Optical Wireless Communications for Beyond 5G Networks and IoT
Prof. Anand Srivastava
Department of Electronics and Communications Engineering
Indraprastha Institute of Information Technology, Delhi

Lecture - 10
Part - 2
Atmospheric Turbulence the Scintillation Effect

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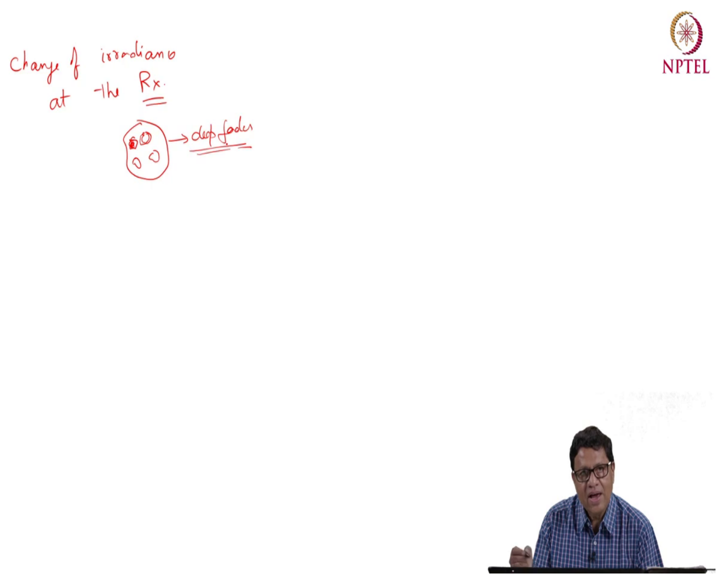

The next is Scintillation Effect. This is change of irradiance at the receiver or fluctuation irradiance at the receiver and this is caused by the eddies which are of same size as beam size. So, basically you get patterns of dark and bright spots here which are called as speckle pattern.

So, this is nothing but scintillation effect and it basically introduces at some places deep fades. If your receiver is close to a place where you have you know the beam, there is a destruction destructive effect of the beam then it will result into deep fade and you may the performance of the system may get affected.

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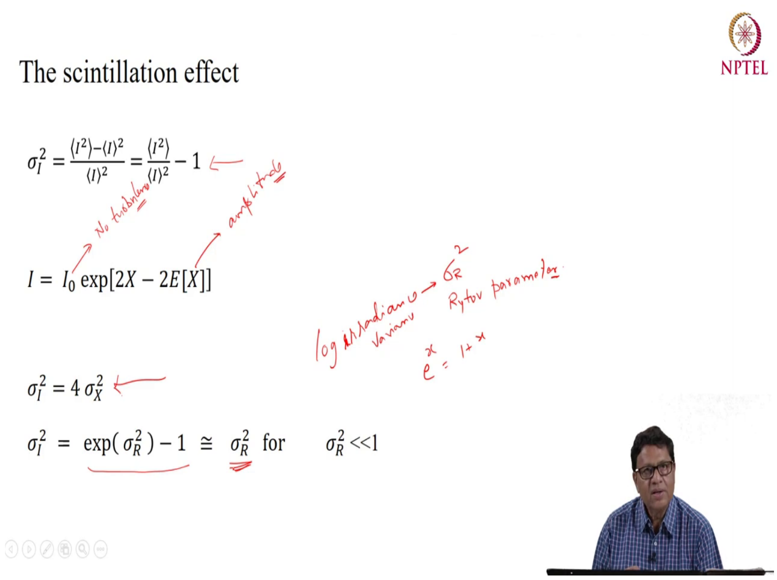

So, Scintillation index this is the whole thing is defined by scintillation index which is intensity square or the irradiance square average of radiance square minus you take the average of radiance and then square divided by average of I and then squared. So, this can be written in this fashion, so this is how scintillation index is defined.

And if you see the intensity or the irradiance is defined by I and X is the amplitude and I 0 is when there is no turbulence, no turbulence intensity when there is no turbulence. So, I will be defined as I 0 exponential 2 X minus expected value of X; where X is the amplitude.

And if I change the just want to see the effect of scintillation index that is sigma, I square in terms of sigma I square that is the because of the change in the amplitude or the variance in the amplitude, they are related by sigma I square is equal to 4 X square.

And sometimes you take log variance of the irradiance or log irradiance and then that variance log irradiance variance if I take the log of irradiance rather than only the irradiance that will be defined by a term which is called as sigma R square, which is nothing but Rytov experience Rytov parameter.

So, this the scintillation index can also be written in terms of log irradiance variance and this is generally given by exponential sigma R square minus 1 and if this value is 1 this can be equated to sigma R square. Because if e you have e is over x and if x is small this can be written as 1 plus x. So, this will be equivalent to sigma R square for low value of right of experience and this is actually weak turbulence effect.

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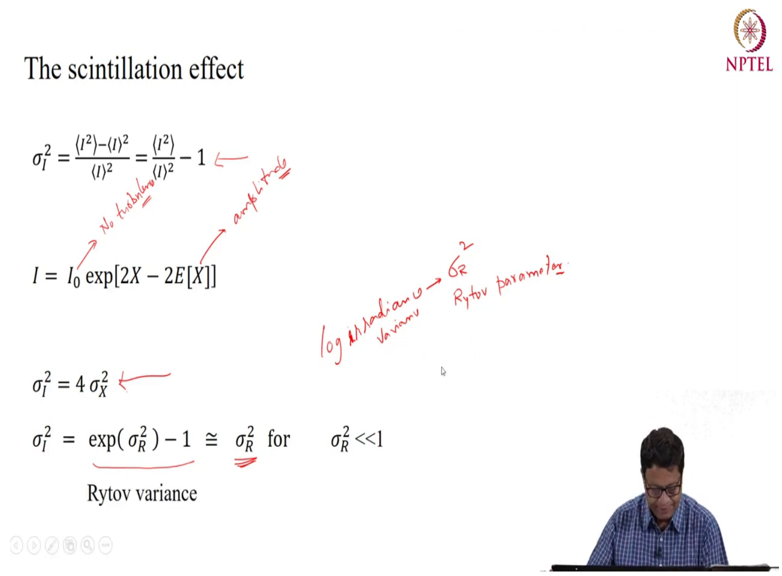

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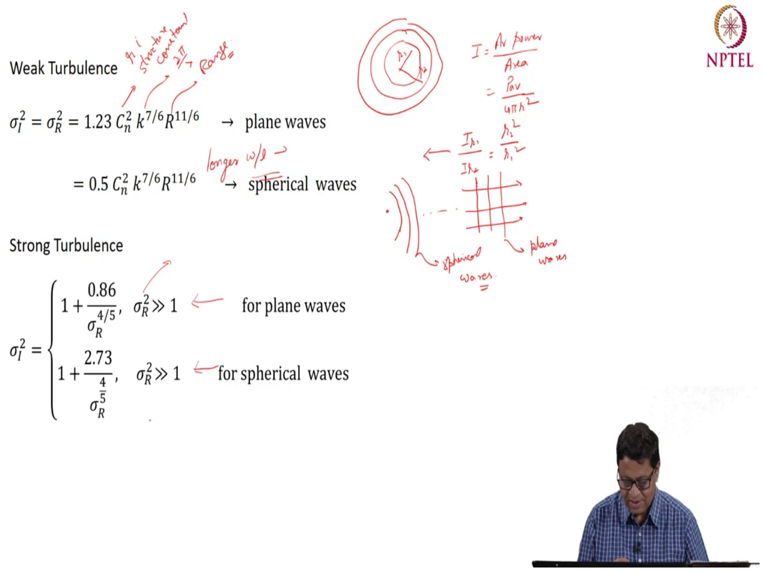

So, there are 2 types of waves plane waves and the spherical waves. So, this scintillation index has been defined both for plane waves and spherical waves. So, when you have a source and it is emitting in all the directions. So, this is say for example, r 1 this could be r 2 let me write a bigger. So, this is say r 1 this could be r 2.

So, if you see the intensity is average power divided by area. So, p average over area if the if the radius is r in general, so this will be 4 pi r square and if I see you know power at different points defined by this r 1 and r 2. So, I at r 1 or divided by I r 2 or I 1 I 2 will be r 2 whole square divided by r 1 whole square.

So, when the wave front is close to the source it is actually a spherical wave front and the energy distribution is given in with as per this formula and as wave moves these, they tend to become parallel. So, these are the rays, so they have plane wave front. So, this is plane waves, this is spherical waves.

The scintillation index sigma I square as we have seen in the last slide is equal to relative experience of weak turbulence is given by 1.23 C n square this is the structure constant refractive index structure constant, which we had defined earlier and k is the wave number which is given by 2 by lambda and R is the range at distance r. So, if you put 2 by lambda here then you can see that for longer wavelength this lesser irradiance function.

So, longer wavelength will have lesser scintillation index or lesser sigma i square, this is for plane waves and this is for spherical waves which is actually 1 by 4 times less as compared to plane waves.

So, this is C n square its again refractive index structure constant wave number and distance. So, these are the 2 expressions of scintillation index for plane waves and spherical waves. If I see at strong turbulence these values are for plane waves and spherical waves for the scintillation index are defined by this and this, is you know when your turbulence is very high.

So, and the sigma R is actually coming at the bottom which is a function of wave number and here we will see that smaller wavelengths will have lesser scintillation index in this case.

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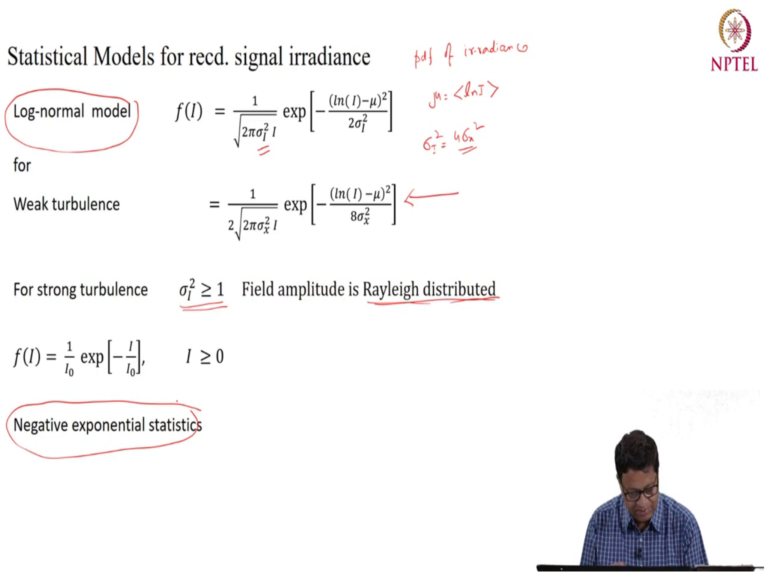

So, now let us go to the some of the Statistical models for received signal irradiance. So, there are some well-known models which are defined how the which actually tells about the PDF of the irradiance which is received which is received irradiance at the receiver.

So, one of the well-known model is Log-normal model, where the radiance the f I the PDF is defined as 1 by 2 pi sigma i square scintillation index this is irradiance exponential log natural of i the average where mu is divided by 2 sigma i square. So, this is the log normal model this is for weak turbulence.

And I can always change this I sigma square if I see the changes in amplitude and take that variance. So, this will be 4 sigma x square and this putting this value of sigma i square by 4 sigma x square i can also write in terms of you know given by this equation; where it is in terms of the variance which is there in the amplitude of the received signal.

So, this is for the log normal model for weak turbulence. This cannot be used for you know high turbulence, because experimentally we have seen that this model fails when you try to use the same model for high turbulence.

So, for strong turbulence the field amplitude is actually Rayleigh distributed and this is much greater than 1. So, for strong turbulence this model is not valid; whereas, the model which is called as exponential model or the negative exponential statistics model that is valid, because the field amplitude at the receiver is rayleigh distributed when you have the strong turbulence. So, this F I the PDF is given by 1 by I naught this is without turbulence exponential minus i by I naught. So, this is another model.

So, we have seen one model which is a Log-normal model which is generally valid for weak turbulence and then for strong turbulence we have another model which is called Negative exponential model or Negative exponential statistics.

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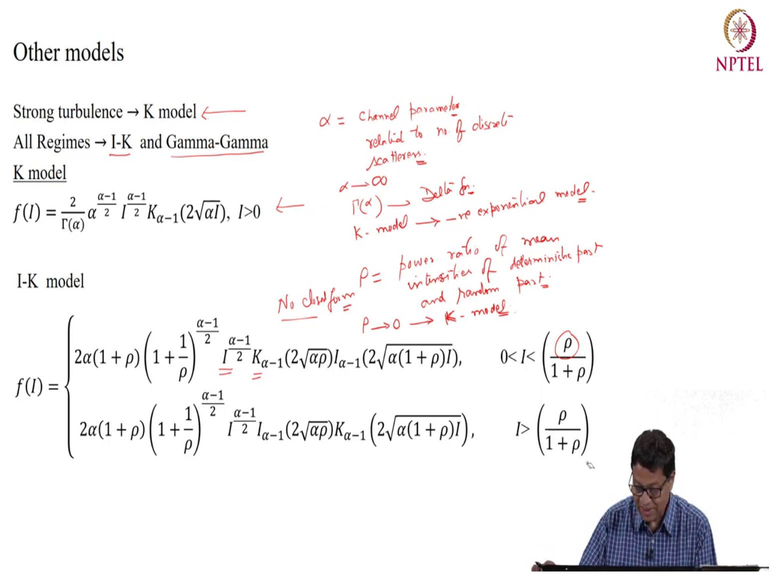

There are other models because these models are specifically for some you know regime, for example weak regime or high regime, but we should have a model which should actually span across the regimes; the same model should work for weak or moderate or high turbulence.

So, there are for stronger turbulence there is a K model and for models which are valid for all regimes you have 1 I-K model and Gamma-Gamma model. So, let us first see the K model, K model the PDF is given by this which is where I and K are the Bessel functions of order alpha second order Bessel function of order alpha and alpha is actually defined as this is a channel parameter and this is related to number of discrete scatteress.

So, this K model is actually only valid for strong turbulence and at when alpha is very high this gamma function here when alpha tending to infinity. For example, this gamma function actually becomes a delta function.

And this K model will reduce to under this condition when alpha is infinite or number of discrete scatterers are many. So, under this condition this gamma alpha will change to will go to delta function and this K model will become a negative exponential model which we had discussed in the last slide negative exponential.

So, this is K model and which is which has a very small regime for high value of turbulence this model is valid and this is similar to your K model when you know alpha is tending to infinity. So, it becomes similar to your K model. The other model which is valid for all the regimes is I-K model this here again you see this I-K they are the modified Bessel function of order alpha and also there are alphas and beta's.

And alpha and beta are the parameters for alpha is basically the eddies which are corresponding to a small size and beta are the eddies which are corresponding to the bigger size and this rho you see here this rho is actually power ratio of mean intensities of deterministic part, that is the fixed part of the average part, deterministic and random part.

And so, these are the this is the PDF for I-K model and as this rho tends to 0, which means you have high random part this model will also reduce to K model K this reduces to K model and they do not have any no closed form equation no closed form. So, sometimes becomes difficult to handle mathematically.

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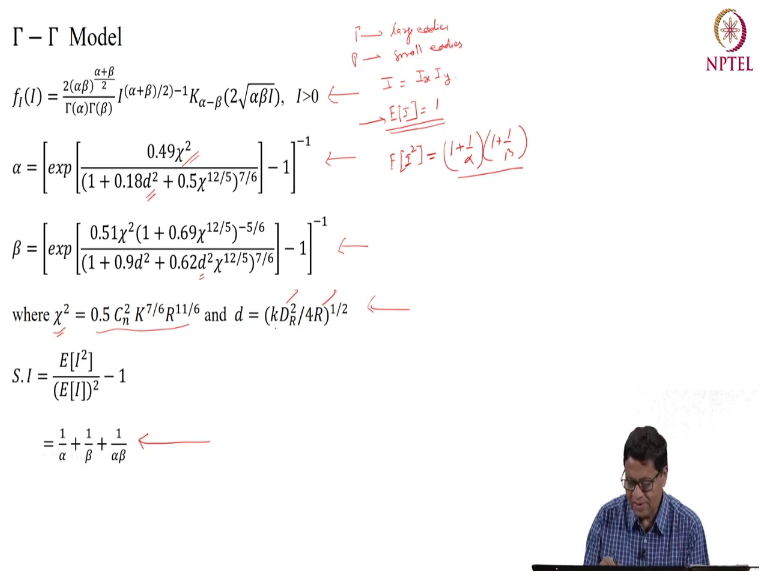

So, in order to have a model which is which has a closed form there is another model which is called as Gamma-Gamma model and Gamma model is basically the PDF is defined by this expression and where alpha and beta are defined by this.

And as you see alpha is function of chi square and d square and these 2 functions, the chi square is defined by this with this is important parameter refractive index structure constant this is k is the wave number and R is the distance and this d has wave number the distance part as well as the D R that is the aperture of the received lens.

So, it involves the C n square the R i refractive index structure constant and also indirectly involved lambda because k wave number is function of lambda the distance and the D R that is the receiver aperture diameter. So, this alpha and beta are components which can be you know calculated based on these parameters given by this equation.

So, and this model actually basically depends on you have two components which is a you know. Why it is called Gamma-Gamma? Because one of the Gamma is for you know large eddies the other is for small eddies and I the radius function is actually a combination of these 2 Ix and Iy and this E expected value we can take as 1.

So, I want to calculate the scintillation index which by definition is given by this and if I take you know expected value of I is equal to 1, then this will change to an E I square so I am not calculating this, but there are some standard textbooks they have calculated this.

So, expected value of I square is 1 plus 1 over alpha into 1 plus 1 over beta. So, by putting E expected value of I as 1 and expected value I square as this we can calculate the scintillation index which is actually 1 by alpha plus 1 by beta plus 1 by alpha beta. And alpha beta you know is a function of psi and psi is function of C n square K r and D. Similarly, you have d that is small d which has receiver aperture lens and wave number and the distance.

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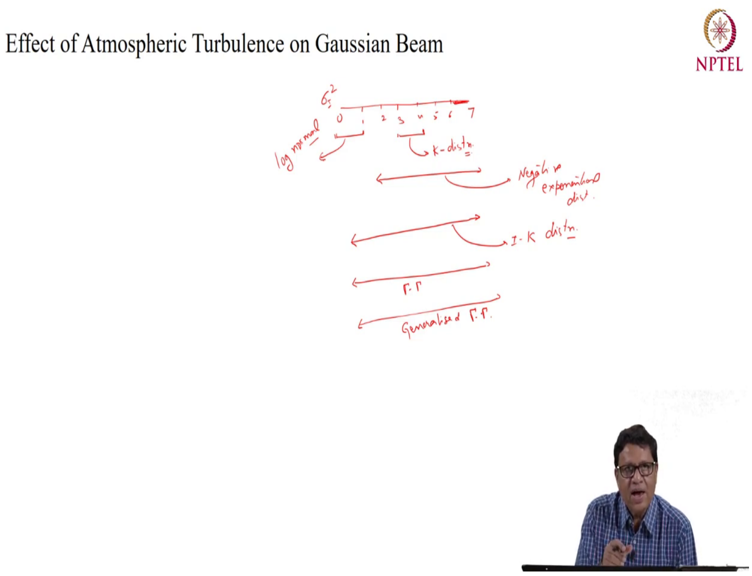

So, now let us try to understand the Effect of Atmospheric Turbulence on a Gaussian Beam. So, what happens in the before that let us just calculate just you know sum up the different distributions which you have studied. So, if you see those distributions suppose I have a scale where rho I square is starting from say 0 to say to 7, I mean this is the range because 7 is considered as highly turbulent and 0 is considered as low turbulence. So, you have 0 1 2 3 4 5 6.

So, in this range which is say 0 to 1 in this range weak turbulence you have the log normal distribution and if you see between 3 and 4 somewhere here roughly which is a high turbulence area when k distribution is valid. And negative exponential is actually only valid for high turbulence, so basically starts from somewhere here 1 and so this is a negative exponential distribution. And I-K basically covers all regimes. So, this is I-K distribution and it covers high all regimes starting from weak to high.

And similarly, Gamma-Gamma which has a closed form solution others have they do not have closed form solution, so difficult to handle mathematically. So, this is Gamma-Gamma which again spans across different regimes and it has a closed form solution, so this is widely used. Nowadays one I mean recently one more distribution has come which is Gamma-Gamma generalized Gamma-Gamma. Again, it is valid for you know all regimes of the turbulence.

So, this is how different models can be used depending upon the value of scintillation index that is sigma I square. So, now let us try to understand what happens to a Gaussian beam as it travels through a turbulent medium?

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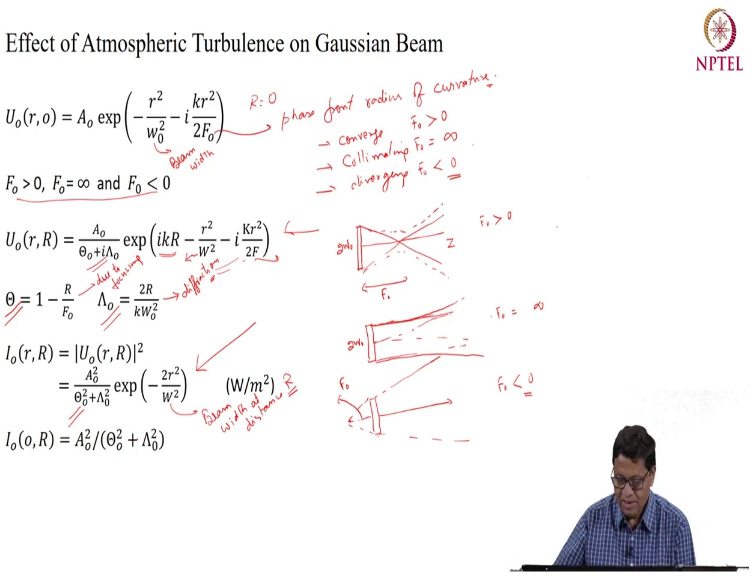

So, Gaussian Beam can be represented by say U r at r is equal to 0 this is capital R that is initially a 0, A 0 is the amplitude exponential minus r square this is the beam width minus ik r square k is the wave number and F naught is given by is a F naught is a phase front radius of curvature phase front radius of curvatures, that depends how your beam forming.

So, there are basically 3 types one is you have converged beam converged converge, the second is collimating, the third is diverge diverging beams. So, this F naught actually is for converging beam is greater than 0 and this F naught for collimating is infinite and is for diverging it is less than 0. So, this is the phase front radius of curvature which actually specifies the beam forming. So, just to understand this you know different structure which will converge the beam or collimate the beam or diverge the beam.

So, this is for example, your beam width say 2 omega 0 and this is moving in say z direction this is z direction. So, the converging beam will behave like this and beam ultimately will take this path this is how the beam will look like and this is actually F 0 for converging.

So, this here it is F 0 greater than 0 and this is first converging and then as it travels it starts diverging. So, this is converging type of beam. The other option is that collimating, collimating again let us see this is your beam width 2 omega naught and collimating in this it goes parallel.

This is so your beam will something will look like this. It is sort of collimating almost goes you know parallel throughout the distance, here F 0 is infinite and the third would be let me write omega square 0 here which is diverging. So, this is the z direction and the beam is diverging and this is actually f 0 which is negative. So, this f 0 is less than 0 here. So, this is how different types of beam forming approaches can be used.

So, this F naught basically defend depends whether you have we have converging or collimating or diverging beams. So, this is mentioned here and at distance r the field is equal to a naught it gets attenuated and then there is also a phase difference which is introduced which is I-K given by this I-K r this is at a distance r that Gaussian beam and where this capital theta is defined by 1 minus r by F naught.

This is due to focusing this is due to focusing I mean whether you are using converging or collimating or divergence due to focusing and the other capital lambda 0 is because of the diffraction. And here this omega this w is actually the beam width after a distance r and this is f this is at the receiver aperture this value of phase front radius of curvature at the receiver.

So, this gets modified as a beam travels a distance of r and this is represented by e naught r capital R and given by this expression. Where these are the beam parameters capital theta and capital lambda naught defined by these expressions. And if I want to calculate the intensity it is amplitude square. So, I 0 is given by amplitude square of the received beam and if I put you know U naught r, R in and into u conjugate r, R and try to see.

So, this becomes watts per meter square exponential minus 2 r square omega square this is the beam width at the at distance r, which has actually increased beam width at distance r of the Gaussian beam which was energized in the beginning or at r is equal to 0 and this is the reduction in the amplitude which is happening.

So, I 0 r, R at r is equal to at R is equal to 0 is given by this formula.

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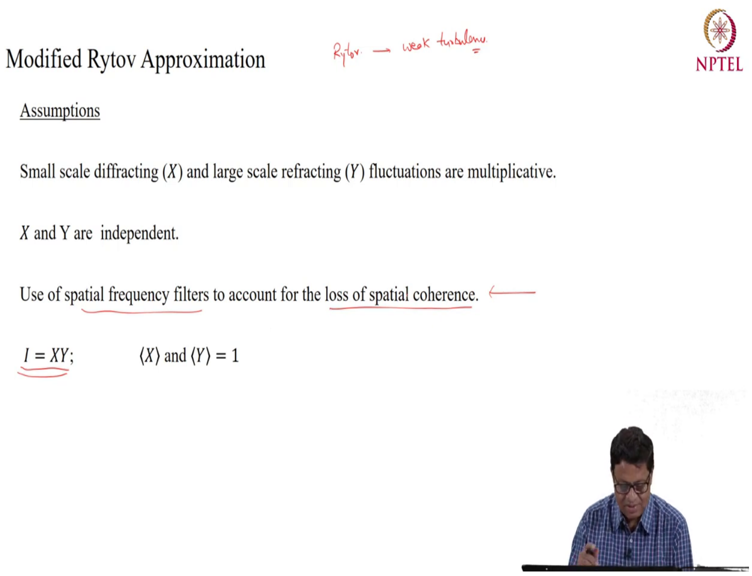

And if I see let me now draw the here itself how the Gaussian beam will look like when it has travel a distance of r. What are the parameters changing what is the power which is received at a distance r let us try to understand some of those quantities.

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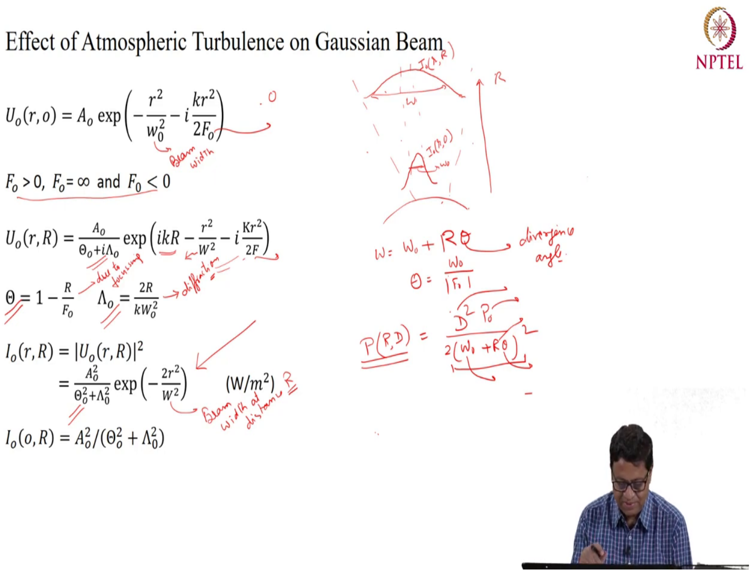

So, as you suppose this is your input Gaussian Beam and this is given by say beam width omega 0 and as it travels a distance R. So, this beam will actually spread and this will become omega that is the distance of R it has the beam width has become omega and so this is the I 0 r, R and this was I 0 E 0.

So, there is a because of atmospheric effect this beam is the beam width size has increased and this beam width size actually is roughly you know is given by this expression, that is m W 0 plus R that is the range into theta and theta is your divergence angle.

So, roughly it is the initial beam width plus R theta and this theta is given by initial beam width size divided by mod value of F naught and if I calculate the power at distance r which is a function of P R and also it is function of the d which is the receiver aperture diameter of the receiver aperture let me write that as capital D. So, the power received P R, D at a at a range r at a distance r is given by D square P naught was the initial power divided by 2 into omega 0 plus R theta whole square.

So, basically depends on receive power depends on your receiver aperture diameter this was the initial power with which the Gaussian pulse was transmitted and this is the initial width and this is the range part and this is the divergence angle this whole thing actually gives you increase in the beam width size and the power received is given by this.

So, basically at high at a long distance this beam width gets expand. So, effectively it is introducing a pointing error loss here and the average power which is sensed by the receiver also reduces.

So, this is how a Gaussian beam behaves under the effect of atmospheric turbulence. Now, this is one more concept which is Rytov approximation is valid for weak turbulence. So, how to modify this for? You know so that it can cover high turbulence as well. So, there are some assumptions which are made and we can calculate the Rytov approximation or use Rytov approximation for any kind of regime whether low turbulence or high turbulence or medium turbulence.

The assumptions maintained are that the assumptions are that those large scale and small scale fluctuations which are happening, because the large scale eddies or small scale eddies the diffraction happening because of the small scale and the refraction happening because of the large scale they are independent and they are multiplicative; which is a valid assumption that is X and Y are independent and the total effect is multiplicative.

So, the total effect is I is equal to XY and let us assume the average of X and Y and Y is their normalized to 1 and we are also periodically using a special frequency filters to account for loss of special coherence, because as it travels beam there is a loss of special coherence happening. So, this has to be you know contained and use of special frequency filter will make the special you know coherence nearly constant.

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$$\langle I^2 \rangle = \langle X^2 \rangle \langle Y^2 \rangle$$

$$= (1 + \sigma_x^2)(1 + \sigma_y^2)$$

$$\sigma_I^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1$$

$$= (1 + \sigma_x^2)(1 + \sigma_y^2) - 1$$

$$= \sigma_x^2 + \sigma_y^2 + \sigma_x^2 \sigma_y^2$$

$$\sigma_x^2 = \exp(\sigma_{\ln x}^2) - 1$$

$$e^x = 1 + x$$

$$\sigma_y^2 = \exp(\sigma_{\ln y}^2) - 1$$

$$\sigma_I^2 = \exp(\sigma_{\ln I}^2) - 1 = \exp(\sigma_{\ln x}^2 + \sigma_{\ln y}^2) - 1$$

In case of weak fluctuation... $\sigma_I^2 = \sigma_{\ln I}^2 = \sigma_{\ln x}^2 + \sigma_{\ln y}^2$

So, I square if I take because X and Y are independent this is ensemble value of X square and Y square and this is given by 1 by 1 plus sigma X square and for y it is 1 plus sigma Y square. And if I put these values in the scintillation index that is I square ensemble values of ensemble value of I square average divided by ensemble average value of I whole square minus 1 will give me 1 plus sigma square plus 1 plus sigma y square minus 1.

And some expanding this will give you sigma square plus sigma square Y square plus sigma square into sigma Y square. Now, I want to change this variation in X with log variation in X. So, this can be seen that variation in X is related to e raised to power variance in log natural X or log X minus 1.

So, this can be equated like this and similarly Y square can be equated like this. And if I put these values sigma i square that is scintillation index is equal to exponential the log variance of scintillation index that is sigma square l n i this log natural minus 1 and putting the value of sigma square log natural for total irradiance is given here minus 1. And this is the formula for you know which handles both small scale as well as large scale turbulence effect.

So, in case of weak turbulence basically this sigma square is nothing but sigma square l I, because this becomes you know this is e raised to power X which can be written as if X is small as 1 plus X. So, using the same thing this can be written as scintillation index for weak fluctuation is sum of sigma square l n X plus sigma square l n Y.

Where this is X, Y is for the large scale turbulence effect and Y is because of small scale eddies or small scale turbulence effect. So, this is how you know this modified expression can be used for all the regimes which includes low weak turbulence as well as high turbulence.

So, this is so far we have understood the effect of turbulence and then let us try to understand how these turbulence effects can be mitigated. There are different schemes for mitigating the effect of this atmospheric turbulence. So, now we will try to understand some 4 or 5 methods by which we can reduce the, or mitigate the effect of atmospheric turbulence.