Modern Computer Vision

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Lecture-63

But then what is this? What is its algebraic, algebraic is a representation right, is what is what is what is important and this leads to what is called a fundamental matrix. I mean you must have heard about it somewhere in some context or something which is a subset of this which is called the essential matrix okay. Both these things are very common okay, fundamental matrix so whenever somebody talks about a stereo or and and anyway I read one more thing let me just tell you that the the the one that is like multi view case right where you have where you have what we call a structure from motion right, all these things simply carry on there okay. So most of the work that we do here we will simply utilize it there okay, there will there will be certain subtle changes there with respect to what we are dealing with but but otherwise the reason for doing stereo first is because all the things that we do here simply carry over there. Because finally everything is about what you take a camera you move, finally it is like one point and then it will move to another point what kind of R and T do you have, what is the relation and how do you relate the image points right, again comes back again and again to the same thing okay. So this fundamental matrix and this essential matrix.

Now now to now to in order to understand right the the way okay we want to do it will be will be as follows and it makes sense right, the the the idea it is as follows, for a point in the left image I am starting from the left okay, you can also repeat the same if you want you can do it the other way, back project back project array that is from the center to the point right that that gives you the gives you the back projected right. So it is like this, so we have so we are here right, so we are here and and we have a point right and we have the center C and this is this is the this is the back projected array going through some X, for a for a for a point right for a point X in the left image back project array to choose 2 points on the ray choose 2 points in the ray and project them project them on to the right image right image plane yeah that is a right camera right image plane or right camera okay I will write right camera, camera plane, camera image plane okay. So what this means is that it select 2 points okay and we will see I mean which are the points that are most convenient to select ideally anything should work but there are certain things that are more easy to deal with and what this means is that I have this other thing so this guy maybe my C ‘ is here and okay and what we are saying is these 2 points right we want to get a C project them to to C ‘ okay. So what this means is that right between these 2 points there is supposed to be a line now right because that will be the that will be the say epipolar line right.

So 3 compute the line through the 2 image points but that is what we want right finally we want to know the know what is that epipolar line how do we get there compute the people compute the line through the 2 image points okay this gives you the epipolar line in the right image or what is called the right epipolar line and use the fact that this is the fact that any point on this line should obey the equation of that line right. So in a sense right what we are saying is we will we will start from a point and we want to kind of find out where is that line and under the same time right we want to be able to also what happens is this fundamental matrix right needs to be solved for okay it has to be it has to be courtesy it has to be found out this is to be courtesy computed this is not automatically coming but we want to know under what constraints right what is the constraint that we can use in order to arrive at the fundamental matrix because once you know the fundamental matrix then you know for example right given any point on the on the left image the fundamental matrix will tell you where where it goes okay to which line it goes on this right that is why it is called a it is called a fundamental matrix okay and but in order to in order to do that we have to be able to first compute that because we do not have it and this whole procedure is in fact aimed at that I mean it is not not just computing the you know epipolar line but in but then eventually leading to leading to what is what is what is called what is called a relation involving a fundamental matrix a point on the left guy and a point on the point at the right image plane. So fundamental matrix in a way I see ties up both something on the left with something on the right which is supposed to be a corresponding point okay not something arbitrary which is supposed to be a corresponding point these two how they kind of tie up and then the geometry is embedded in F okay so this F kind of elegantly captures all that geometry. Sir, it let us see left image point to the right image point and right to be called a line. So I will tell you where is that where is this epipolar line that is why I said that eventually it all boils down to search okay.

You have to search on the line. You have to search on that line if you want to reduce a search to one dimensional search the epipolar geometry is just that so only to know that when you do this it is not like things are going you know arbitrarily wherever they feel like they cannot go like that right even if you have an arbitrary 3D scene right even if an arbitrary 3D scene and also that you should also note that all through the scene never influenced any of this any of this epipolar geometry. It is independent of the geometry right this whole epipolar geometry is not is not dependent on the scene at all it only depends upon what you call it depends upon the camera intrinsic right and then it depends upon the the camera extrinsic that depends on the pose of the camera okay that that is that is important right how the two are mutually related and what their what their you what their own intrinsic are it is not dependent on how the scene is okay the scene can be whatever it is does not matter okay. Now alright so so let us now let us start with what are called the homogeneous representation of representation of lines and points in the in the 2D plane because finally our search is all in actually a 2D space right so the line is in 2D and points in this is a 2D plane or 2D space. Now we know that right a point I mean homogeneous representation for a point right we know that I can write this as x tilde and then I can write this as a vector x y 1 for a line right I mean a representation goes like this you can write this as L and then we write this as L1 L2 L3 okay what this means is that this kind of homogeneous representation means that L1 x so a point to lie on this line will effectively mean that L2 y + L3 = 0.

This you can write also in the form of y = mx + c and the only thing is instead of the ratios being between first and last this will be between between what first and second and this is third and second I mean you can write this now y = - L2 y = - L1 x - L3 and then you will get L2 by what is it L1 L1 by L3 and then L1 by L2 and then L3 by L2 right so it mnc can be actually related to this but then this is more clean I mean you know because we want to stick to homogeneous representation this is more ideal for us to work with okay and this also you know effectively means that L transpose x it should be equal to 0 right if x is a point on the line then a . product right I mean you know . product of that of that of that point right with respect to the line should be 0. And also and also note that right any any alpha right if you multiply this with any alpha right it is also okay it is also the same line by the way right you can actually multiply it by by because you know just has to satisfy this = 0 therefore multiplying bit with by any alpha right does not really matter that is why we call this a homogeneous representation. Then or the other way right x transpose L or you can just simply write this x . L or whatever x transpose L = 0 either way right both should go through and then a line through the points let us say P and Q all this if you can see that we need this know this is the whole idea right because we wanted to take 2 points in the line right I mean you know drop them on the other plane and then look at the line through that plane at a line through the points P and Q is given by L = P X Q this is a X product this and all comes from your whatever previous math days right. And then okay and and this and this follows because of the fact that right what is what is the proof and why is this correct because if you take if you take a right L . P right that will be equal to P X Q . P which we know is 0 and similarly if you do L . I mean L . means I mean a . product right and then if you do L . Q that is again P X Q . Q and that is again orthogonal right so that is also 0 right and therefore therefore right L is the line that goes through actually P and Q. And the intersection I will I will actually take an example okay I will take an example to just set this up and once we set it up then we can move kind of see quickly to the fundamental matrix and so on the intersection of 2 lines L and M is the is the point how is that X equal to L X M we will show this I mean if you are wondering as to why is this true and so on and I will take an example and show L X M only thing is only thing is right the I mean third coordinate you should scale of X okay if it is not 1 we should scale it okay I mean scale the third coordinate scale by the third coordinate not scale the third scale by the third coordinate if not 1 okay.

Now having said this I think maybe right we will stop here because I have to start an example.