Modern Computer Vision

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Lecture-64

Okay, so I said that right let us take an example, so the idea is to kind of get to the fundamental matrix today. So let me kind of say to take an example to just try to illustrate, just a few things which we will actually use down the line. So suppose I have xy on the xy plane, suppose I have two lines imagine, we call this line okay, now this coordinate is let us say 1 and this is 2, this line is l and this line is m, okay then of course as far as l is concerned we know that its equation is like y equal to 1, this is for line l and for line m right we know that its equation is x equal to 2. So in terms of the homogeneous representation right, we can say that l is, you know that it is like l1x + l2y + l3 equal to 0, so you can maybe take it as 0, 1, - 1 and it is anyway valid up to a scale factor, so it does not hurt if you take 0, - 1, 1 also and then m is equal to, so we have got like 1, 0, 2 right, sorry - 1, 0, 2 because x equal to 2 okay. Now if you kind of look at the point of intersection right, that is some P, that is your point of intersection then P is l cross Q, sorry l cross m, the lines, the two lines l and m and if I represent l as l1, l2, l3 which is how we did last time and m as m1, m2, m3 and you all know that this cross product right between these two will take the standard form l2, m3 - l3, m2, then l3, m1 - l1, m3, l1, m2 - l2, m1. So in this case right you get this as, so what is l2 by the way, l2 is 1, so 1 * m3 is 2 - 1 * 0, then - 1 * - 1 - 0 * 2, then 0 * 0 - 1 * - 1.

So this is what is it 2, 1, 1, 2, 1, 1 right and if you have taken like 0 - 1, 1 maybe you would have got like - 2 - 1 - 1 and then if you scale by the last value right what you get should be the point of intersection okay that is your, that is this guy right which is 2, 1 okay. Now we will actually need these things okay down the line and you can also verify that other points right lie on that line and all that right. Okay one of the things that we do is you know there is, so we do what is skew symmetric matrix representation. So for example, if you wanted to take you know cross product between okay this is cross okay, this is not X, this is a cross product okay let me put this as Y right.

Then we write the same thing right that we wrote here, we can also write in terms of a matrix vector multiplication right and that is the standard form. So let us say this is V1, V2, V3 and we are looking at Y as Y1, Y2, Y3 then this you know this okay we have got a special kind of a representation for this we call this as V and then cross not X okay this is called cross you would have seen this right earlier, you guys seen this so V cross right that simply a skew symmetric matrix which looks like 0, - 3 V2, V3, 0, - V1 and - V2, V1, 0. I hope this is skew symmetric right yeah fine. So this is a skew symmetric matrix and you know it is just that you know this actually gives you a way to kind of say when we go further right we will use this to say represent a cross product as a kind of a matrix multiplying. So for example, if you multiply this with Y1, Y2, Y3 right so that will give you like - V3, Y1 sorry what happened no - V3, Y2 + V2, Y3 and that is the same as what I wrote before right and then you will have like V3, Y1 - Y3, V1 and then the last one will be like - V2, Y1 + V1, Y2 right that is what we had no is that correct this is okay right okay.

Now so the idea is that right you want you have a cross product okay then you can actually express it as a kind of right multiplication with the skew symmetric matrix and yeah and let us now write I mean and of course you know and you can show that right and it is easy to see that V cross I mean V is actually you know V will be in the V will be you know a null vector of this matrix right because I mean V cross V right dot V right I mean if you do that that will be 0. So V is a null vector of this matrix and therefore it has a rank 2. Yeah let me let me come back to the come back to that matrix business right to the fundamental matrix because this is all that we need in terms of what we want to use okay. Let us come back to the F matrix right in stereo. So let us say that the left camera right we have P is equal to k I0 and right matrix right camera P dash is equal to let us say k dash I do not know where we using R and something okay yeah okay well let us use that or whatever it does not matter.

Let us say PL then KL and we use PR and this will be like kR and then RT okay. Now let X ~ so it is like this right so we want again again right go back to this arrangement where we have a 3D point right which is seen by 2 camera centers right C and C dash and see right as opposed to the homography right when you had a homography matrix what was it trying to relate it was trying to relate 2 points right one point in one image with the other point at another point in the other image. Whereas if there is a fundamental matrix even though right it may also look like you know it is governing 2 points but actually it is mapping it is establishing a relation between a point in one image and it is a epipolar line in the other image okay. So in that sense the two are going to see different homographies between points whereas a fundamental matrix establishes a relation between point in one image with its epipolar line in this other image okay. So let us say that X ~ right is this image coordinated where this guy now intersects and X ~ as we know is P and then PL right in this case and then we have let us say a 3D point everything in everything is homogeneous coordinates and then we will say that a corresponding point right for this guy is let us say PR acting on X ~ the same guy okay.

Now there are a few things right that we have to do as you see manipulation so let me write this as K left okay. So K left and then we have I0 at X ~ and this we will represent as KL and then XYZ right so X ~ of course right we so the X ~ is like XYZ 1. So I0 when you multiply with X ~ you will get like KL * XYZ okay so this is like your X ~ or XYZ right we can write as KL inverse right because as I said right this is a this is an upper triangular matrix and its diagonal entries are nonzero so you can always invert it so KL inverse X ~. And if you divide both sides by Z and if you do like 1 by Z right XYZ then you get like 1 by Z KL inverse X ~ okay or K inverse X ~ yeah okay so this is equal to KL inverse right X ~ so you can actually think of this as you know XY1 in the sense that X ~ has let us say something right in it which is like you can think about it as X ~ Y ~ some Z some Z right which we are trying to divide and this is like the actual right image coordinates. Well yeah there is some abuse of notation along the way so I am just trying to be careful otherwise you know we can easily slip up here so KL inverse right XY so all that I am doing is I am scaling this guy right X ~ and you know that the last coordinate of actually X ~ is in fact Z KL has 0 0 1 right in its last row so you know that the last coordinate of X ~ is actually Z therefore if you scale by that Z then you will get the actual image coordinate right in the first reference the first camera plane so that is why I am writing X and Y as the actual image coordinates X ~ is like this homogenous form this is like any multiplication any factor of X any like alpha times XY1 but specifically when you when you divide by in this case right we are specifically dividing by Z and that is for a reason okay.

So what this means is that right let me just follow it up here so what I can then do is write XYZ right is equal to I will write this as Z okay there is a small abuse of abuse of notation okay this most people do so I am also doing it here but then right but then I have told you clearly what it is so what they do is typically this one is written okay now till this point right this is okay we can write this as XY1 I am just transferring Z on to the other side okay but then you actually define something like X of Z okay some quantity I mean I will tell you right what is X of Z is okay and this is like Z kl inverse I am going to write this as okay right here is where the sort of slight abuse of notation comes okay X ~ and then 1 okay but this is this one okay this is not the okay this is actual image coordinate okay. So always remember because you know see earlier we had used X ~ where there was actually the last coordinate was actually containing the depth we scaled it right and then see people just use it I mean right in order to get a say represent that you know it is actually the image coordinate but then they still indicate it as X ~ but just remember that this X ~ is XY1 okay now yeah so I think till this point it is okay. X ~ yeah X ~ is actually a homogenous coordinate but yeah I mean see the idea is that no homogenous does not mean that the last coordinate should be 1 homogenous simply means that any scalar multiple of that vector is all the same point only okay and then if you want the image coordinate then you should scale by the last one I mean if you see it is like saying that I might have alpha X alpha Y alpha for all alpha it is called a homogenous coordinate because right I mean they are all they are all simply related to if you scale they all map to XY1 but I am saying this XY1 right I am saying it is actually the XY is actually the image coordinate now because you have scaled it already. It is more specific in the sense that this is referring to the actual image coordinate itself yeah so this step right yeah so I just wanted to spend some time on this okay so where X ~ is to be interpreted as XY1 right origin 0, 0 is here yeah that that and all I have not changed at all I mean we are we are following the exact thing so 0, 0 so this is the origin and C dash right you have to get to that I mean so these are the other reference is rotated and whatever translated right okay alright. So then okay let me go to the next page okay so think of X okay so you can actually think of XZ which is actually is actually XYZ1 right why that is because okay right that is coming from here that is oops that is coming from the previous one for example if you kind of read look at this so if you have like Z and then you have like KL inverse and if you kind of go back to where I mean right if you go back to this one right I mean XYZ so this XY so what do you have here right XYZ ~ see here right so in this equation where this is still this is not scaled right as yet okay in this place right it is still not scaled and at the end and right and then you know in order to get the scaled version right you will have to do like KL by Z that is why I said it is a little confusing okay but it is not so bad so it is like XYZ you get to see that right I mean that is why okay this this requires a little amount of careful attention right see this I am writing this XZ right is but actually XYZ1 okay but then why is it like a XYZ1 is because of course 1 is that probably right just follows from here right that is XYZ is in fact this but then you can also show that you know X ~ written in this form is actually XY1 that is why I keep saying there is XY1 and therefore right here this is not XY1 this is still the X ~ with that Z sitting right at the bottom therefore if you push this cable to the other side you will get KL XYZ but then the last coordinate of X ~ is still Z and therefore if you divide it by Z right that is when you will get XY1 if you divide it by Z then the Z and Z will cancel KL KL inverse is 1 and identity and then you get XYZ okay.

So this so this XYXZ right that is a it is of course and it also follows from what we have written at the top but I just wanted to let you know that this interpretation that XY1 right is is critical otherwise you will have an extra Z sitting there right and then your cancellations would not be right this is okay all that I am saying is this X ~ and that X ~ right let us be careful about it that is all because in the slide it is not when if you look at the slide I think you will go wrong okay. Then which is actually okay XYZ1 okay you can think of which is actually XYZ1 as the as you see back projected ray as the back projected ray from camera center center through right I mean here is here is why I feel that this XY1 is very very important through XY1 because XY1 is on the image plane right. So it is like saying that it is like saying that that right you have the camera center right you have the image plane this is the back projected ray and this coordinate is actually X, Y that is the actual image coordinate and this projected back projected ray that is why that X ~ that is sitting there has that form XY1. So it is like saying that you have a back projected ray which you can take from the center of the camera push it through XY1 and then and then you have the back projected ray and then how do you show that it is actually the back projected ray then what you should do is you know any any the P the P left right the P left if you act on X of Z you should be able to get this XY1 because any point on that ray right because it can have any Z it had like Z no see this this one right I mean this is a form that they use to sort of convey the fact that this is a back projected ray. So see the Z sitting there right this is an arbitrary Z right.

So what it means is that any Z that you pick on that ray if this if the if the camera matrix P L acts on that it should actually exactly project XY1 that effectively means that you can have a back projected ray going wherever it wants at whatever Z it be, but then if I apply my P on that it will always come to come to XY1 which is on the image XY which is on the image. No XY you should get on the image that is why that is why. So, I am saying that let us be clear about this you see it is like I mean see it is like I have a camera center and then I have the image plane and I am kind of doing I am doing at this one right back back some projection, but now this back projection I am doing from the camera from the image plane now I have a coordinate there no I have a point on the image plane that is XY that is not some homogenous coordinate and all that is my actual image coordinate. So, we are going to say looking at looking at a ray that is going from C and then and then hitting XY1 and going right. Now, if I take this ray which is a back projected ray and if I apply my P L whatever be the Z corresponding to this it should come back to this X , Y.

Ok, then Z column is always be 0. No Z coordinate no , I am saying the image coordinate should be X , Y let us not talk about Z for any back projected for any point on this back projected ray if P left acts on it, it should it should map to image coordinate X , Y image coordinate is this clear. It is not it is not so difficult, but just I am just saying that you should understand why I am why I keep insisting about this XY1 and why actually they have it there, but then they do not actually explicitly tell it right. So, it is like it is like you have a back projected ray that is going and the Z can be whatever it be, but then the moment I apply P on it, it will all come back and hit me hit the image plane at X , Y ok. So, so, so, so, right.

So, in that sense this where did it go. So, so, so, in fact, in fact, so, so, in fact, that is what I wanted to write this. In fact, X Z is expressed in terms of in terms of XY1 precisely for this reason I mean it is not explicitly said precisely for this reason precisely for this reason. You see that you see that X ~ sitting there as XY1 that is precisely sitting for this reason that that you have this back projected ray and act PL on it every point there irrespective of its Z coordinate will come and sit at X , Y ok. So, you can show this right.

So, what you can do is you can take PL then and then it you can act it on this X Z and PL of course, was just identity and then 0 act it on X Z which is actually what is that I mean so, X Z yeah K Z Z K inverse X ~ right and then 1 right and then this will mean that this will mean that right you get * say X Z right * X Z wait a minute no no * X Z is already here right. So, I one minute K is there I forgot K. So, K is here right and so, what happens. So, right so, then this is like K Z K inverse X ~ right this is like Z X ~ ok and and and this and this X ~ is XY1 that is why I am saying this is not XYZ this is XY1 right and therefore, and therefore, it if you actually scale this by Z right you will you will end up with end up with XY1 right irrespective of the irrespective of the Z for any Z. So, the Z right can be basically anything when you say X of Z whatever whatever be that Z right that that that Z right moment you scale.

So, I mean so, so you have the Z X ~ right X ~ is XY1. So, what you are getting is ZX ZY Z you scale it by Z you will get XY1. So, irrespective of whichever Z it be right irrespective of whatever be the Z the final coordinate that you get that you will get on the image plane will always be XY1. Is this thing clear? Ok. If this is clear then I then then right then we can go ahead ok.

Now, so, so any ok right let me just make it even more clear any 3D point on the on the back projected ray on the back projected ray maps to the same image coordinate X , Y X , Y ok. Now, now right I mean earlier at what did we say right when we said that right we will try to get to the fundamental matrix right. One of the things that we said was we will actually take what did we say we said that we said that right we will actually take take the 2 points exactly right. So, we said that so we said that right we will take this back projected ray we will take under 2 points on this ray right and then and then and then right you have your second camera maybe right somewhere here ok. You have a camera second camera here with its image plane ok whatever it is optical axis and then and then we said that we said that right we will see ok where it comes ok.

And then and then the point was if you can actually get the get this point of intersection which we can get through P right. We know we know we can find out right where this point maps here where this point maps here and then if you if you know these 2 points then we know the what should be the epipolar line the cross product should give us epipolar line ok. That is why that is the that is the motivation for doing right what we did before. So, now, the idea is to be able to take is it 2 point 2 points on this on this on this is the back projected ray because the because epipolar line right is basically coming from that line right epipolar line in the right epipolar line is coming from this ray right. And all that we need are actually 2 points right on that ray if you can actually project them to P R and then if you take the take this cross product of those of those of those 2 points right then you will actually get the line ok that is the idea ok.

Now, to now to in order to get right 2 points. So, we need to select 2 points on right on the A this is always.