

**Optical Wireless Communications for Beyond 5G Networks and IoT**  
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**Lecture - 10**  
**Part - 01**  
**Atmospheric Turbulence**

Hello everyone, so, today we are going to discuss about Atmospheric Turbulence. In the last class we had studied about by how the laser beam is affected by the path loss and also how it is affected by the pointing, pointing loss, pointing error loss.

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Atmospheric Turbulence

by



So, today we will understand about atmospheric turbulence, what is the effect laser beam will have depending upon what kind of turbulence you have in the atmosphere.

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$$Y = RX + N$$

AWGN

$$H = H_i H_p H_a$$

IM/DD  $\rightarrow$  amplitude

Coherent  $\rightarrow$

Viscous fluid

laminar      turbulent

$Re \equiv$  Reynold number



So, if you model the system the output is  $Y$  is equal to say responsibility and channel state the input information plus the noise, which can be assumed to be AWGN. Now, this channel state  $H$  will have actually three components, one because of the path loss which let us call that as  $H_i$ . The other could be because of the pointing error loss let us call that  $H_p$ , the third part is because of the turbulence let us call write as  $H_a$  atmospheric turbulence.

So, we have studied about  $H_i$  in the previous class also we had some discussion around the  $H_p$  or about the point the loss because of the pointing error. So, today we are going to discuss the atmospheric turbulence part. So, what actually happens in atmospheric turbulence? When you have you know warmer air in the ground it is less dense, it goes up and mixes with the cold air surrounding it.


And because of this mixing there are certain in-homogeneities which are formed in the atmosphere and they act as a refractive prisms. And these in-homogeneities are of different orders, small in-homogeneities or large in-homogeneities and actually they are called as eddies. So, they are of different size, they have different temperature, they may be traveling at different velocity, they may have different refractive index.

So, when a laser beam passes through these eddies or different sizes of turbulence which is there in the environment, it suffers phase and amplitude fluctuation. Now, if you are dealing with IMDD systems that is Intensity Modulation Direct Detection system, in that case we will be concerned only with the fluctuation which is having which is happening in the amplitude.

Because, this modulation scheme basically depends on the amplitude of the signal or the electric field or the intensity of the signal. And if you are dealing with coherent communication, then I should be worried about both amplitude as well as phase fluctuation.

So, atmosphere is basically consists of if you can call this as a viscous fluid and this viscous fluid the flow is the two types of flow, one is laminar the other one is turbulent. The laminar flow is something like this and turbulent it is some random flow of the air. And how it the how the transition is done from laminar to turbulent flow is defined by a dimensionless quantity which is called as Reynolds number  $Re$ .

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$$Re = \frac{V l_f}{\nu_k}$$

*Handwritten notes:*  
 Characteristic velocity m/s  
 Dimension of flow (m)  
 Kinematic viscosity m<sup>2</sup>/s


$Re < 10^3$  Viscous forces dominate

$Re > 10^4$  Inertial forces dominate

Near the ground

$l_f = 2 \text{ m}$   
 Wind velocity = 1 m/s  
 $\nu_k = 0.15 \times 10^{-4} \text{ m}^2/\text{s}$   
 $Re \sim 10^5$

*Handwritten notes:*  
 $Re < 10^3 \rightarrow$  less turbulent  
 $Re > 10^5 \rightarrow$  high turbulence



So, this  $Re$  is defined as  $V$  into  $l_f$  divided by  $\nu_k$ ; where,  $V$  is the characteristic velocity. This is given in meter per second and  $l_f$  is the dimension of the flow dimension of the flow which is given in meters and  $\nu_k$  is the kinematic velocity viscosity this units are meter per second. So, this is a dimensionless quantity and the first part that is in the numerator basically tells about the inertial forces and the quantity which is in the denominator is actually viscous forces or viscous forces.

So, sometimes initial force will dominate sometimes viscous force will dominate. So, depending on the value of these forces the Reynold number is defined and the Reynold number is actually if that number is  $Re$  is less than  $10^3$  this is called less turbulent atmosphere is less turbulent. And if you have  $Re$  is greater than  $10^5$  it is high turbulence.

So,  $Re$  less than 10 to the power 3 viscous forces they nominate, when the Reynold number is 10 raised to the power 4, initial force is dominate and the atmosphere is considered highly turbulent and if you just to get some idea that near the ground; if I assume  $l$  as 2 meter which is a dimension of the flow.

And when velocity let us assume as 1 meter per second square 1 meter per second, then and  $V$  k the value is 0.15 into 10 to the power minus 4 meter square per second. And if you calculate the value of Reynold number it comes out to be of the order of 10 raised to the power 5 which is near the surface which means at this velocity near the surface the atmosphere is turbulent.

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#### Atmospheric Turbulence

$P_e(a) = P_e$  for the signal strength  $a$

$$\langle P_e \rangle = \int_0^\infty P_e(a) p(a) da$$

$$\text{Var } P_e = \int_0^\infty (P_e - \langle P_e \rangle)^2 da$$

$$\langle P_e \rangle = P_e(\langle a \rangle) + \frac{1}{2} \int_0^\infty P_e^{(2)} \{ \langle a \rangle + \theta(a)(a - \langle a \rangle) \cdot (a - \langle a \rangle) \}^2 p(a) da$$

$$Y = RHX + N$$

$$H = H_1 H_2 H_3$$

JM/D → amplitude

Celestial →

Viscous fluid



Now, let us now try to understand why atmospheric turbulence will cause reduction in or more increase increasing the bit error rate or the probability of error. So, as the signal which is getting received at the receiver is random, because you require certain number of minimum

certain number of photons in order that receiver should be able to detect the signal. Now, this is fluctuating, this the number of photons which are arriving, because of the turbulence that is fluctuating it is a random variable.

So, the probability of error actually depends on the signal strength. So,  $P_e$  let us define  $P_e$  as the probability of error for the signal strength  $a$ . Now, the signal strength which is getting received as a PDF which is defined by  $P_a$ , because it is changing with time and turbulence is you know changing this parameter; so, it has a PDF.

So, the property of this  $P_e$  the probability of error basically is decided by 2 parameters, first one is what is the mean value of the probability of error and the second moment which is the variance in the probability of error. So, the mean is defined as  $P_e$  as  $P_a$  that is the PDF of the received signal strength and variance is  $P_e$  minus ensemble average of  $P_e$  whole squared  $da$ .

Now, if I use a Taylor series for expanding  $P_e$  around ensemble value of my received signal strength  $P_e$  as  $a$ , I get this expression which is written here that is  $P_e^2$ . This is second derivative with respect to  $a$  that is  $d^2 P_e / da^2$  into ensemble average of the received signal and  $\theta$   $a$  this value is actually from 0 to 1, it can take any value from 0 to 1 and then rest of the quantities.

And if you see that  $P_e$  average  $P_e$  will always be greater than  $P_e$  at some average signal strength, because this function is positive. Why this is positive? Because, you have the second derivative of probability of error with respect to signal strength so, if you plot any typical see any typical curve suppose this is signal strength. Or signal to noise ratio or and on this side you have the probability of error it is a non-linear curve with a positive second order differential.

So, so, because of this nature this quantity is going to be this quantity which have it is going to be positive. So, my average probability is actually will change with the presence of turbulence in the atmosphere. So, this atmosphere turbulence will have a effect on the performance of the link, it is going to degrade the probability of error.

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Kolmogorov Model

$$r = |\vec{r}_1 - \vec{r}_2| \quad l_0 \leq r \leq L_0$$

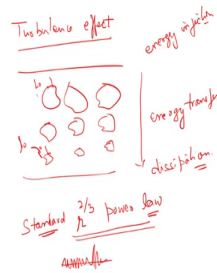
Structure-function for a r.v.  $x(r)$

$$D_x(r(\cdot)) = D_x(f(\vec{r}_1, \vec{r}_2))$$

$$= \langle |x(\vec{r}_1) - x(\vec{r}_2)|^2 \rangle$$

If  $x(r)$  r.v. having mean and fluctuating component

$$x(r) = \langle x(\vec{r}) \rangle + x'(\vec{r})$$



Now, let us to understand the turbulence effect or which is a laser beam will suffer, we need to model this and the very important model which is used for modeling turbulence effect is Kolmogorov Model or Kolmogorov Theory. So, in this theory basically it explains about how the energy is transferred from large eddies to smaller eddies, how much energy is contained in the eddies, how much energy is converted into or dissipated into heat.

So, there are bigger eddies there are smaller eddies and those eddies let us you know these are some smaller eddies and these are some mid size eddies in that one. And these are you know smaller eddies and let us have the outer scale diameter for this bigger eddy they say  $L_0$  and for smaller eddy say small  $l_0$ ; so, this will be small  $l_0$ .

Now, the large size eddies they are unstable; so, they you know break into smaller eddies. And again these smaller eddies will further breakdown into further smaller eddies you know

and this process goes on until the Reynold number is small and eddy motion is stable in the atmosphere. And molecular viscosity is effective in dissipating the kinetic energy in the atmosphere.

So, basically what is happening here in this is there is a energy transfer happening from bigger eddies to smaller eddies. So, energy injection this point can be energy injection and in the end there is some dissipation; so, this can be explained using energy cascade flow. So, the energy from the you know bigger eddies they are transferred to small they break into smaller and then further you know smaller unless you have a stable system or you have low Reynolds number, and then finally, the energy is dissipated.

Now, this effect as per Kolmogorov is actually you know follows a standard  $r$  raised to power  $2/3$  power law I will explain this what is this? So, if I take points two points, two position vectors  $r_1$  and  $r_2$  which are separated by  $r$  and this  $r$  is between the outer scale of eddy that is  $L$  naught and smaller scale of eddy that is  $L$  naught. So, this effect you know follows its typical or standard this has been proved experimentally as well  $r$  raised power  $2/3$  power law.

Now, let us define a structure function, because we will ultimately we are modeling the turbulence effect in the atmosphere. So, we need to define because the temperature is changing, the refractive index in the atmosphere is changing or it is different at different places, velocity is different places. So, let us define a typical standard function, a structure function for a random variable  $x_r$  which is actually function of  $r_1$  and  $r_2$ , the two position vectors which I had mentioned here separated by  $r$  as given here.

And this is equal to  $x_{r_1} - x_{r_2}$  mod square and then the average is taken; so, this is a this is a definition of a structure function. Now,  $x_r$  will have two components, one is mean component which is sort of average and then you have the fluctuating component. So, basically  $r$  can have you know some mean component and then there is over it there is some fluctuation fluctuating component.



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Therefore

$$D_x(r) = \underbrace{[\langle x(\bar{r}_1) \rangle - \langle x(\bar{r}_2) \rangle]^2}_{\text{mean part}} + \underbrace{[\langle x'(\bar{r}_1) \rangle - \langle x'(\bar{r}_2) \rangle]^2}_{\text{fluctuating part}}$$

Structure function for wind velocity

The structure function in the inertial range satisfies the universal 2/3 power law

$$D_v(r) = \langle (v_1 - v_2)^2 \rangle = c_v^2 r^{2/3} \quad l_0 \leq r \leq L_0$$

$$D_t(r_t) = c_t^2 r^{2/3}$$

velocity structure constant



And if I denote this  $x r$  as the mean component and  $x$  prime  $r$  as the fluctuating part and put this value in this expression and do some calculations, then I will get this structure function as given this. So, where I have you know grouped the mean part and then this is the fluctuating part; so, this is the fluctuating part and this is the mean part.

Now, we are able to understand the structure function of different quantities or for example, if I want to define the structure function of wind velocity. And as I mentioned that this is in the inertial range because this 2 by 3 power law is actually valid in the inertial range. Because, there are two types of ranges viscous range and inertial range, but this has this is valid when you are in the inertial range.

So, the for the wind velocity, the structure function will be  $v_1$  minus  $v_2$ , where  $v_1$  and  $v_2$  are the velocities at two points, in earlier case we had  $r_1$  and  $r_2$ . So, at those points separated

by a distance  $r$ , which will be given by a structure constant  $c_v$  square, velocity structure constant, this is velocity structure constant into  $r$  raised to power 2 by 3, this is the that famous 2 by 3 power law under inertial range.

And similarly, this can be done for temperature where  $D_{tr}$  is equal to  $c_t$  squared and  $c_t$  square is temperature structure constant into  $r$  raised to power 2 by 3. So, this is the variation of temperature at 2 points defined by you know  $r_1$  and  $r_2$  vectors, this is for velocity and this is for temperature.

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$$\begin{aligned} n(r) &= n_0 + n'(r) \\ &= 1 + n'(r) \\ &= 1 + 79 \times 10^{-6} \left( \frac{r'(r)}{r'(r)} \right) \\ D_n(r_n) &= C_n^2 r^{2/3} \\ \underline{C_n^2} &= \left[ 79 \times 10^{-6} \frac{r'}{r'^2} \right] \underline{C_t^2} \end{aligned}$$



And similarly, the refractive index is also changing from one position to another position. And let us assume that it has two parts, one is the constant part which we will take as 1 for air and then  $n$  dash which is a fluctuating part. So, this is 1 plus  $n$  dash  $r$  and if I do some calculations then this can be because this is function of pressure and temperature.

So, this can be written as  $1 + 79 \times 10^{-10} P^6$  divided by  $T^6$ . And  $D_n$  is defined by  $C_n^2$ , this is refractive index structure constant into  $r$  raised to power  $2/3$ . So, we have defined all the three quantities, the temperature, the velocity and the refractive index. And using some other equations or which we shall not be discussed here; so, you can relate  $C_n^2$  and  $C_t^2$  using this formula; so,  $C_n^2$  and  $C_t^2$  can be related.

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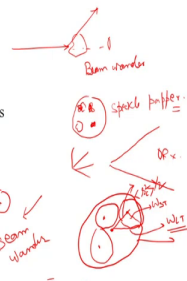
### Turbulence Effect

Beam wander...Eddies larger than the beam size will deflect  
 Beam scintillation...Eddies same size as beam size will act like lens  
 Beam spreading...Eddies smaller than beam size will result in diffraction and scattering

#### Beam wander model

$$W_{LT}^2(R) = W^2(R) + W^2(R) T_{ss} + W^2(R) T_{Ls} + \langle r_c^2 \rangle$$

Handwritten notes:  $W_{LT}^2(R)$  is labeled "long beam with turbulence at distance R".  $W^2(R)$  is labeled "small size eddies".  $T_{ss}$  is labeled "large size eddies".  $T_{Ls}$  is labeled "Beam wander".  $\langle r_c^2 \rangle = \frac{1}{2} W_{LT}^2(R) - \frac{1}{2} W^2(R)$ .



So, now let us see the effect of turbulence, what actually happens because of turbulence? So, the first effect which happens is beam wander. Now, there are eddies which are small which are size and then you have a beam size. So, if the eddy size is larger than the beam size, then the whole beam is actually deflected and in the receiver there is a possibility that you might miss the beam.

So, what happens actually if this is a beam going in this direction? Because of the eddy, it might get deflected and your receiver is for example, here. So, you may miss the receiver; ideally, you should have gone like this; so, you will miss the receiver, the beam will miss the receiver; so, this is called as beam wander. The other is scintillation, the eddies which are of similar size as the beam size, these eddies will act like a lens.

So, sometimes they will be focusing of the radiation, optical radiation or sometimes it will defocus. So, because of this focus and defocus, there will be some, you know you will have some constructive and destructive kind of pattern onto the receiver. And if you see in the plane of the receiver, you will get some dark spots, some bright spots. This is also called as speckle pattern; so, this will result this is what is called as a beam scintillation and it is because of the eddies which are of similar size as the beam size.

So, there are ways of handling, you know these anomalies and how do you mitigate this effect that we will discuss later. So, this is the another effect which turbulence atmosphere will have, that is a beam scintillation. The third is beam spreading, this is because of the eddies which are smaller than the beam size. So, what happens? Because of these eddies, the beam will get scattered; there will be redistribution of energy in the wave front.

So, and because of these redistribution of energy because of this scattering, there will be loss. The light will get or the laser beam, the power will get spread over a large area and on the receiver you will get much less area; so, this is called as beam spreading. So, basically what happens? If your receiver size is this much and beam has for example, spread this much; so, only part of the energy is captured by the receiving area.

So, the turbulence, the atmospheric turbulence has these three effects and let us we will study in detail about these effects; so, let us start with the beam wander. So, in beam wander what happens when the laser beam traverse through the atmosphere, the hot spot where the max the energy is maximum actually keeps changing, there is a you know sneak like movement.

So, sometimes you know hot spot is here, sometimes the hot spot is here, sometimes this is here. And if I see the profile for example, corresponding to this hot spot, it will be something like this is a Gaussian beam, where we are having maximum at this point. And this will give me a maximum at this point; this will give me maximum at this point. So, if you see the overall effect, there will be a Gaussians you know spread out Gaussian or the Gaussian profile is skewed; so, this is what you get at the receiver.

So, this is actually the movement of hot spots as the beam travels in the atmosphere, this happens because of the turbulence. So, the other effect it can have is you know there will be beam wander; so, if I make a model try to model this beam wander. So, basically this is one effect which will happen, because of the movement of the hot spot. And also the another issue will be that when you have for example, say these are say; so, this is say let me redraw this.

So, what happens? This is a center of this circle, this these are the small hot spots where let me denote this as the beam width size here is  $\omega$  a short term and because it is changing. So, it will ultimately result in a bigger circle of this radius and let me denote this as a  $\omega$  long term.

So, the beam size this is a short term because of the hot spots and then the combined effect of this hot spot and then and the centroid moving of the beam will result in a bigger circle which will have a radius of  $\omega L$  or this is  $\omega$  long term. And this is the actually the beam wander; so, this will have some average value  $r_c$  square; so, this is the beam wander effect.

So, what actually if I try to model a beam wander; so, this is the long term beam width at a distance  $R$ , this is long term beam width at a distance  $R$  which is happening because of the turbulence. And this beam width  $\omega^2 R$  is actually because of the space, because the wave is traveling through the space and the space will increase the beam width at a distance  $r$  which is given by  $w^2 R$ .

And this particular component is because of the because of the small size at eddies. And this is because of the large size eddies; this width increase is because the large size eddies and this is the beam wander part which I had explained. So, you can say this part whole is short term that is short term. So, basically it gives you  $r^2 c^2$  half will be  $\omega^2 l^2 r$  minus width  $W^2$  square this is  $W R$  and this is short term. Or you can write this as and the beam this is a beam wander; so, beam wander can be written as this.

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### Turbulence Effect

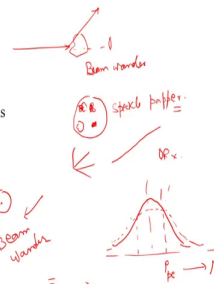
Beam wander...Eddies larger than the beam size will deflect  
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#### Beam wander model

$$W_{LT}^2(R) = W^2(R) + W^2(R) T_{ss} + W^2(R) T_{Ls} + \langle r_c^2 \rangle$$

Handwritten notes on the slide include:

- $W_{LT}^2(R)$  is labeled as "long term beam width at distance R".
- $W^2(R)$  is labeled as "small size eddies" and "large size eddies".
- $T_{ss}$  is labeled as "scintillation".
- $T_{Ls}$  is labeled as "beam wander".
- $\langle r_c^2 \rangle$  is labeled as "beam wander".
- A handwritten equation:  $\langle r_c^2 \rangle = \frac{2}{\omega_{LT}^2(R)} - \frac{W^2(R)}{s^2}$ .



So, let me again explain this; so, you have hot spot moving and also the centroid of the beam is also moving around its central position. And because of this the long term effect will be a bigger you know hot beam size or spot size of dimension  $W L T$ . And this  $W S T$  is for the smaller because of the hot spots and then this gives you the beam wander part which is and the whole beam wander model can be modeled by this equation.

And the overall effect of this beam wander model will be if you launch a Gaussian pulse like this, then because of this beam wander at a distance  $r$  the pulse will become flattened in the middle. So, near the bore side it will get flattened; so, this is the. So, if you see the power in this range it has actually decreased.

So, it has introduced some sort of pointing error in the system which is you can say this can be defined as the  $\rho$  sigma or  $\rho$   $P_e$  pointing error. So, because the beam has flattened, you have less power and also it is less power across this whole width. So, effectively you know the system has a pointing error introduced into this because of the turbulence effect; so, this is the effect which the beam wander will have.