Optical Wireless Communications for Beyond 5G Networks and IoT
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Lecture - 09
Part - 02
Range equation of FSO link

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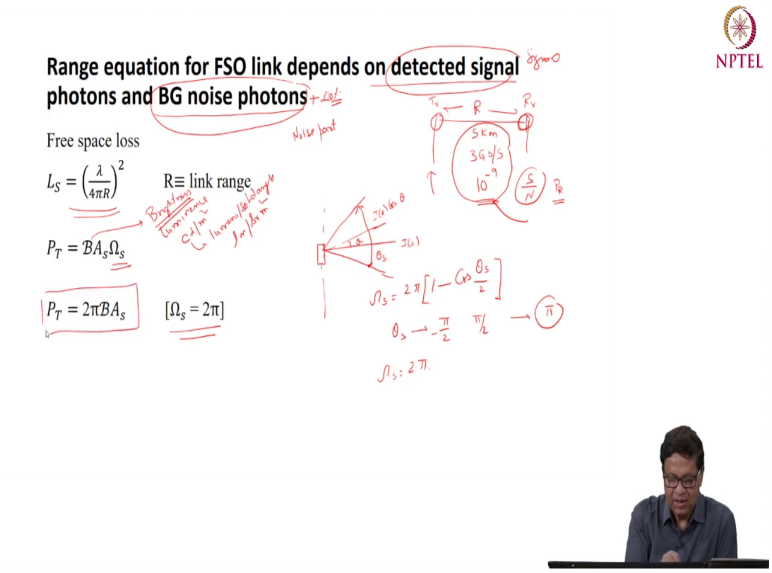

Now, let us now try to calculate the range equation for FSO link. This is my T x. So, how do I design a system? Suppose I want to design a system say for example, 5 kilometer and I want to design a system for say 3 gigabits per second and my quality of service or the reliability or the probability of error say for example, of this order.

So, how do we design what should be the level the transmit power? What kind of source I should use a laser source and what kind of divergence angle? What kind of receiver? What kind of antenna gain at the transmitter antenna gain at the receiver right when these parameters are given.

Or there can be; there can be a system which has thousands of kilometer for example, space links geo satellite from one geo satellite to another geo satellite then what kind of components I need or what kind of antenna gain at the transmitter receiver I require. So, we will try to understand this using range equation for free space optics link.

So, the idea is to how much photons I detect at the signal and how much as a background noise photons, it can come from solar radiations, it can come from noise of the receiver. So, basically this gives me the signal part and this will give me the noise part. So, this also includes detector noise.

So, this is how I because for meeting this requirement I need some singular to noise ratio at the receiver or some received power which will satisfy this requirement of 5 kilometer 3 gigabits per second or and beta ray of 10 to the power minus 9. So, let us now build the theory part first and then we will understand this using two examples. So, free space loss is generally given by L S is equal to lambda by 4 pi R whole square where R is the link range this is R for example.

So, this is a free space loss lambda by 4 pi R whole square. Now, let us see that this is the transmit power which can be written in this form where B is actually your brightness or we can use another term which is luminance and then luminance is actually given as candela per meter square where candela is your lumens per solid angle. So, the units are actually lumen for solid angle it is steradian into area.

So, this B is brightness or luminance which is actually candela per meter square and candela we know is a lumen per steradian. So, this is you can see here as well B is brightness which is luminance into the area into the solid angle. So, P T A s omega s is the transmit power. Now, let us assume a Lambertian source, Lambertian source is basically it is emitting like this and this varies as intensity here will be say for example, I 0 and this angle is theta.

So, this will be I 0 cos of theta. So, if I convert this angle this whole angle let us call it as a theta s 2 solid angle because ultimately the power is going into the solid angle. So, this is of course, converting into this angle into a solid angle we can use this formula 2 pi 1 minus cos of theta s divided by 2. So, this theta s actually the limits are you know minus pi by 2 2 pi by 2.

So, the total angle is actually pi, Lambertian source can emit anywhere in this range. So, the maximum limit of theta s is pi. So, if I put this theta as pi here I get this solid angle as 2 pi which I have used here and I have put this omega s as 2 pi. So, my PT becomes 2 pi into luminous function into the area.

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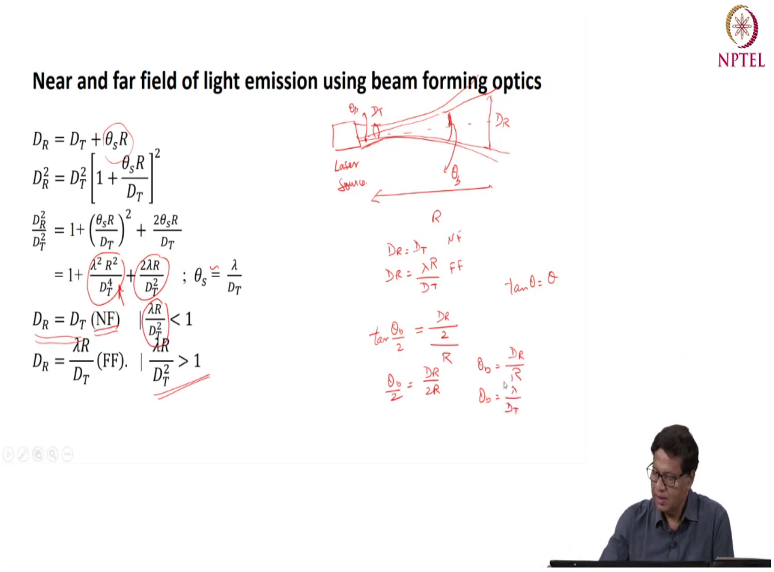

So, now let us understand near and far field of light emission using beam forming optics because I am using telescope or lens arrangement in the transmitter in the receiver. So, let us understand the concept of near and far field of light emission using beam forming optics. So, for example, this is your laser source and you have beam forming lens here optics here.

So, the beam from the laser source will come gets converged and then starts diverging this is what happens. So, let us define this as midpoint. So, let us define different angles here. This is for example, your theta s this small angle divergence let us call it theta s and the transmit aperture is let us called as D T and similarly you have the receive aperture at distance R.

So, this is a range for example. So, this will be let us call it as D R or the planar beam diameter or the receive aperture. So, D T and D R they are defined and then let us called as a beam angle as this and let us call this as a theta b. So, theta b is the beam angle and this is using this lens I am able to beam form, but as it travels in the environment it diverges and this is defined by this D R which is a planar beam diameter.

So, this D R which is the receive beam diameter is actually given by D T plus theta s theta s is the divergence angle into the R. So, as R increases this quantity will increase and gets added up and then there is a expression which is D R is equal to D T plus theta s R. Now, I am squaring these terms D R square into D T square I have taken D T square common. So, this becomes 1 plus theta s R divided by D T whole square. Let us take this D T whole square here.

So, this becomes expand this this becomes 1 plus theta s R divided by deta T whole square plus twice of theta s R or D T. Now, I am using that theta s the divergence angle is actually lambda by D T to be precise it is 1.22 lambda by D T, but let us not worry about that 1.22 factor we can take this as 1. So, theta s is approximately lambda by D T. So, putting these values here lambda by D T we I get this.

Now, when your r is a small that is a near field so, this quantity and this quantity is actually less than 1. Then D R square D T square is equal to 1 then D R is equal to D T. So, this is the near field equation here. So, basically this this quantity lambda R by D T square is less than 1.

So, you have D R is equal to D T. The far field you cannot neglect this term and the second term is actually square and the third term is actually you know under root of this.

So, basically will be dominated by this this second term here. So, this equation will be D R is equal to lambda R by D T when under this condition when lambda R is equal to D T square is greater than 1. So, this is far field. So, let us write here. So, D R is equal to D T when you have then near field and D R is equal to lambda R divided by D T when you have far field.

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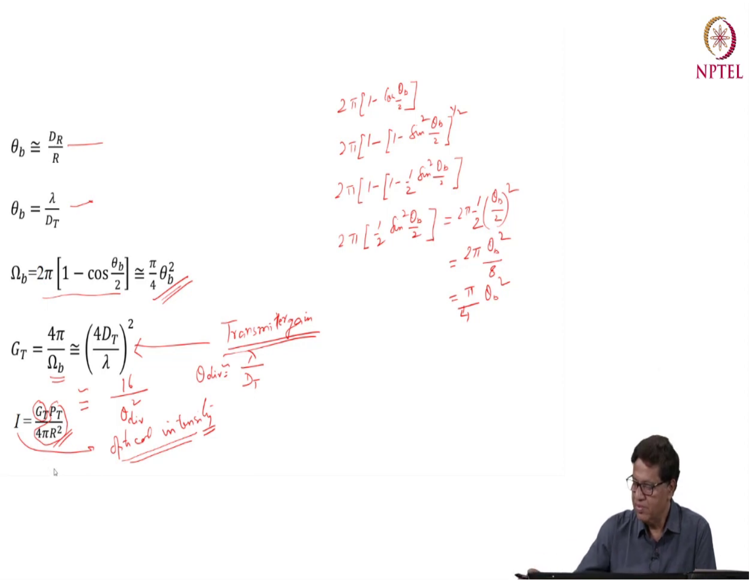

The theta b the beam angle actually is given by D R by R. This you can see from here because if you take you know tan theta by 2 I take half the angle theta by 2 here this angle for example, theta by 2 is actually can be written as D R divided D R by 2 divided by R and this angle is generally small.

So, this can be taken as tan theta when the angle is small can be taken as theta this is 2 up to 15 degree. So, this will give me theta by 2 is equal to D R over 2 R or theta b is equal to D R over R and theta b is also equal to lambda by D T. So, this theta b is equal to approximately equal to D R by R and theta b is equal to lambda by D T.

So, I can convert this solid angle omega b using this formula 2 pi 1 by cos theta b by 2 and this expression gives pi by 4 theta b square because what you can do is I mean if I write this equation this is 2 pi into 1 minus cos theta b by 2 is actually 2 pi 1 minus I can write in this fashion sin square theta b by 2 raise to power half.

So, this will be 2 pi into 1 minus and this theta b is very small. So, this can be written as 1 minus half this half can come here sin square theta b by 2. So, this will give me 2 pi this 1 and minus 1 will get cancelled and what you get is half sin square theta by b by 2 again I you know invoke the same thing that theta b is small.

So, this will become 1 by 2 pi by 2 pi into theta b over 2 whole square sin theta is equal to theta when theta b is small. So, if you do this what you get is 2 pi into theta b square over 8 which is nothing but theta b square pi divided by 4 pi by 4 theta b square. So, let us write it clearly here this is pi by 4 theta b square.

So, this is what I have written here pi by 4 theta b square and let us define the G T as that 4 pi the whole solid angle divided by this solid angle omega b and putting these values of theta b and using these expressions what I get is G T is equal to 4 D T by lambda square this is a transmit gain. So, this is expression for transmit gain.

This also you can because we know that theta divergence is equal to lambda by D T is 1.2 to lambda by D T, but for approximation we can take lambda by D T and I if I put this D T in this expression this can also be written as 16 over theta divergence whole square. So, this is another equivalent expression for gain if theta divergence is given you can calculate straight away you can calculate the transmit gain and if transmit aperture is given then you have to put the value of D T here and then get the value of the transmit transmitter gain.

So, this is the transmitter gain and if I calculate the optical intensity which is power divided by area. So, I get this is the gain part and this is the transmit power divided by 4 pi R square gives the optical intensity.

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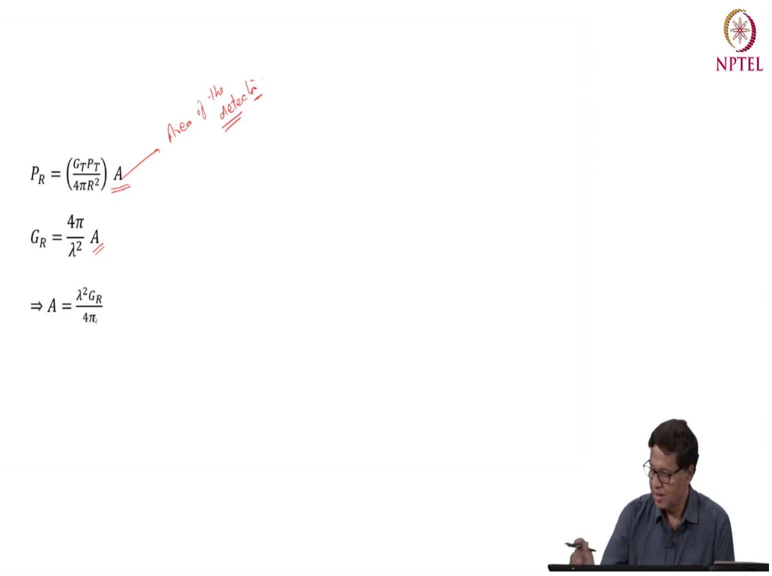

So, this is expression for I optical intensity which is G T into P T divided by 4 pi R square. So, P R power the received power now it is falling on some area of the detector this is area of the detector or area of the receiving device area of the detector. So, G T P T divided by 4 pi R square is the transmit part G T is the transmitter gain, P T is the transmit power divided by the 4 pi R square and this falls on to the area detector.

So, the basically it will be G T P T 4 pi R square into A and the G R the gain at the receiver can be written as 4 pi by lambda square that area of the detector.

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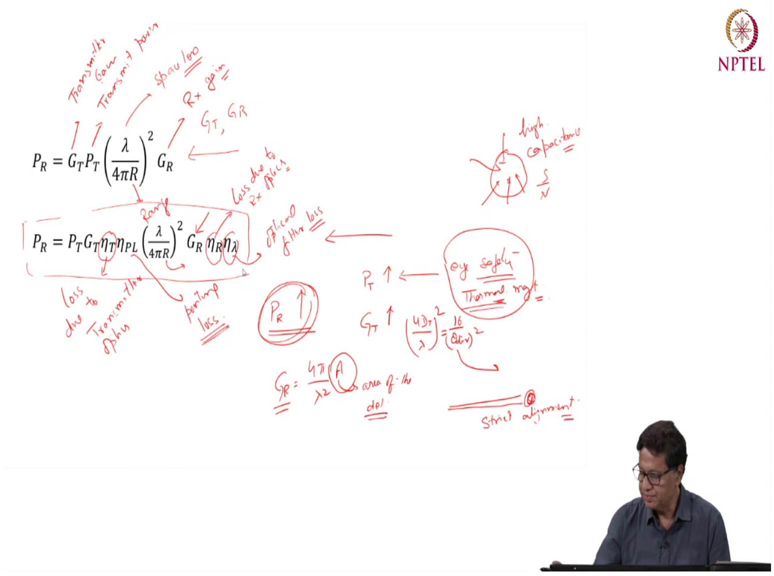

So, from here you can get A which is this lambda square G R 4 pi and if you write that now the complete equation which has G T transmitter gain and the G R the receiver gain and the space loss this is the space loss parts because the light will lose its intensity as it travels. So, this is because the space loss. So, this is the received power you get at the receiver. This let me again write here this is transmitter gain because of the optics or that which I am using at the transmitter this is a transmit power.

This whole thing is space loss which is given by lambda by 4 pi R, R is range equation and G R is the receiver gain again using some optics at the receiver so, G R is the receiver gain. So, in order to make this complete I need to consider other factors also for example, I am using some optics there may be some loss. So, this is loss due to transmit optics loss due to transmit optics or transmitter optics.

This is the pointing loss because the transmitter and receiver if they are not aligned or if there is some mismatch there will be some loss of power. So, I need to factor in this this this also which is called as pointing loss pointing loss and then this is the space part which is taken from here.

G R is the receiver gain and then you have the n R this is the loss due to receiver optics loss due to receiver optics or at the receiver and this loss could be because of the filter optical filter which you might be using in the receiver. So, optical filter loss or gain depending upon what it does right.

So, the complete expression is P R is equal to P T into G T, neta T into neta P L pointing loss space loss lambda by 4 pi R whole square G R is a receiver gain and loss due to receiver optics neta R and neta lambda is because of the filter loss because of the optical filter. Now, let us try to understand this received power how idea is to increase this received power.

Because more than a received power then I will be able to decode the signal correctly I will get higher data rate. So, idea is to increase the received power. Now, for increasing the received power this P R I can either increase P T, but increasing P T I cannot go beyond a certain limit, because issues related to eye safety for example, because it is outdoor you know where you have you may have some human beings right.

So, eye safety requirement one has to meet, then there with some thermal issues because high power means you require a source which generates, high power you require large kind of current which will produce heat. So, there are thermal issues there. So, these are the constraints I mean you cannot go on increasing P T in order to get high P R. So, one has to working at these limits of eye safety and thermal management.

The other option is I increase G T. As we know this G T is equal to 4 D T divided by lambda whole square or this is also equal to 16 theta dive whole square. So, if I am doing this G T; that means, I have to decrease theta divergence, theta divergence meaning I have to make the beam very very narrow. If I make the beam very very narrow and there is a receiver here, this alignment becomes very very critical even if there is a mismatch of the alignment you will not get any signal.

So, there is a limit up to which I can you know decrease theta divergence. So, lower value of theta divergence means I require strict alignment, which may not be possible in in practice. So, again you know one has to do within a certain limit. These anyway you have to keep small the losses which are happening because of the optics one has to keep very very small. Pointing loss is also has to be kept loss it kept low and some time you have tracking mechanism so that this pointing loss is kept minimum.

So, whenever it goes you know it misaligns there is some feedback mechanism and then there is alignment is established. So, you can reduce this pointing loss, space loss it is you know depending upon the distance you cannot do much about it. Then you have the G R and we know the G R is actually 4 pi by lambda square into area of the detector or the area of the receiving device area of the detector or area of the receiving device.

Now, I want to increase G R means I have to increase this area, the moment I increase the area of the receiving device. First of all increasing the detector area will give you high capacitance then it may not respond to high deteriorate because the capacitance is high and also if I increase the area you will have more background noise come into it. So, your signal to noise ratio will go down. So, again there is a limit to which you can increase your area if you want to increase P R.

And similarly one has to use optics which has loss less loss at the receiver and then same logic is for filter which are using you should have minimum loss. So, you see here the trade off in order to get to high P R I can play with different components, but every component has some limit or every parameter has some limit. So, this is how becomes a design problem, ok.

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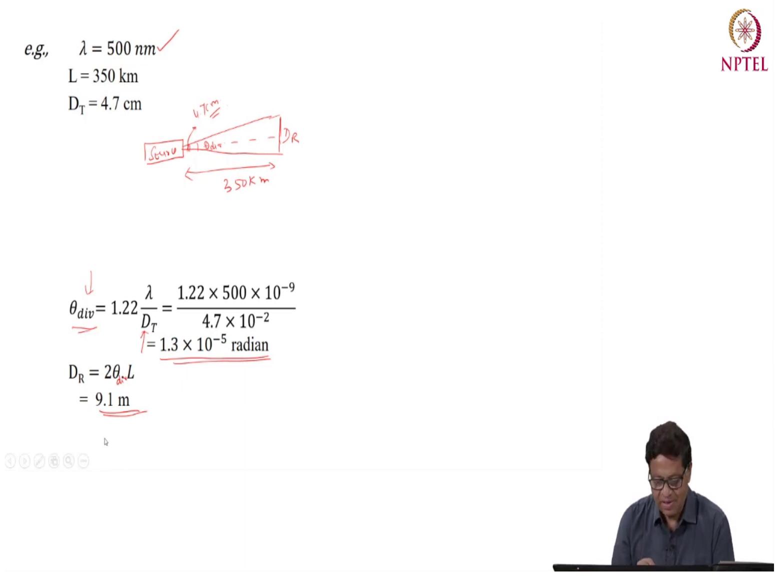

So, let us do one example just to understand you know what kind of values we get. So, suppose I have a source at say 500 nanometer this is a wavelength I have selected this is arbitrary wavelength I have selected and distance is say for example, 350 kilometer and I want to calculate you know theta divergence and the received aperture. So, a beam will become something like this.

So, this is a D R and this is a theta dive theta divergence and D T I use here is let me show here D T or the transmit aperture lens I have used is has dimension 4.7 centimeter and this length is 350 kilometer. So, I want to know what is theta divergence required and what is a D R I will get.

So, theta divergence here I am using exact formula 1.22 into lambda by D T earlier I have been using lambda by D T, but just to get some feel of the values I have used 1.22 lambda by

D T just putting these values what you get is 1.3 into 10 raised to minus 5 radian. So, it is in micro radian 0.13 micro radians and this D R this is D R by 2 and theta dive by 2.

So, you can calculate right D R is equal to 2 theta L actually this is dive divergence into L is 350 kilometer and so, what you get here is a 9.1 meter area of beam with beam spot. So, just to give you some ideas about this so, if you want to get less D R for example, you have to decrease theta divergence, for decreasing theta divergence you have to increase the D T. So, this is all trade off here.