Optical Wireless Communications for Beyond 5G Networks and IoT

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Lecture - 12
Part - 1
Modulation Schemes for OWC, BER for OOK

Hello everyone. So, today we are going to discuss different modulation schemes, which I which will be used in Optical Wireless Communication systems.

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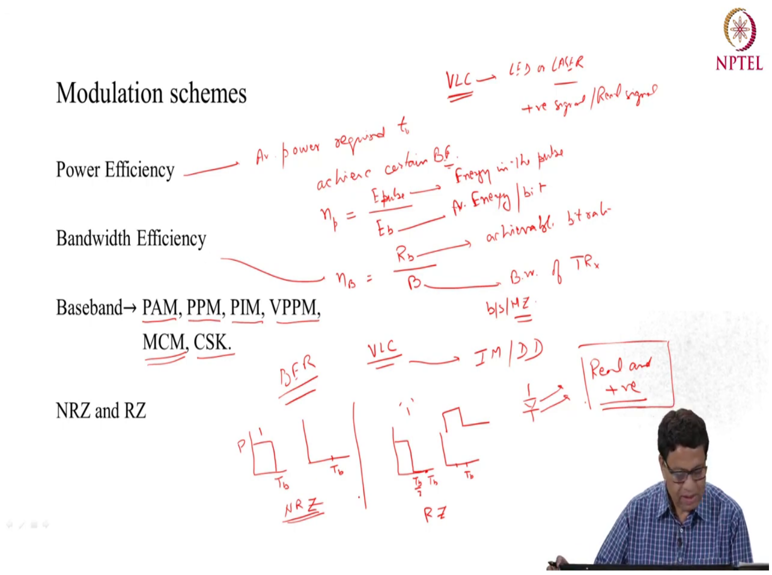

Now, in systems which are based on light for example, visible light system normally they use LED or laser as the source and these source they require positive signal and real signals. This is different from what we have in RF, where the signal could be bipolar positive as well as negative and also signal can be complex. It has both real parts and imaginary parts.

So, for optical wireless systems there are some specific modulation techniques which are used not all RF techniques which have been used are directly you can be used in optical wireless systems, there may be some modification required. Some of the you know advantage we have in RF scheme using a particular modulation scheme those advantages may not be there in optical wireless communication systems.

So, we need to understand the modulation schemes which are specifically useful for optical wireless communication systems. So, before we start the discussion on modulation schemes, we want that the power either from the transmitter or from the hand held device using optical wireless technology should be as low as possible.

So, you should look for modulation schemes which is you know power efficient and also we know the bandwidth offered by different sources also limited and optical wireless channel bandwidth is also limited. There maybe you know huge bandwidth available in the spectrum visible spectrum or I-R spectrum, but it is limited by the bandwidth of the devices. So, we need to have modulation scheme which is bandwidth efficient.

So, before we start let us understand the different a parameters which we will try to calculate for different modulation schemes for example, power efficiency. So, the power efficiency is average optical power required to achieve a certain BER. So, this is average power required to achieve certain BER. This is how we define the power efficiency.

And the for power efficiency there is a term which is called eta power efficiency which is defined as E pulse that is energy in the pulse and energy in the bit or average energy per bit average energy per bit. This is energy in the pulse. This is how we define. So, for different modulation schemes we will calculate this power efficiency parameter.

The other important parameter which is to be discussed or which is to be calculated for different modulation schemes is bandwidth efficiency. Now, bandwidth efficiency is defined as eta B is R b that is achievable bit rate divided by the bandwidth of the transmitter receiver. So, this is bandwidth of transmitter receiver and this is the achievable bit rate. And the units are bits per second per hertz.

So, we need to calculate these two important parameters for any modulation scheme the power efficiency and bandwidth efficiency. And in VLC for example, I am taking one of the optical wireless communication technology that is visible light communication. Normally, we use intensity modulation direct detection.

So, you modulate the intensity that is whenever you know there is logical one the device is lighted whether it is LED or laser you get light here. And whenever there is a zero this device is off. So, this is called as intensity modulation you have modulating the intensity. And signal to the laser diode has to be real and positive.

So, if I am trying to use some RF signal which generally is bipolar and complex, I cannot directly give it to the LED transmitter or laser transmitter. I will have to modify that signal to make it real and positive. So, under this modulation schemes we will discuss about some basement modulation techniques such as pulse amplitude modulation, pulse position modulation, pulse interval modulation and also, we will discuss about variable pulse position modulation.

So, and also subsequently we will discuss the multi carrier modulation like OFDM. And also, we will discuss about color shift keying. This is another modulation scheme which is used in optical wireless communication systems. So, we will try to analyse, understand these modulation techniques, try to calculate BER for each one of them and also see which one of them are power efficient, which of them which are of which are bandwidth efficient and also, we can use you know OFDM. It has to be modified to suit our optical wireless requirement.

So, we will study all these modulation schemes one by one. So, before understanding before going into the modulation, let us try two understand two types of signalling which is NRZ signal and RZ signalling. So, in NRZ signal you have one which is denoted as this is one, this is duration of the bit and this is some power P. And for 0, this is Tb there is no power. So, this is an RZ signalling. And for RZ that is written to 0, for half of the duration in 1 it is pulse is high.

So, this is RZ. So, this is logical 1 for example, and this is Tb by 2 and then you have Tb here. So, this is how RZ signalling look like and for 0 it is same. So, this is TB. So, there is no power at 0. So, that is the difference between NRZ and RZ which we will be using when we discuss these different modulation schemes. And also let me tell you the what is the power spectral density for NRZ and RZ.

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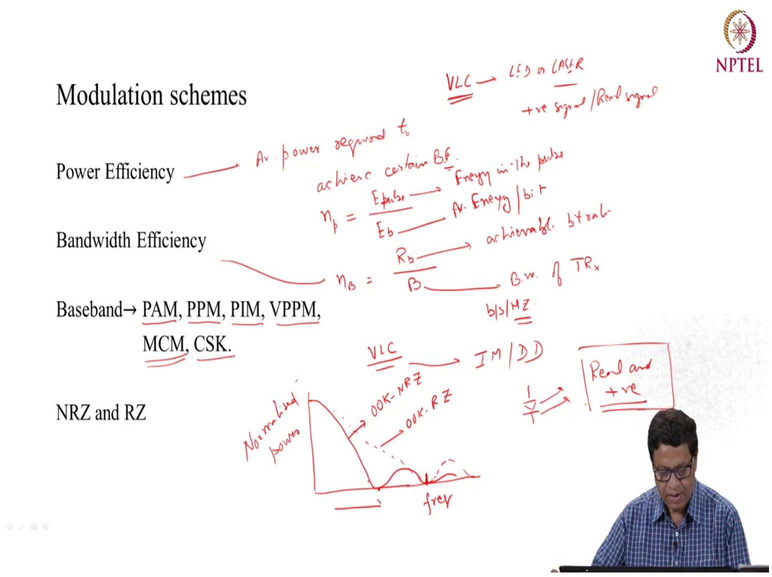

So, for example, if I plot the normalized power on this side, normalized power and this side is say normalized frequency. So, for NRZ, this is the power spectral density for NRZ. So, this is you can say on off keying NRZ OOK on off keying NRZ. And for on off keying RZ, I am drawing with a dash and this will be 0 here.

So, this is on off keying RZ. So, the first 0 occurs here in for OOK and RZ somewhere here and for OOK RZ which require higher bandwidth. So, first 0 requires occurs at point A. So, this is a power spectral density for NRZ and RZ signalling.

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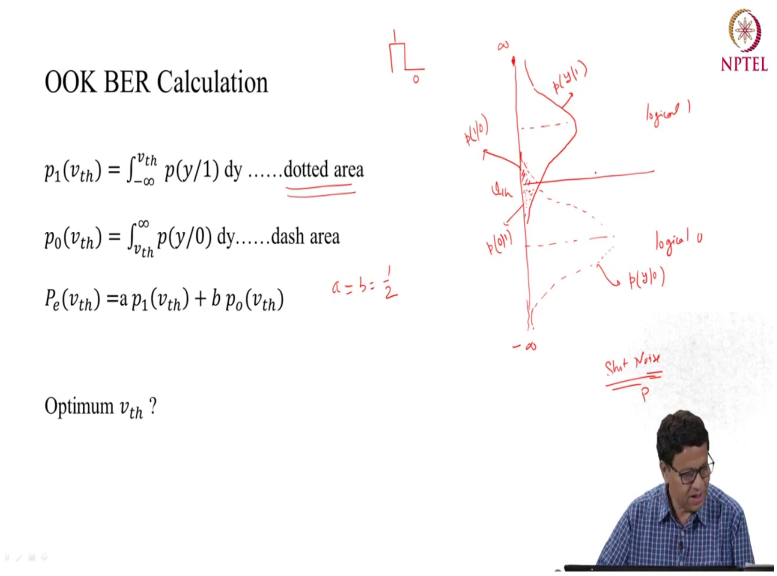

Now, let us take a simple on off keying system and try to calculate the BER for such a system. So, as I mentioned on off keying is you have this is a digital pulse and you modulate your LED or laser diode. So, whenever there is one, the LED is forward biased and emits power, whenever it is 0, there is no power coming. So, basically you are getting on off the device and off of the device, there is on off keying.

So, we will try to calculate bit error rate for such a system. So, the received signal which is received after it has gone through the channel optical wireless channel, the signal is actually the received signal is Gaussian in nature. So, let me draw how the spectral density of the received signal. So, say suppose this is the middle point and this is a logical 1, this follows Gaussian distribution, this is a logical 0.

So, logical 1 is say for example, here logical 0 is say somewhere here and this will have distribution like this, this should go to 0 from and for logical 1 same. So, it will go that I am drawing with a dash. So, this is also dash here. So, let us call this as probability received signal when one was transmitted. So, this is pi p y given 1, when one was transmitted and this is the probability of received signal y when 0 was transmitted. And this is some sort of threshold I have put here, let us call it as v th.

So, anything I do not know what this v th, whether this will be in the middle or it is towards you know towards 1 or towards 0 that depends you know what is the probably PDF of the received signal for 1 and received signal is 0, because this can be different in optical communication systems depending upon the power and the short noise. So, this is how this quantity is defined. So, it is not necessarily that v th will be exactly in the middle, it may be towards p y1 that we will see or towards p y given 0.

So, right now I am putting here. So, the error component in this is for example, this area, this is this area, the dash area is the error and the probability that I received 1 and when 0 was transmitted. So, this is given by this dash area and if you see this dotted area which I am showing here as dots is p the probability that you got 0 when 1 was transmitted and this is minus infinity, this is infinity. So, we will try to calculate the BER for such a system.

So, if you see the probability that there is an error is given by this dotted area, this dotted area here this one and as you see this is p y1 that is the probability, the received function when 1 was the received signal when 1 was transmitted and the limits are from minus infinity to infinity because this will extend to minus infinity. So, this dotted area is the error part. So, this is denoted by p 1 v th and similarly for 0 v th that is p 0 v th, it is the dash area, the dash area is this here somewhere here.

So, this goes from v th to infinity all the way to the infinity here. So, this dash area is the probability that in the 0 and p 1 v th is the probability in the 1. So, we need to calculate these two areas. This is actually error in the system because I had transmitted 1 and received 0. In the other case I had transmitted 0 and received 1. So, I need to calculate these two areas which will define the probability of the total probability of error or bit of error rate.

So, the total probability of error will include both these areas and A and B are the probability that 1 is coming and the B is the probability that 0 is coming. So, for simplicity we can assume the probability that 1 is coming is half I mean same as 0. So, a is equal to b is equal to half.

The 0s and 1s which are coming in the (Refer Time: 16:44) stream and they are in equal numbers, the probability that of 0 occurring is half, probability of 1 occurring is half. So, the total probability of error will be basically half p 1 v th plus half p 0 v th. And what is that v th? What is that optimum v th? Where I should push put my threshold so, that the BER is minimum.

The idea is I want to have as low as BER. So, what is the optimum v th? If these two probability distribution functions are same then probably then the v th will be exactly in the middle. But if these two are different then it may not be in the middle because if you see in optical wireless system there is something called short noise and that depends on the input power or p at p received.

So, in case of 1 the power is high the short noise is high, it may give you know different kind of probability distribution function. In case of 0 there is no power there is no short noise it may can give you different PSD or power spectral density. So, depending upon this these two PSDs can be different and also the v th the optimum threshold is not in the middle.

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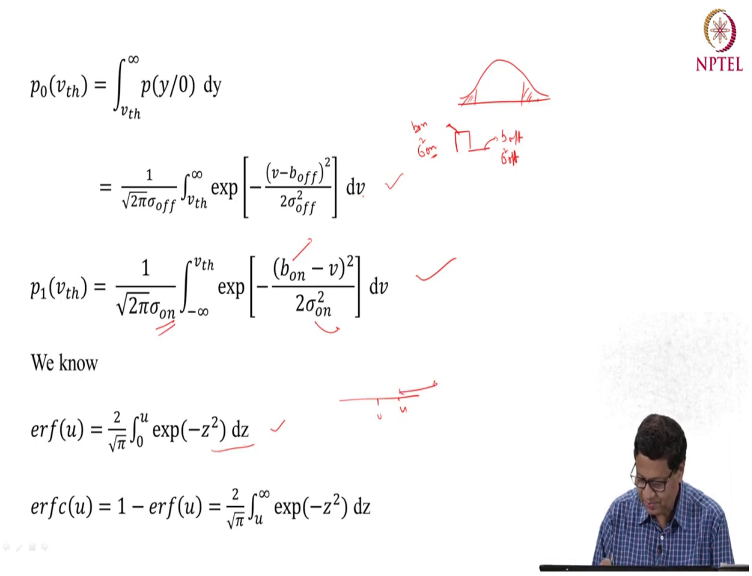

So, let us try to calculate this optimum v th. So, as I mentioned the that the dash area was p 0 v th the units are from v th to infinity that is a that is the error in 0. And this is given by this area and if you see this area this particular area that the dash of the dotted area depending upon what you consider. For example, if it is p 1 it is dotted area if it is p 0 it is dash area.

So, I am trying to calculate the 0. So, it is from v th to infinity and the PSD when of received signal when 0 was transmitted. And this can be equated as from v th to infinity this is Gaussian this is how you calculate the you know area you know if the it is like this and you want to calculate this area.

So, it is represented in this equation where b off is the mean that is for the this is b off for example, and this has sigma variance for the off state. And similarly, you have the b on average b on and this will be sigma square on. So, this is for the 0 1 by root 2 pi 0 off case, the variance in the off v th to infinity exponential minus the received signal v minus b off whole square divided by 2 sigma square off.

Similarly, we can write p 1 v th the limits will be from minus infinity to v th you can refer the same diagram which was there earlier. So, the limit will be minus infinity to v th and if you have sigma on that is the variance for the on state and this is the average for the on state and this is the variance this is sigma on this is root of variance. So, this is the p 1 v th and this is a p 0 v th. So, basically, I have calculated both of these areas. So, there are standard expressions for error function.

So, erf u is defined as 2 root to pi 0 to u exponential minus z square dz and if you see you know these expressions here these two are of this form if I slightly change you know I can write in error function form. So, doing this an error function complementary is 1 minus error function u which is given by same expression here.

So, the unit the limits are changed in case of error function is 0 to u, but when I say complementary function, the units are from u to infinity. So, for example, this is 0 to u and if I take 1 minus then it becomes u to infinity in this part.

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$$p_0(v_{th}) = \frac{1}{\sqrt{2\pi}\sigma_{off}} \int_{v_{th}}^{\infty} \exp\left[-\frac{(v-b_{off})^2}{2\sigma_{off}^2}\right] dv$$

$$= \frac{1}{2}\left[1 - erf\left(\frac{v_{th}-b_{off}}{\sqrt{2}\sigma_{off}}\right)\right]$$

$$= \frac{1}{2} erfc\left(\frac{v_{th}-b_{off}}{\sqrt{2}\sigma_{off}}\right) \text{ —— (A)}$$

Similarly,

$$p_1(v_{th}) = \frac{1}{2} erfc\left(\frac{b_{on}-v_{th}}{\sqrt{2}\sigma_{on}}\right) \text{ —— (B)}$$

So, p 0 v th is given like this v th to infinity exponential v minus b off whole square divided by 2 sigma square off and this can be written in the error function form as discussed earlier. So, this in error function form is half 1 minus erf v th minus b off divided by root 2 sigma off.

So, this basically has come from this equation. So, basically this and this you know have been used to calculate the p 0 v th in the erf form or error function form and if you want to write in error complementary form. So, this 1 minus erf will be erf c 1 by 2 v th is given by this expression. Similarly, I can calculate for p 1 v th. So, p 1 v th on same the way we have done for p 0 v th half error function c.

So, this will be not v th minus b off it will be b on the on part minus the threshold part minus v th divided by root 2 sigma on similar lines. So, I have calculated p 0 v th this area which is error in the system and p 1 v th again this is error for the 1 and this is error for the 0. So, the total error will be summation of this and this that area both the shaded area as well as the dotted area. So, that is the becomes the total error.

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$$P_e = \frac{1}{2}[A + B]$$

$$\text{Define } \frac{v_{th} - b_{off}}{\sigma_{off}} = \frac{b_{on} - v_{th}}{\sigma_{on}} \equiv Q$$

$$v_{th} = \frac{\sigma_{off} b_{on} + \sigma_{on} b_{off}}{\sigma_{off} + \sigma_{on}}$$

$$P_e = \frac{1}{2}\left[\frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) + \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left[1 - erf\left(\frac{Q}{\sqrt{2}}\right)\right]$$

So, which is written as half A plus B and A and B are in the as shown in the previous slide. So, let us define a quantity which is there in this that is v th minus b off divided by sigma off for B1 minus v th divided by sigma on as some quantity Q. So, let us define this as Q and from here I can calculate the v th.

So, from these two I can calculate v th which is given as given here sigma off into b on plus sigma on into b off divided by sigma off plus sigma on. And if I want that write that earlier expression in terms of Q. So, the probability of error will be half, half error function Q by root 2 plus half error function Q by root 2.

So, basically, I have replaced this Q in the earlier expression. So, this is the probability of error in the form of Q. And so, this can be added and what finally, we get? The probability of error as half error function complementary Q by root 2. And this can be written in the form of error function also so, 1 minus erf Q by root 2.

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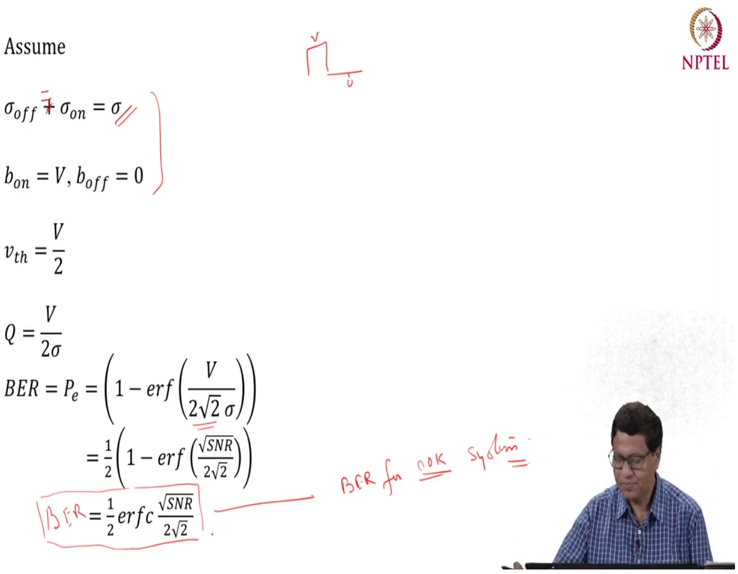

Now, let us assume that sigma off plus sigma on is sigma some standard deviation. And this is actually yeah and b on is some voltage V this is the on voltage and b off is 0. Then v th will be in the middle because there are similar curve actually this is not plus this is equal.

So, the variance in sigma off and variance in on they are equal to sigma and b on is voltage V and it is 0 here. So, if I put the value of v th in the expression which had all these four quantities I get v th v by 2 that is exactly in the middle. So, if my threshold is in the middle under these conditions then the BER will be minimum. And Q the factor using this is V by 2 sigma.

And if I put this value of Q in the expression which I had calculated so, the BER will be 1 minus error function V by 2 root 2 sigma. And this is nothing but signal to noise ratio under root because B is if you take the V square it gives you some sort of power. And if you take sigma square it gives you the noise variance.

So, V by sigma is nothing but root SNR 2 root 2 is coming because of this. So, half 1 minus erf root SNR 2 root by 2 or it can be written in complementary form half error complementary function root SNR 2 root 2. So, this is the BER for a OOK system. So, this is let me write here BER for on off keying system. And this is you know valid when I have these the variance in the off and variance in the on they are equal to sigma and b on is defined by v and b off is divided by 0. If these values are different then the earlier expression is valid where I had specifically written sigma off and sigma on.

So, if you know the variance for the off and variance for the on is different then we will have to use that equation. But if in case this is equal and then your threshold where you have to put the threshold has to be V by 2 and the BER for such a system is half error complementary function root SNR divided by 2 root 2.

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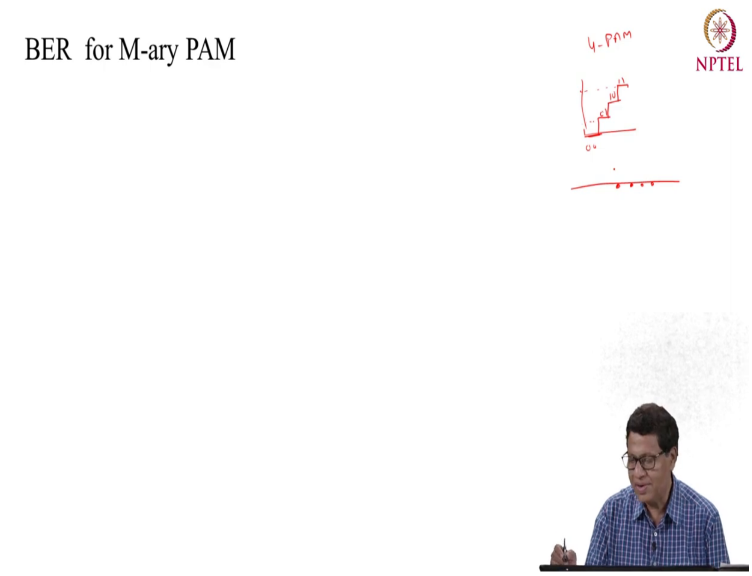

So, this was for OOK scheme. Now, let us try to calculate BER for a M-ary PAM system. So, M-ary PAM system basically consist of so, I am taking a very general case normally as I mentioned you earlier the voltage the signal which is given to the LED transmitter receiver is always positive.

But I am doing a analysis for a signal which may have both positive and negative. But the analysis will not change our concept of VLC modulation. But for a better understanding I am just using typical M-ary PAM system. So, a PAM system is basically you have different amplitudes. For example, if you have a 4 PAM system.

So, for example, this is represented at 0 0 and this may be this may this amplitude is for 0 1 and this may be for say 1 0 and this may be for 1 1. So, there are different levels. So, this is a case of 4 PAM system and this I can use a constellation diagram to analyse the BER.

So, basically, I if I change this system the constellation form it will be something like this. You have this point here, then you have one point here, you have one point here, you have one point here and all of these they represent either 0 0 0 1 or 1 0 or 1 1.

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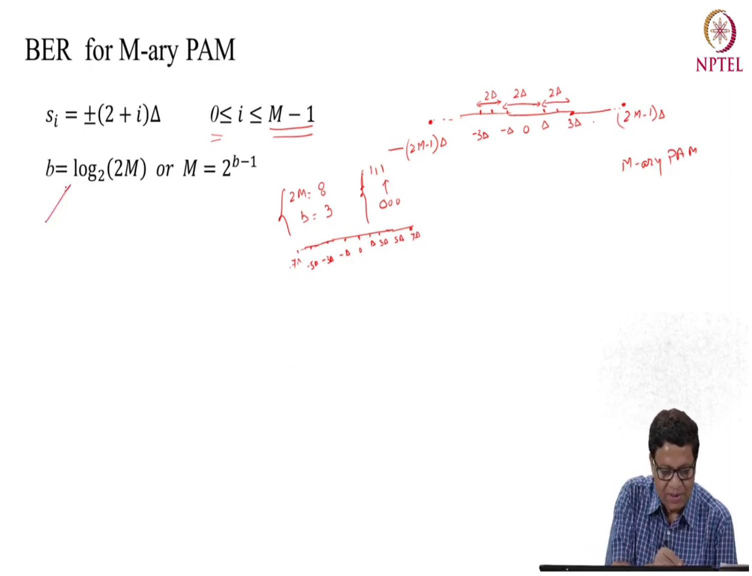

So, we will try to calculate for a generic system which is M-ary PAM system. So, let me draw the constellation of a M-ary PAM system it will be. So, say this is 0 and say this is delta, which is away delta point of delta distance away and this is say minus delta. So, this whole thing between two constellation points the spacing is 2 delta. The next will be at 3 delta.

So, this is spacing if you see is again 2 delta. Similarly, I can see on the negative side again I want to make it clear that for VLC system you require a positive signal only, but this is for understanding of M-ary PAM. So, which is both valid for you know system which is bipolar as well as unipolar. So, we can change this you know coordinate system to make it appropriate for universal system as well.

So, 2 delta and this is this will be minus 3 delta. So, again the spacing is 2 delta. So, I can go on and then my last one will be 2M minus 1 delta. So, this is the last point and on this side. So, let me put dash here. So, this is 3 delta and similarly last point here will be minus 2M minus 1 delta. So, this is a typical M-ary PAM system. This is say the last point here 2M minus 1 delta. So, as you see here s i is 2 plus i delta where i goes from M minus 1 to 0.

So, as you see i is equal to 0 this becomes plus minus 2 delta. Which is shown here plus minus 2 delta and so on and so forth and the last point is M minus 1 and this is written here as 2M minus 1 over delta. So, if you see the this M-ary PAM is actually corresponding to b number of bits. So, where b can be written as log 2 of 2M or M is equal to 2 b minus 1. So, for example, 2M is say 8 and then b will be actually 3 and this will be there will be 8 levels.

So, this is starting from 0 and this will be say delta this will be 3 delta and this will be 5 delta and 7 delta. Similarly, on this side you have minus delta minus 3 delta minus 5 delta and minus 7 delta and this can correspond from all triple 0 to 0 0 1. So, all sorry all once. So, this will have from triple 0 to triple 1's. So, this is how you know it can be represented for a system where 2M is 8 and so, number of bits will be 3 and you can calculate from here as well from this formula.

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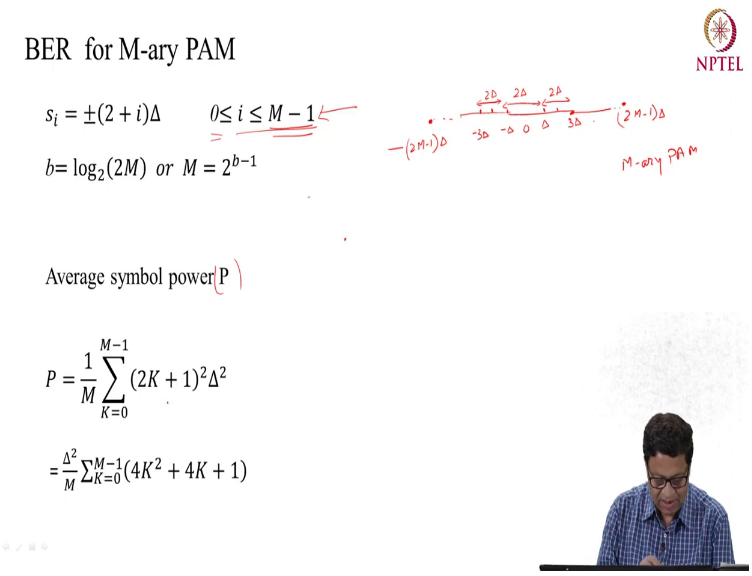

So, now let us try to calculate the BER for a such a system. So, the average symbol power if I want to calculate the average symbol power because each will have you know different power levels. So, the average symbol power will be given by 1 by M this is a total number of levels summation K is equal to 0 M minus 1 2K plus 1 whole square delta square. So, this is the average powers for example, if K is equal to 0. So, this becomes delta square.

So, that is the power in the system so on and so forth K is equal to 1 M minus 1 and it is divided by M which is i is the total number of levels in a PAM signal or M. So, this is the average power. So, this is you can write as average power or P. This is average symbol power and this can be expanded here. So, del square by M can come out and this is 2 K plus 1 has been expanded here.

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$$= \frac{\Delta^2}{M}\left(M + 4 \times \frac{M(M-1)}{2} + 4\frac{M(M-1)(2M-1)}{6}\right)$$

$$= \frac{\Delta^2}{M}\left(M + 2M^2 - 2M + \frac{2}{3}(M^2 - M)(2M-1)\right)$$

$$= \frac{\Delta^2}{3M}(4M^3 - M)$$

$$= \frac{\Delta^2}{3}(2^{2b} - 1)$$

$$\Delta = \sqrt{\frac{3P}{(4M^2 - 1)}}$$

$M = 2^{b-1}$

$b = \log_2(2M)$

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And you know further simplifying this becomes del square by M and these quantities which are there in the bracket and rearranging some other terms, it can give a expression like del square divided by 3 M into 4 M cube minus M. So, this M can be taken common this M can be cancelled and you are left with del square by 3 where I have used M as 2 b minus 1 because b is nothing but log 2 of 2 M.

So, putting this value of M here. So, then we get 2 raise to our 2 b minus 1. So, this is in in terms of number of bits and from here I can calculate this the delta which actually decides how what is my power. So, delta will be root 3 P 4 M square minus 1 for a M-ary PPM system.