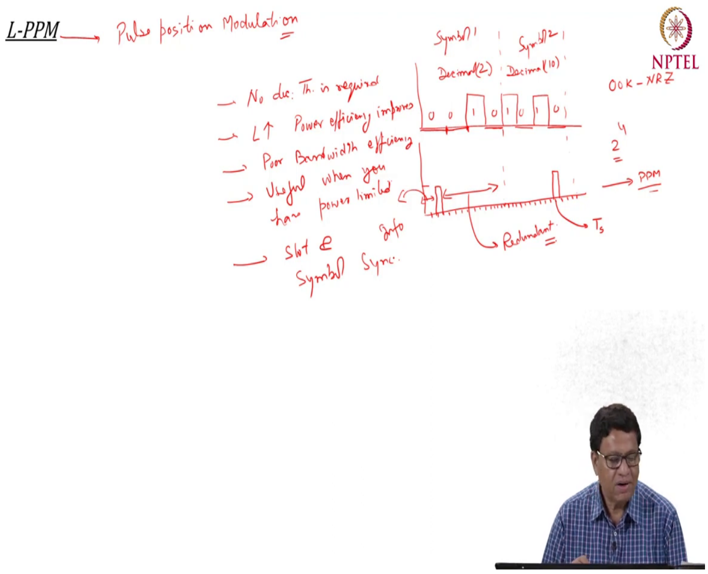

So, now let us discuss about next modulation scheme which is called as L-PPM. So, what is PPM? So, let us first draw a normal OOK NRZ stream and then we will draw the L-PPM version of it and then that will explain the difference that will explain what is L-PPM. So, PPM as you know is pulse position modulation pulse. So, the information is actually in the position of the pulse.

So, if I take for example, a set of streams is 0 0, this is 0 0 and then I have 1 0 and then I have 0, this is say 0, this is 0, this is 0, this is 0 and then I have again 1 0 1 0. So, 1 0, 1 0. So, this is 1 0, this is 1 0, and the symbol is for example, 0 0 1 0 0 0 1. So, this is one symbol suppose, this is symbol 1 and the second symbol is for example, this is a second symbol which is 1 0 1 0. Now, this is a case of on-off keying NRZ. So, I have made NRZ here.

And this is and if you want to have the PPM, then there are 4 bits in the symbol. So, I will divide each duration, each symbol duration into 16 parts that is 2 raised to power 4. So, if I divide this, so this is actually if you see there is the decimal value of this is 2 and the decimal value of this will be 2 raised to power 3 plus 2 raised to power 1. So, the decimal value is this is 2 here and this decimal value is say 10 here.

So, I will divide this into 16 slots, there is 2 raised to power 4 to represent any other combination, there will be 2 raised to power 4 combinations. So, I will divide this into 16 parts 1, 2, 3. So, there are 16 parts here. Similarly, I will do it here for the symbol 2. So, this is 16 parts divided here.

So, decimal 2, that means, I will be sending a pulse after 2, so 1 2. So, I will be sending a pulse here of this duration. And similarly, this is decimal 10, so I will be sending from here the 10th pulse. So, 10th pulse will be 1, 2, 3, 4, 5 6, 7, 8, 9, 10. So, I will be sending this pulse here. So, this is a PPM version of this data stream.

So, you transmit; so, this is the info part and these are the redundant space. So, this is the info part, information part info and this is the redundant. So, basically you should see the position of this, right. So, this is the PPM version of OOK NRZ signal shown here. So, there will be slots 16 slots and each slot has say width T s.

So, what is the advantage of pulse position model or what are the disadvantage of pulse position modulation? So, first of all you do not require any decision threshold because you want to detect the position of the pulse. You are not detecting the amplitude of the pulse. So, either the pulse is present, at what location it is present that is that matters to us.

So, no decision threshold is required and as L increases, this is a case of 4 PPM, as L increases the average power improves. The power efficiency improves though. You require less amount of power, power efficiency improves. But the cost one has to pay is poor bandwidth efficiency because the pulses are becoming narrow, they require more bandwidth, poor bandwidth efficiency.

So, PPM is useful where you have a power limited system, useful when you have power limited system. And other issue is because you have slots as well as symbol. So, you have to maintain the synchronization of slot and symbol. So, slot and symbol synchronization is required. So, these are the some you know plus points or negative points of PPM system.

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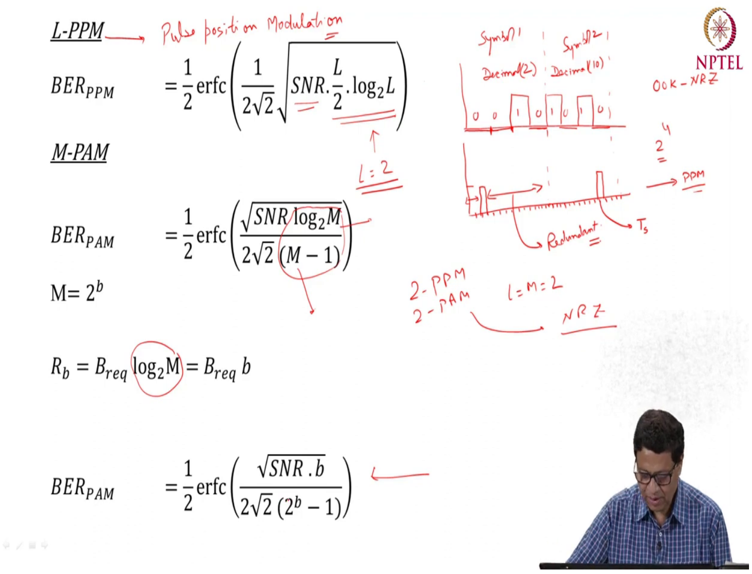

And I think I will have to remove this. So, the BER of PPM can be calculated in similar fashion as we had done for earlier cases OOK NRZ. So, it can be shown that the BER of PPM is half error function 1 by 2 root 2 SNR. And there is a additional factor of L by 2 into log 2 L.

So, if I for example, if I have L is equal to 2 for example, which is a case of NRZ signal only, OOK NRZ; that means, there are only two levels, two pulses 1 and 0. So, this will reduce to you know our standard OOK NRZ case. And for M-ary PAM system, this is given by half error function SNR. And then you have this additional factor coming log 2 M divided by M minus 1. So, this is expression for BER PAM. So, if I take 2 PPM or I take 2 PAM that is L is equal to M is equal to 2, both of them they reduce to NRZ.

And you get same probability of error as we got for OOK NRZ. So, for example, here log 2 M if I take 2 this becomes 1. So, M minus 1 is 1. So, this reduces to root SNR divided by 2 root 2. Similarly, here L is equal to 2. So, special case of 2 PPM and 2 PAM is nothing but OOK NRZ.

And M as we saw earlier is in terms of bits if you want to write then M is equal to 2 raised to power B. And now let us also understand the bandwidth required. So, the R b is the data rate. So, the data rate will be actually increased by log 2 M in case of PAM signal. So, B required increased by a factor of log 2 M.

And this can be written as B required into B because, and then BER PAM if you want to write in terms of B, this M can be written in terms of B. And then it reduces to half error function root SNR into B 2 root 2, 2 B minus 1.

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$$\frac{SNR_{8PPM}}{SNR_{2PAM}} = \frac{\left\{\frac{\sqrt{2}}{\sqrt{3}}\, erfc^{-1}(2BER)\right\}^2}{\left\{2\sqrt{2}\, erfc^{-1}(2BER)\right\}^2} = \frac{1}{12}$$

8PPM requires -10.7dB less power as compared to 2.PAM

	16 PPM	8 PPM	4 PPM	2 PPM
16 PPM	(1) 0 dB	(8/3) 4.5 dB	(8) 9.03 dB	(32) 15.01 dB
8 PPM	(3/8) -4.2 dB	(1) 0 dB	(3) 4.77 dB	(12) 10.79 dB
2 PAM	(1/32) -15 dB	(1/12) -10.7 dB	(1/4) -6.02 dB	(1) 0 dB

L: PPM
M-PAM

So, now let us try to calculate the SNR for different schemes and just compare with you know different schemes. So, for example, in this table what I have done here is I have 16 PPM here, I have 8 PPM here, 4 PPM here, 2 PPM here. And then on the y axis I have 16 PPM, 8 PPM and also, I am trying to compare with a 2 PAM system.

So, one can calculate for different combinations. So, this I have done only for few combinations and this is how I have done. So, SNR 8 PPM you can get the expression from the earlier equation which I had given you in the last of few slides. It was in the form of BER, but you can always calculate in terms of SNR. So, SNR 8 PPM from those expressions is given by this. And similarly, if I calculate SNR for 2 PAM, 2 PAM, which is nothing but NRZ, OOK NRZ is given by this expression.

And some of these things will get cancelled and what I get is 1 by 12. So, if you see for example, 8 PPM here and 2 PAM, 2 here, 8 PPM is here so, this is actually 1 by 12; and if you can convert this into minus 10.7 dB. So, what it means that 8 PPM requires minus 10.7 dB less power than 2 PAM for any desired BER.

So, this particular example shows that 8 PPM requires minus 10.7 dB less power as compared to 2 PAM. Similarly, you can compare any these any of these 2, any modulation schemes, any L-PPM modulation scheme with any M PAM modulation scheme and can you know find out what is the you know power, whether it is more power required or less power required with respect to you know different schemes. So, this table can be you know extended and can cover all possible combinations.

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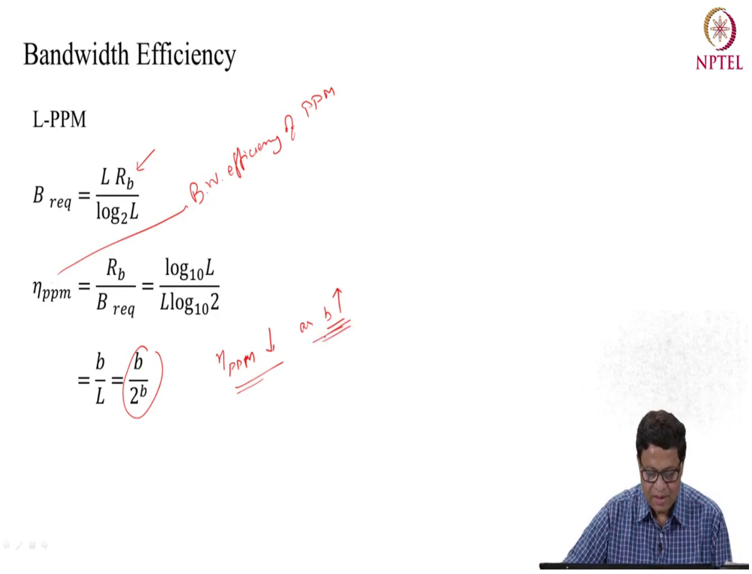

Now, let us try to understand the bandwidth efficiency. So, we have seen the SNR part and then later on we will see the power efficiency, but before that let us understand the bandwidth efficiency of these systems. So, for example, L-PPM the bandwidth required is L by log 2 L into R b. R b is the data rate and this is because of the L-PPM. You require more bandwidth which is which R b is to be multiplied by L by log 2 L.

And if I want to calculate the efficiency of the PPM bandwidth efficiency, this is bandwidth efficiency of PPM is R b required of B required. So, R b will get cancelled and what we get is log, I mean you can write this log 2 L in a different form. So, log to the base 10 L divided by L into log 10 L. And this can be further simplified which will be given as b by 2 by b.

Now, if you see here the NPPM that is sorry eta PPM that is a bandwidth efficiency of PPM, PPM decreases as you increase as b increases because this is 2 raised to power b. So, this you know increases at a much faster rate. So, it reduces the bandwidth efficiency of the system. So, in PPM L-PPM depending upon number of bits or number of levels L, the efficiency eta, bandwidth efficiency eta of the PPM decreases as b increases.

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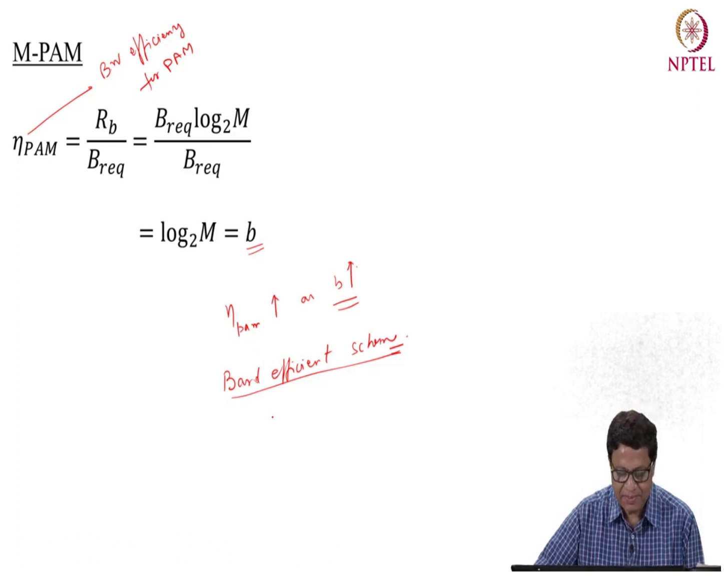

And let us now try to calculate the bandwidth efficiency for the M-ary PAM. So, efficiency this is bandwidth efficiency bandwidth for PAM which is given by R b required by B required from our initial discussion. And B required is different here. This is his B required is given by R b is actually B required and B required in this case will be log 2M divided by B required. So, this gives you log 2 M which in terms of number of bits can be written as b.

So, in this case for MPAM the efficiency increases as; so, in this case the eta that is a bandwidth efficiency PAM increases as b increases. So, it is a bandwidth efficient scheme, so bandwidth efficient scheme. So, you want more bandwidth efficiency or better bandwidth efficiency one can increase the value of b number of bits and can get a good which is good bandwidth efficient modulation scheme.

So, one of the scheme was good as far as power efficiency was concerned and PAM is a good scheme for bandwidth efficiency and PPM as we saw in earlier discussion is good for power efficiency or is a power efficient scheme.

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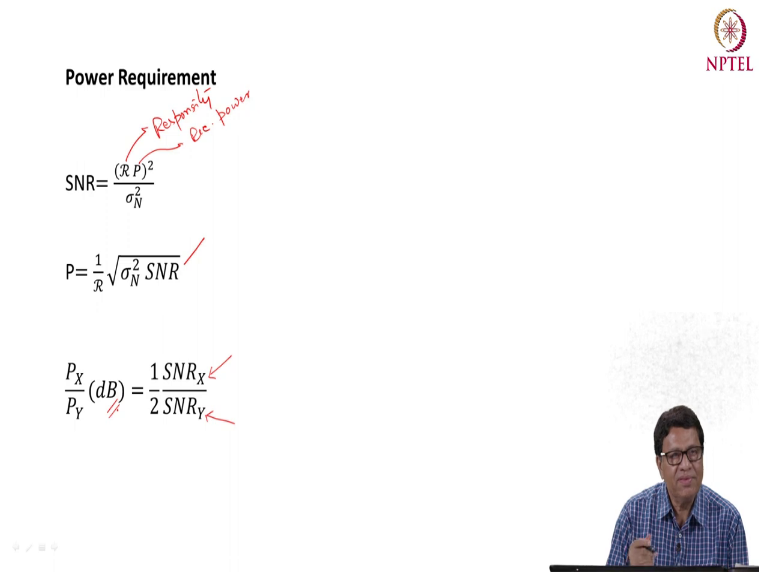

So, now let us understand the power requirement. So, we had done some calculation for SNR, let us do for power requirement. So, SNR is actually given by signal power divided by noise power, signal power actually is current as this is a responsibility into power is current. So, this is some sort of signal power. So, this is a responsibility. This is for a optical wireless base system and this is a receipt power. So, this gives you the SNR.

And if I calculate P, then it is given by 1 by responsibility into square root of sigma n square SNR which means P is directly proportional to root SNR. And if I calculate if I take the ratio the power required for a scheme X as compared to power required in a scheme B in terms of dB, this will be given by half.

The SNR required for you know X modulation scheme and SNR required for Y modulation scheme. So, this half is coming because of the square root function here in this. So, this is the ratio of power for different modulation schemes. So, basically, I need to calculate SNR X, SNR Y, and then using this formula I can find out the ratio of you know one power requirement in one scheme with respect to the power required in the other scheme.

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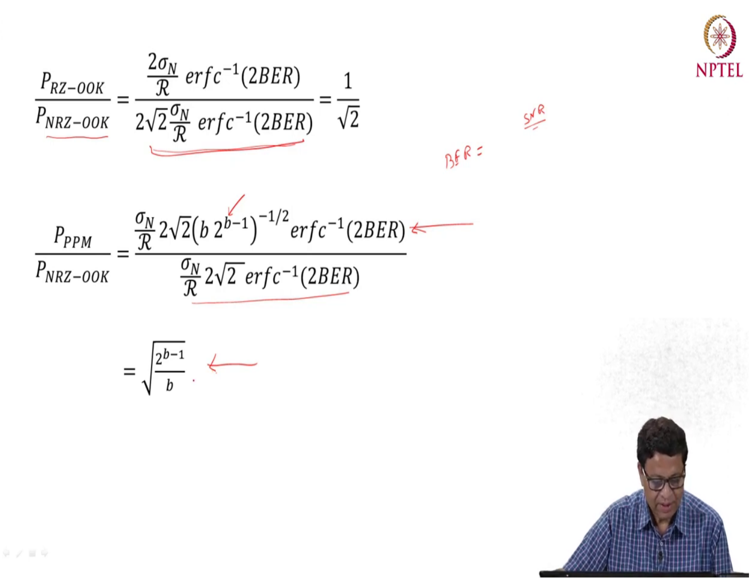

So, this is P RZ OOK. This is the power required for OOK. This we can calculate from our earlier formula which I had given for a BER. And so, the power for RZ OOK is given by 2 sigma N divided by responsibility error complementary function minus N within bracket 2

BER. And similarly for N RZ OOK it is given as 1 by is given by this expression and if see the ratio it is going to be 1 by root 2.

Now, if I calculate the power required for a PPM system and I am normalizing with power required for OOK system, P and RZ OOK then it is given by this expression. So, this is the expression for I mean from the earlier expression which is you know BER we had calculated, and it was in the form of SNR. And SNR and power relationship we have seen in the earlier slide.

So, using all these formula we can get power in the PPM in terms of BER, in terms of BER in terms of number of bits. So, and it is normalized to P NRZ OOK which is a standard expression which I am using here which is same as you know this expression. So, this is the power ratio, and this lot of things will get cancelled and what you get is square root 2 raised to the power b minus 1 divided by b.

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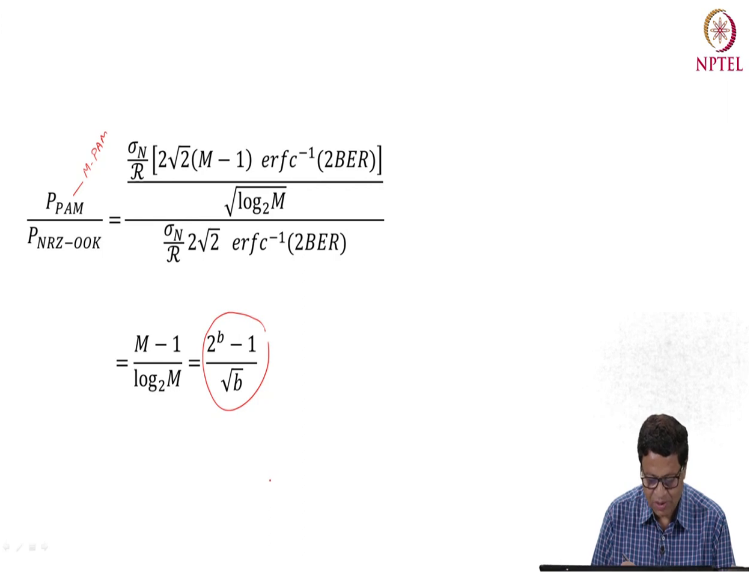

And similarly, I can calculate the power required for a M-ary PAM system. This is actually for a L; I mean this is basically for L-PPM system. I am not using L here; it is in given in terms of b. And this is for a M-ary PAM system, so this is for a PAM. So, using the PAM be better a bit error rate expression, I can calculate the power required and it is again normalized to NRZ OOK.

So, what we get is this ratio. And you can simplify this and this is the ratio which we get of power required in PAM system with respect to power required in NRZ OOK system, 2 raised to the power b minus 1 root b.

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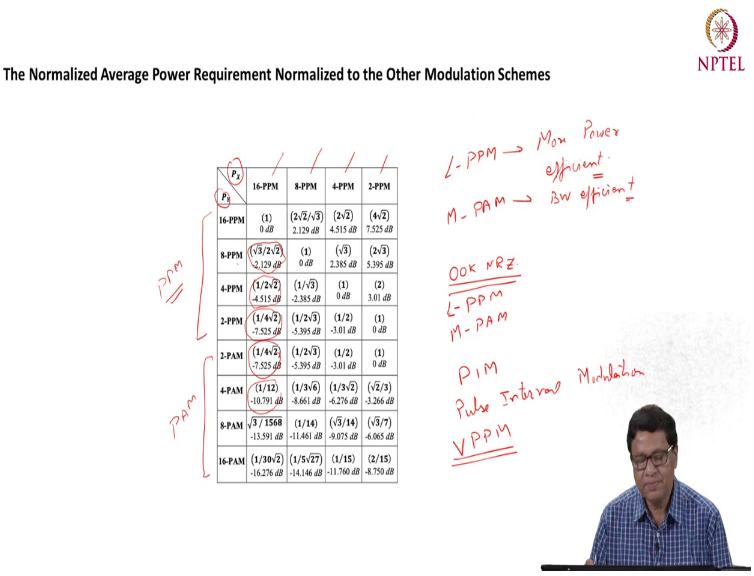

So, again, the way we had plotted the earlier SNR values for different modulation schemes, here this is the PX value the power required value has been plotted for different modulation schemes. So, here you have 16 PPM, 8 PPM, 4 PPM, 2 PPM, and on this scale again you have you know different PPM system and also there are you know different PAM systems. So, these are for PAM and these are for PPM.

So, as you see here 16 PPM requires less power as compared to 8 PPM by this amount. It requires less power as compared to 4 PPM by this amount. And for 2 PPM this amount, and then this is 2 PAM, 4 PAM and so on and so forth. So, you can you know compare any modulation schemes with any of the PAM scheme, any PPM scheme with any of the PAM scheme for you know different amounts of power required for each scheme by looking at this table.

So, this is in short, the whole conclusion is that L-PPM is more power efficient. So, we use in these techniques wherever the power is limited in optical wireless communication systems, and M-PPM, M-PAM is bandwidth efficient. So, this is a scheme which can be useful when you have when you want to consider bandwidth as a prime factor. So, M-PAM is bandwidth efficient and L-PPM is power efficient.

So, in this lecture, we have discussed about how to calculate BER of OOK NRZ. We have also discussed about PPM in general, different levels of PPM and also calculated BER. Similarly, we have done M-PAM system, M-ary PAM systems, and also, we have seen comparison of these two systems with respect to SNR requirement or with respect to power requirement.

So, and next we want to start with another modulation schemes which is PIM that is pulse interval modulation. And also, we will discuss a new technique which is there in the standards of for example; VLC is variable pulse position modulation. So, we are going to discuss in detail about these two PIM and VPPM modulation schemes.

So, thank you very much.