Modern Computer Vision

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Lecture-80

Now, there is something called a brightness constancy assumption, but once we do the math right then it will become more clear, brightness constancy assumption. This is an assumption that actually says that when you go from one frame to another right then within a distance right which is very small the right amen intensity can be kind of right assumed to be the same right amen. So let me just write that down. So for a small space time step for a small space time step that means both in space and in time right you can assume that I of I is your frame I of I is your intensity I of x + ∂ x y + ∂ y and then t + ∂ t right is equal to I of right x y comma t. That means right in the in the in the right immediate neighborhood and both temporally and spatially right. So you can assume that the intensities have been changed much which is okay it is a kind of a reasonable assumption to make right.

And where ∂ x is u ∂ t and ∂ y is v ∂ t and u v is actually the optical flow. This is the optical flow at that particular location okay like I said this u comma v is going to change at at right I mean at every point at every x comma y. So u v is is your is your optical flow and this is how it is now if you do if you do a Taylor series expansion a first order Taylor series expansion then the left right you can write it write this as I of x y comma t + ∂ x and then I will write this as I x right by I x I mean partial derivative of so dou I x y comma t with respect to dou x with respect to x okay that is I x + let us say ∂ y I y + ∂ t okay I t now I t I t will be will be a temporal this one a derivative okay these are spatial derivatives this will be a temporal derivative okay this will come from across the frames okay is equal to I of I of x y comma t okay. So this I of x y comma t right we can actually remove from both sides and if you now if you now write divide this by this time okay ∂ t so what we get is ∂ x divided by ∂ t I x + ∂ y divided by ∂ t I y + I t right is equal to 0 right this is what we get now in the in the I mean limit that right ∂ t tends to 0 right then then this becomes actually ∂ x by ∂ t is your is your u velocity right that is your u component ∂ y by d y by d t is your v component therefore right this becomes u I x + v I y + I t right is equal to 0 this is called the brightness okay right so this is called the optical flow equation based upon the brightness constancy okay u I x + v I y or in other words right you can even write this as I x as actually a dot product I x I y u v is equal to - I t.

See it is one thing that I x and I y and I t right we can actually find out I think I have a slide I mean right which we know how to do actually I mean because spatial derivatives are all you have already seen and right this is simply a matter of you know computing temporal so we know that right if you have an image say I x will come from let us say within the image right because this is like a spatial derivative I y will also come from let us say within the image it is just a kernel right all the several kernels that we that we did when we did edge detection or any of them but then I t is going to be ∂ I by ∂ t so it is like it is like right I mean you know if you want to want to find out right I mean what is this difference at this location right then you just subtract the two and then you will get like whatever 0 0 0 corresponding to this and then 10 10 10 - 1 1 1 that will like 9 9 9 and so on right so you have a spatial derivative and then you also have actually a temporal this one a derivative now the point is right you have only a single equation right but then the unknowns are 2 see I x we know I y we can compute I t I can compute right between successive frames correct I mean so this has to come from 2 successive frames so I have this but then but then right and I x and I y are at a point right again everything is at is at a location okay U V is at a location is that a pixel location I x I y is actually the gradient at that location I t is a temporal gradient right for that location now you seem to have a line like this right now if you really plot this right then what this means is if I say that this U and that is V right then then right then this equation is like U I x + V I y + I t is equal to 0 right so where I know I x I y and I t but the point is right when you so I know that I know that right I mean I so if you tell me that if you give me I x I y and I t I only know that my optical flow value lies on this line where on this line right I am not able to tell right because I only only one equation right now I can only say for sure that it lies on this line but then where it lies on this line that I do not know I mean right it could be here it could be there I do not know where it lies right so kind of there is an ambiguity but then right but then okay things are not so bad so what you can do is so what we can do is you know suppose I take this equation and suppose I scale it by √I x square + I y square so in a sense what I am going to do is okay or else I will just write this as I x by √I x square + I y square and then I y by √I x square + I y square and then I have U V and then I also scale on the right – I t by √I x square + I y square. Now what is this guy? So now so not it I mean I can think of this as some vector dot product U V right earlier also it was that but then but this is now a special vector what is this vector? See this is I x and I y this is not any old vector right this is I x which is the gradient I y which is a spatial intensity gradient and you are kind of right normalizing it by its length so what will this be this will be the unit vector in the in the direction of the brightness gradient right at that location right so this is the unit normal this is the unit normal in the direction of the in the direction of the brightness gradient brightness gradient. And this equation right and this kind of dot product this dot product gives you what does it give you this kind of a dot product what is it kind of giving you? It is giving you the component of the optical flow along the along the brightness gradient direction right that is what it is no so this dot product gives you the gives you the component of the optical flow in the in the event direction of the of the see brightness gradient brightness gradient brightness gradient. But this has a special significance on the right what is that? This component is this right − I t by √that is the component right I am taken the dot product I get a value that is my component which is gives you the component this component is − I t by √I x square + I y square that is what I have on the right that is my component. But what is this with respect to that figure what is it with respect to this figure where does it point where does this lie this is not the this is not the this is not a perpendicular distance from the origin to a point on the line that is how you do know a x + b y + c equal to 0 that is your line then the perpendicular distance from the origin to that line is mod c by √a square + b square that is exactly what this is right.

When you have you have this is the equation of the line right and this one right so this is actually orthogonal this is actually perpendicular. So what this really means is that so right but here is a here is the key point so what it is telling is see if you if you get a right think about the optical flow I do not know where it is right I told you that I do not know where the optical flows it could be anywhere suppose it is here at this location. So I can kind of think about it as having one component right which is this and this is another component which is this it is a summation of these two components right two vectors. So I can call this as p and I can call this as q this is a vector this is a vector. So the summation of these two vectors is my is this is this optical flow vector but what you can clearly see is that this p is always the same see even if I am here right I mean I would I would have I would I can again decompose it as this component and that component.

Do you see this I mean this is very very important right you can see that the bright the component along the brightness gradient you can always find out without ambiguity. The ambiguity comes only along the parallel flow the parallel flow you can never find out with this with this with this one equation you cannot find out the component along the right edge direction. What does it mean right see that is that is the orthogonal component no I have a I have one component going along the brightness gradient I am saying that I am saying that the other component is orthogonal to it right that I am saying I can never find out what does it mean that means that that means that when an object moves along the edge if there is a motion I cannot find it actually this happens in your in your in your human visual system this actually this actually happens I mean this is not a this is not something mathematical alone this is true this this happens and I am going to show you an example where where where I am going to fool you right and you will know that right such a thing. So this is called the aperture problem this this is this this fancy thing is called the aperture problem what why we why do we call it as an aperture problem is because it is like saying that if you see why do we call it an aperture because I am looking at a point that I am trying to compute optical flow at a point right that means you are down to a very very local motion and you are trying to see what is that motion right. So if I sort of if I sort of expand my aperture and if I if I if I just make my aperture bigger and bigger and bigger then I can see a lot more and I can say a lot more I might I may not even have this ambiguity.

The ambiguity is coming because you are looking at optical flow at at a pixel that means you know you are the the window right through which you are watching it is so narrow then and through that window right whatever you are seeing you are trying to you are trying to base your opinion on that and that is why it is called the aperture problem and this is exactly that and the the next slide right will hopefully right right demonstrate that let us see. Now now you tell me ok based on this motion which way which way is the motion I will play again so it looks like it is going this way right but let us see the actual thing this is how it moved the motion was actually horizontal right this is I mean right all of us got got fooled right I mean this is nothing this is exactly the aperture problem because what you are doing I mean see it is like saying now I have a larger aperture right the outer one it is a larger aperture now I can see oh it is right and all but the moment I kept it aperture very small I reduced it to this then you could not make out right this is exactly the point so what this means is that the or the the but the the parallel flow cannot be found at all with this one equation of course there are ways to kind of right get around this but this is a fundamental problem called the aperture problem. Ok so right you saw an example of this ok so right let me just write this down so the right ok so here is here is the most important conclusion right so only only the normal flow ok that that is called that is called the normal flow ok normal flow that is the optical flow in the direction of the of the of the brightness gradient in the direction of the brightness gradient yeah there is a there is even this other illusion right what is called the barber barber pole illusion let me see that if you can show you that see this right this is called this is called the barber pole illusion look at the left right that is a barber pole I mean I do not think our barbers have it but right places there are places where they have it so it is like it is like it is actually it is actually rotating like this the motion field is this but then if you look at the optical flow it looks like it is all going upwards actually actually if you look at this bar right it is only it is only rotating like this so the optical flow the motion field is actually this so if you see the actual motion field it should be this but then the optical flow is exactly orthogonal to that. So so so there are several illusion in fact in fact there is one more illusion let me also show you right while we are on this that is at the end I mean right this will this is watch this do you see something moving but you but you just focus on any one pixel nothing will move there is no change right can you can you can you see that I mean I am trying to see I think I can see that it looks like something is moving but you just focus on one area and nothing is moving anywhere you focus nothing is moving but your but your but your but your brain is tricked into believing that these are called rolling snakes or something and there are several such illusions okay this is actually this is actually called some drift illusion or something so our our so we so that is why I read this this motion business we get fooled many times I mean right it is not like every time even we even we get we are we can be in trouble okay anyway so that is the optical flow in the direction of the of the can be can be exactly exactly exactly determined determined and and also right this is the only component that you can actually can be I mean determined the component along the component along the edge along the along the edge direction what do you call that is parallel flow it is called the parallel flow cannot be cannot be determined based on the single equation okay cannot be determined means right based on that based on one the one equation okay.